

PHYSICS

Unit – 8. Electromagnetic Waves

Class – 12th

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8. Electromagnetic Waves

Introduction -

Oersted experimentally predicted that the electric current produces a magnetic field. Faraday from his experiment on electromagnetic induction found that a changing magnetic field produces an electric field.

At this time the clerk Maxwell state that the magnetic field is not only produced by electric current but also by a time varying electric field.

Maxwell formulated a set of equations involving electric and magnetic fields and their sources (the charge and current). These equations are known as Maxwell's equations. These equations along with Lorentz force formula, explain mathematically all the basic laws of electromagnetism.

Maxwell through his equations come out with a most important prediction i.e. the existence of electromagnetic waves, which can propagate in space with a speed very close to the speed of light (3×10^8 m/s). By this the Maxwell conclude that light is a electromagnetic wave.

Maxwell's Equations and Lorentz Force -

(1)

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

This equation is Gauss's Law in electrostatics. This gives the value of total electric flux in terms of the charge enclosed by the surface S.

(2)

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

This equation is Gauss's Law in magnetostatics. This law show that the number of magnetic field lines entering a closed surface is equal to number of magnetic field lines leaving it. It confirms the absence of magnetic monopole.

(3)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S}$$

This equation is Faraday's Law of electromagnetic induction.

This law gives a relation between electric field and a changing magnetic flux. It shows that the line integral of electric field around any closed path (i.e. emf) is equal to the time rate of change of magnetic flux through the surface bounded by the closed path.

(4)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 I_D$$

$$I_D = \epsilon_0 \int_S \frac{d\vec{E}}{dt} \cdot d\vec{S}$$

This equation is generalised form of Ampere's Circuital Law, modified by Maxwell and is known as Ampere'- Maxwell law.

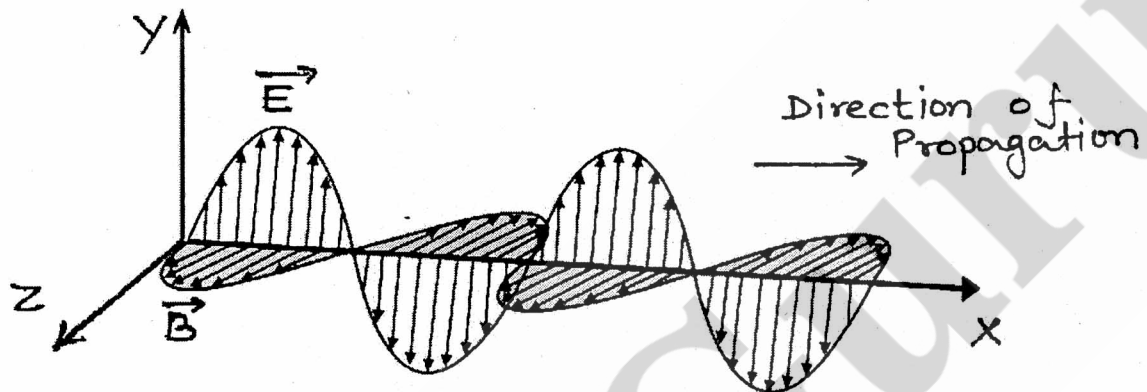
This law shows that the line integral of the magnetic field around any closed path is related to the conduction current (I) and displacement current (I_D) through the path.

(5) Lorentz Formula

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Electromagnetic Waves -

According to Maxwell, the electromagnetic waves are those waves in which there are sinusoidal variation of electric and magnetic field vectors at right angles to each other as well as at right angles to the direction of wave propagation.



Both these fields ($\vec{E}$ and $\vec{B}$) vary with time and space and have the same frequency.

The electromagnetic waves can propagate through the space with the speed of light (3×10^8 m/s).

Maxwell also found that the electromagnetic waves should travel in free space (or vacuum) with a speed given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

where μ_0 and ϵ_0 are permeability and permittivity of the free space respectively.

$$\mu_0 = 4\pi \times 10^{-7} \text{ Wb/Am}, \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

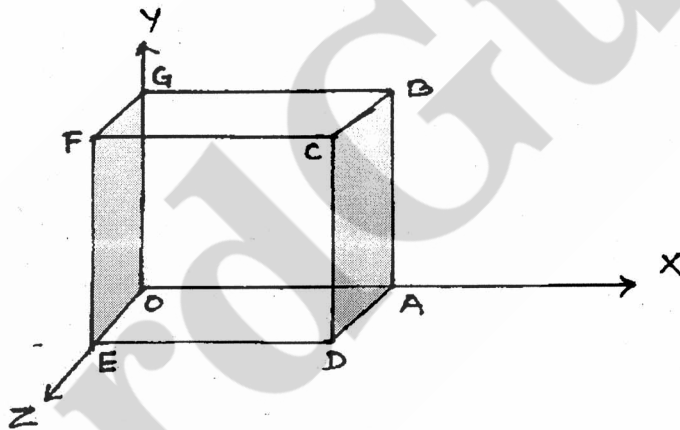
$$\text{The velocity of EM waves in medium} = \frac{1}{\sqrt{\mu \epsilon}}$$

where μ and ϵ are the absolute permeability and permittivity of the medium.

Transverse nature of Electromagnetic Waves -

Consider a plane electromagnetic wave travelling in the x -direction and a rectangular parallelepiped $OABCDEFG$ placed with its edges parallel to the three axes. The wave front of electromagnetic wave is in the y - z plane and $ABCD$ is a portion of it at time t .

The value of electric and magnetic fields will be zero to the right of the face $ABCD$, but to the left of this face the values of the two fields depend on x and t but not on y and z , since we are considering a plane wave.



To establish transverse nature of electromagnetic waves we will establish that $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of wave propagation.

If the rectangular parallelepiped does not enclose any charge, the total electric flux across it must be zero, according to Gauss's Law -

$$\oint \vec{E} \cdot d\vec{s} = 0$$

$$\text{or } \left(\int_{ABCD} \vec{E} \cdot d\vec{s} + \int_{OAGE} \vec{E} \cdot d\vec{s} \right) + \left(\int_{BGFC} \vec{E} \cdot d\vec{s} + \int_{ODEG} \vec{E} \cdot d\vec{s} \right) + \left(\int_{DCFE} \vec{E} \cdot d\vec{s} + \int_{OACF} \vec{E} \cdot d\vec{s} \right) = 0$$

Since $\vec{E}$ does not depend on y and z , therefore the contribution to the total electric flux coming from the faces normal to y and z will cancel out -

$$\text{Hence } \left(\int_{BGFC} \vec{E} \cdot d\vec{s} + \int_{ODEG} \vec{E} \cdot d\vec{s} \right) = 0$$

$$\text{and } \left(\int_{DCFE} \vec{E} \cdot d\vec{s} + \int_{OABG} \vec{E} \cdot d\vec{s} \right) = 0$$

Hence we have -

$$\int_{ABCD} \vec{E} \cdot d\vec{s} + \int_{OGFE} \vec{E} \cdot d\vec{s} = 0 \quad \text{--- (1)}$$

If E_x and E'_x are the values of x-component of electric field on faces ABCD and OGFE respectively and S be the area of each of these face, then -

$$\int_{ABCD} \vec{E} \cdot d\vec{s} = E_x S \quad \text{and} \quad \int_{OGFE} \vec{E} \cdot d\vec{s} = -E'_x S$$

The negative sign arises due to the fact that the area vector at the face OGFE is in opposite direction to that of face ABCD.

Hence, from equation (1) we have -

$$E_x S - E'_x S = 0 \quad \text{or} \quad (E_x - E'_x) S = 0$$

This relation is true only if either $E_x - E'_x = 0$

$$\text{i.e. } E_x = E'_x \quad \text{or} \quad E_x = E'_x = 0$$

Since $S \neq 0$, the possibility, $E_x = E'_x$ predicts that the field is static. But a static field can not propagate a wave of finite wavelength.

Therefore $E_x = E'_x = 0$ i.e. no component of electric field is parallel to the direction of wave propagation.

It means, the electric field is perpendicular to the direction of propagation of electromagnetic wave.

A similar argument holds true for the magnetic field. We thus conclude that the electric and magnetic fields are transverse, in the propagation of electromagnetic waves.

Production of Electromagnetic Waves -

An electric charge at rest has electric field in the region around it, but no magnetic field. A moving charge produces both the electric and magnetic fields.

If a charge is moving with a constant velocity (current is not changing with time), the electric and magnetic fields will not change with time, hence no electromagnetic waves can be produced.

But if the charge is moving with a non-zero acceleration, both the magnetic field and electric field will change with space and time, it then produces electromagnetic waves. Hence an accelerated charge emits electromagnetic waves.

The L-C circuit is an oscillatory circuit where the charge is oscillating across the capacitor plates. An oscillating charge in L-C circuit has a non-zero acceleration hence it emits electromagnetic waves, which have the same frequency as that of the oscillating charge.

In an atom an electron while orbiting around the nucleus in a stable orbit, does not emit electromagnetic waves. Electromagnetic waves are emitted only when it falls from higher energy orbit to lower energy orbit.

Electromagnetic waves are also produced when fast moving electrons are suddenly stopped on the metal target of higher atomic number.

Properties of Electromagnetic Waves -

- (1) The electromagnetic waves are produced by the accelerated or oscillating charge.
- (2) The EM waves do not require any material medium for propagation.
- (3) These waves travel in free space with a speed 3×10^8 m/s (speed of light) and is given by the relation -

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

- (4) In electromagnetic waves the sinusoidal variation in both electric and magnetic field vectors ($\vec{E}$ & $\vec{B}$) occurs, simultaneously. As a result they attain the maximum and minimum values at the same place and at the same time.

The amplitudes of the electric and magnetic fields in free space are related by -

$$c = \frac{E_0}{B_0}$$

The equation of electric field and magnetic field varying sinusoidally with x and t can be given as -

$$B_y = B_0 \sin(\omega t + Kx)$$

$$K = \frac{2\pi}{\lambda}$$

and

$$E_z = E_0 \sin(\omega t + Kx)$$

- (5) The direction of variation of electric and magnetic field vectors are perpendicular to each other as well as perpendicular to the direction of propagation of waves. Therefore, electromagnetic waves are transverse in nature like light waves.
- (6) The electromagnetic waves carry energy which is divided equally between electric field and magnetic field vectors.

In vacuum the average electric energy density (U_E) and average magnetic energy density (U_B) due to static electric field E and magnetic field B which do not vary with time are given by -

$$U_E = \frac{1}{2} \epsilon_0 E^2$$

and

$$U_B = \frac{1}{2} \frac{B^2}{\mu_0}$$

- (7) The electric vector is responsible for the optical effect of an electromagnetic wave and is called the light vector.
- (8) The electromagnetic waves being uncharged are not deflected by electric and magnetic fields.
- (9) The velocity of EM wave in dielectric is less than c ($3 \times 10^8 \text{ m/s}$).

Q. The magnetic field in a plane electromagnetic wave is given by $B_y = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ T}$.
 (a) What is the wavelength and frequency of the wave (b) Write an expression for the electric field

sol. (a) Comparing the given equation with the equation of magnetic field varying sinusoidally with x and t as -

$$B_y = B_0 \sin \omega \left(t + \frac{x}{c} \right) = B_0 \sin \frac{2\pi}{T} \left(t + \frac{x}{c} \right)$$

$$B_y = B_0 \sin \left(\frac{2\pi t}{T} + \frac{2\pi x}{T \times c} \right) = B_0 \sin \left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T} \right)$$

we get - $\frac{2\pi}{\lambda} = 0.5 \times 10^3$

or $\lambda = \frac{2\pi}{0.5 \times 10^3} = 1.26 \times 10^{-2} \text{ m}$

and $\frac{2\pi}{T} = 1.5 \times 10^{11} \Rightarrow 2\pi \nu = 1.5 \times 10^{11}$

so that $\nu = \frac{1.5 \times 10^{11}}{2\pi} = 2.39 \times 10^{10} \text{ Hz}$

$$(b) \quad B_0 = 2 \times 10^{-7} \text{ T}$$

$$\text{so that } E_0 = c B_0 = 3 \times 10^8 \times 2 \times 10^{-7} = 60 \text{ V/m}$$

Here the electric field is varying along z-direction

$$\text{Thus - } E_z = E_0 \sin\left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T}\right)$$

$$\text{or } E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ V/m}$$

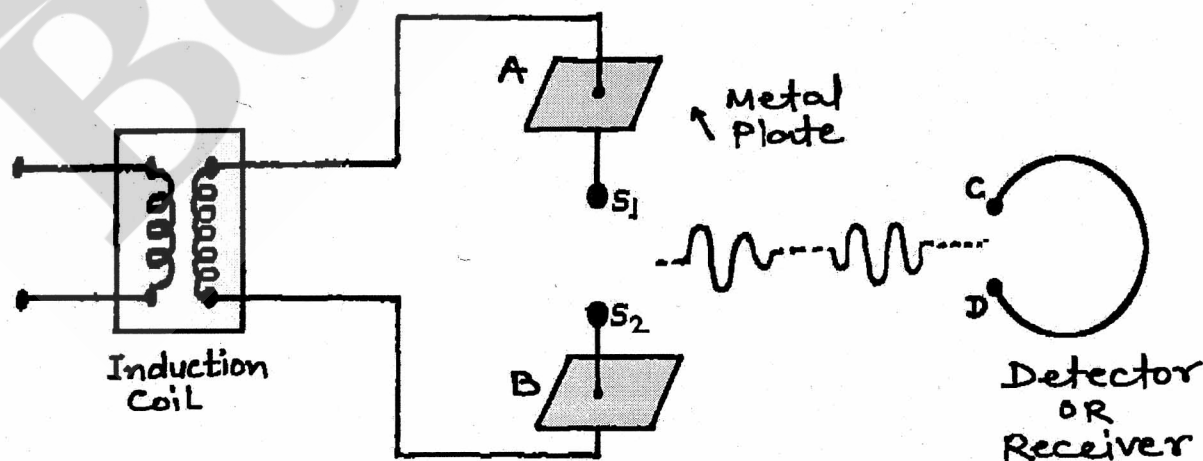
Hertz Experiment -

The existence of EM waves was confirmed experimentally by Hertz in 1888.

Hertz experiment was based on the fact that an oscillating electric charge radiates electromagnetic waves and these waves carry energy which is being supplied at the cost of kinetic energy of the oscillating charge.

Apparatus -

In the experimental setup used by Hertz, A and B are two large square metal plates of Copper or zinc placed about 60 cm. apart. They are connected to two highly polished metallic spheres S_1 and S_2 through thick copper wires.



A high potential difference of several thousand volts is applied across the spheres using induction coil. Due to high potential difference across S_1 and S_2 , the air between the spheres gets ionised and provides a path for discharge of plates. Due to it a spark is produced between S_1 and S_2 and electromagnetic waves of high frequency are radiated.

Here the two plates act as a capacitor having a small capacitance C and the connecting wires provides the low inductance L .

The high frequency of oscillations of charges between the plates is given by -

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

In the given figure the detector is in the form of unclosed metallic ring having small metallic spheres C and D . It is held in a position such that the magnetic field produced by the oscillating current is perpendicular to the plane of ring.

The oscillating magnetic field linked with ring produces large induced emf which causes a spark to appear at the spheres C and D , since the resistance of the metallic ring is very small.

Main Parts of Electromagnetic Spectrum -

(1) Radio Waves -

These are electromagnetic waves of frequency range from 5×10^5 Hz to 10^9 Hz. These waves are produced by oscillating electric circuits having an inductor and capacitor.

Uses -

- (1) The EM waves of frequency range from 530 KHz to 1710 KHz form amplitude modulated (AM) band. It is used in ground wave propagation.
- (2) The EM waves of frequency range 1710 KHz to 54 MHz are used for short wave bands. It is used in sky wave propagation.
- (3) The electromagnetic waves of frequency range 54 MHz to 890 MHz are used in television waves.
- (4) The electromagnetic waves of frequency range 88 MHz to 108 MHz form frequency modulated (FM) band. It is used for commercial FM radio.
- (5) The EM waves of frequency range 300 MHz to 3000 MHz form ultra high frequency (UHF) band. It is used in cellular phone communication.

(2) Microwaves -

These are EM waves of frequency of frequency range 10^9 Hz to 3×10^{11} Hz. They are produced by special vacuum tubes called; Klystrons, magnetrons and Gunn diodes etc.

Uses -

- (1) Microwaves are used in Radar system for air craft navigation.
- (2) Microwave ovens are used for cooking purposes.

(3) A radar using microwave can help in detecting the speed of cricket ball, automobiles, aircrafts in motion.

(3) Infrared Waves -

Infrared waves were discovered by Herschell. These are EM waves of frequency range 3×10^{11} Hz to 4×10^{14} Hz. Infrared waves sometimes are called as heat waves because their absorption causes the heating effect in the bodies and surroundings.

Infrared waves are produced by hot bodies and molecules. These waves are not detected by human eye but snakes can detect them.

Use -

Infrared waves are used

- (1) in physical therapy i.e. to treat muscular strain.
- (2) to provide electrical energy to satellite by using solar cells.
- (3) in green houses to keep the plants warm and for producing dehydrated fruits.
- (4) For taking photographs during the condition of fog, smoke etc.
- (5) in solar water heaters and Cookers.
- (6) in checking the purity of chemicals and in the study of molecular structure by taking infra red absorption spectrum.

(4) Visible Light -

It is the narrow region of EM spectrum, which is detected by the human eye. Its frequency ranges from 4×10^{14} Hz to 8×10^{14} Hz. It is produced due to atomic excitation.

The visible light emitted or reflected from objects around us provides the information about the world surrounding us.

(5) Ultraviolet Rays -

The ultraviolet rays were discovered by Ritter. The frequency range of UV rays is 8×10^{14} Hz to 3×10^{16} Hz. The ultraviolet rays are produced by sun, special lamps and very hot bodies.

Most of the ultraviolet rays coming from sun are absorbed by the ozone layer in the earth's atmosphere. The ultraviolet rays in large quantity produce harmful effect on human eyes.

Uses -

Ultraviolet rays are used

- (1) for checking the mineral samples through the property of ultraviolet rays causing fluorescence.
- (2) in the study of molecular structure and arrangement of electrons in the external shell through ultraviolet absorption spectra.
- (3) to destroy the bacteria and for sterilizing the surgical instruments.
- (4) in the detection of forged documents, finger prints in forensic laboratory and in burglar alarm.

(6) X-rays -

The X-rays are the EM waves of frequency range from 3×10^{16} Hz to 3×10^{18} Hz. These are produced when high energy electrons are stopped suddenly on a metal of high atomic number. X-rays have high penetrating power.

Uses -

X-rays are used

- (1) In surgery for the detection of fractures, foreign bodies like bullets, diseased organs and stones in the human body.

- (2) in Engineering for detecting faults, cracks and holes in final metal products.
- (3) in Radio therapy to cure untracable skin diseases and malignant growths.
- (4) in detective departments for detection of gold, silver and explosives in the body of smugglers.
- (5) in industry for the detection of defects in rubber tyres, gold and tennis balls etc.
- (6) In scientific Research for the investigation of the crystal structure, arrangement of atoms, molecules in the complex substances.

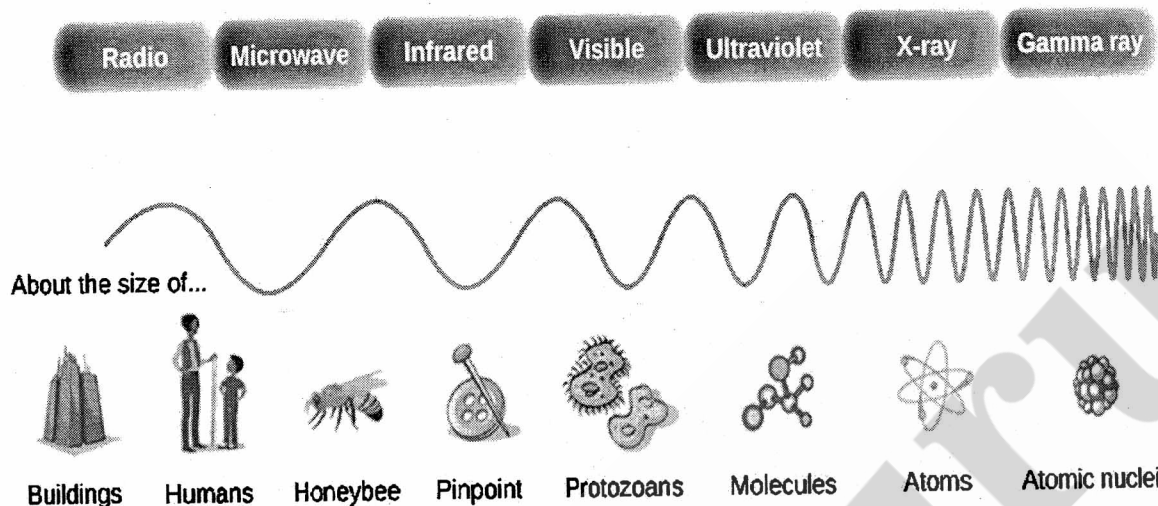
(7) γ -rays -

These are the EM waves of frequency range 3×10^{18} Hz to 3×10^{22} Hz. γ -rays have nuclear origin. These rays are highly energetic and are produced by the nucleus of the radioactive substances.

Uses -

γ -rays are used

- (1) in the treatment of cancer and tumours.
- (2) to preserve the food stuff for a long time as the soft γ -rays can kill microorganisms easily.
- (3) to produce nuclear reactions.
- (4) to provide valuable information about the structure of atomic nucleus.

Wavelength
(meters)Q.
CBSE
2011

A parallel plate capacitor is being charged by a time varying current. Explain briefly how Ampere's circuital law is generalised to incorporate the effect due to the displacement current.

Sol. While dealing with the charging of parallel plate capacitor with varying current, it was found that Ampere's circuital law is not logically consistent because $\oint_C \vec{B} \cdot d\vec{l}$ has not same value on the two sides of a plate of charged capacitor.

The inconsistency of Ampere's circuital law was removed by Maxwell by predicting the presence of displacement current in the region between the plates of capacitor, when the charge on capacitor is changing with time. Maxwell also predicted that the sum of conduction current and displacement current has the property of continuity.

The generalised form of Ampere's circuital law modified by Maxwell states that

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I + I_d) = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

Q.
CBSE
2012

Name the Electromagnetic waves which
(a) maintain the earth's warmth and
(b) are used in aircraft navigation.

Sol. (a) Infrared rays maintain the earth's warmth
(b) Microwaves are used in aircraft navigation due to their short wavelength.

Q.
CBSE
2009

Why is small ozone layer on top of the stratosphere crucial for human survival?

Sol. Because ozone layer traps most of ultraviolet rays coming from the sun and prevents harmful impact of Ultraviolet rays.

Q.
CBSE
2008

In an EM wave the oscillating electric field having a frequency of 3×10^{10} Hz and an amplitude of 30 V/m propagates in the positive x-direction.

- (a) What is the wavelength of EM wave?
(b) Write down the expression to represent the corresponding magnetic field.

Sol. Given that - $\nu = 3 \times 10^{10}$ Hz, $E_0 = 30$ V/m

$$(a) \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3 \times 10^{10}} = 10^{-2} \text{ m.}$$

$$(b) \because c = \frac{E_0}{B_0} \Rightarrow B_0 = \frac{E_0}{c}$$

$$B_0 = \frac{30}{3 \times 10^8} = 1 \times 10^{-7} \text{ T}$$

Hence Magnetic field equation can be given as-

$$B = (10^{-7}) \sin \left(\frac{2\pi x}{10^{-2}} + 2\pi \times 3 \times 10^{10} t \right)$$

$$B = 10^{-7} \sin (200\pi x + 6\pi \times 10^{10} t)$$

Very Important Questions

1 Mark Questions -

Q.1. What are the direction of electric & magnetic field vectors relative to each other and relative to the direction of propagation of electromagnetic waves ?

(CBSE 2012)

Q.2. A plane electromagnetic wave travels in vacuum along z direction. What can you say about the direction of electric and magnetic field vectors ?

(Delhi 2011)

Q.3. Write two uses of microwaves and x-rays.

(RBSE 2011)

Q.4. Write two uses of infrared rays.

(CBSE 2011)

Q.5. What is the frequency of electromagnetic waves produced by oscillating charge?

(Delhi 2005)

Answer - Radio Waves = 10^4 Hz to 10^8 Hz

Q.6. How are Infrared waves produced ? What is the range of their wavelength ?

(Delhi 2010)

Q.7. Which of the following has the shortest wavelength ? Microwave, ultraviolet rays, x-rays.

(CBSE 2010)

Q.8. Arrange the following in decreasing order of wavelength - x-rays, radio waves, blue light, infrared light.

(RBSE 2010, 04)

Q.9. Which part of electromagnetic spectrum has largest penetrating power ?

(Delhi 2006)

Q.10. Which Part of electromagnetic spectrum is used in radar systems ?

(RBSE 2007)

- Q.11. Which part of electromagnetic spectrum is used in radar systems ?
(RBSE 2008)
- Q.12. Which part of electromagnetic spectrum is absorbed from sunlight by ozone layer ?
(CBSE 2002)
- Q.13. Name the electromagnetic radiation used to destroy cancer cells and write its frequency range.
(CBSE 2010)
- Q.14. Express the velocity of propagation of an electromagnetic waves in terms of the peak value of the electric and magnetic fields.
(RBSE 2010, 08)
- Q.15. Name the electromagnetic waves used to study crystal structure of solids. What is its frequency range ?
(CBSE 2005)
- Q.16. Name the electromagnetic radiation which can be produced by a klystron or a magnetron valve.
- Q.17. State the reason why microwaves are best suited for long distance transmission of signals.
(CBSE 2008)
- Q.18. Name the electromagnetic radiation to which the following wavelengths belong (a) 10^{-2} (b) $1 \text{ \AA}$.
(Delhi 2006)

2 Marks Questions -

Q.1. Draw a sketch of a plane electromagnetic wave propagating along the z-direction. Depict clearly the directions of electric and magnetic fields varying sinusoidally with z.

(CBSE 2011)

Q.2. How are x rays, infrared rays, microwaves produced? Write their two important uses.

(RBSE 2010)

Q.3. How does a charge q oscillating at certain frequency produce electromagnetic waves?

(RBSE 2002,09)

Q.4. Name the electromagnetic radiations having the wavelength range from 1 nm to 700 nm. Give its two important applications.

(CBSE 2006,09)

Q.5. Identify the different types of electromagnetic radiations which are used -
(a) to kill germs (b) for physical therapy.

(RBSE 2005)

Q.6. The following table gives the wavelength range of some constituents of this electromagnetic spectrum

S.N.	Wavelength Range
1.	1 nm to 700 nm
2.	400 nm to 1 nm
3.	1 nm to 10^{-3} nm
4.	$< 10^{-3}$ nm

select the wavelength range and name of the electromagnetic waves that are -

(a) widely used in the remote switches and household electronic devices.

(b) Produced in nuclear reactions.

(Delhi 2008)

3 Marks Questions-

Q.1. What is meant by transverse nature of electromagnetic waves? Draw a diagram showing the propagation of an electromagnetic wave along the x-direction, indicating clearly the direction of the oscillating electric and magnetic fields associated with it.

(RBSE 2007, CBSE 2008, 10)

Q.2. What do you understand by electromagnetic waves? Give its four properties.

(RBSE 2002, CBSE 2004)

Q.1. What physical quantity is the same for x-rays of wavelength 10^{-10}m , red light of wavelength $6800\text{\AA}$ and radiowaves of wavelength 500m ?

Q.2. A capacitor has been charged by a dc source. What are the magnitudes of conduction and displacement current when it is fully charged and during charging?

Q.3. Welders wear special goggles or face mask with glass windows to protect their eyes from electromagnetic radiations. Name the radiations and write the range of their frequency.

Ans. Ultraviolet radiations, Freq. range $\rightarrow 10^{15} - 10^{17}\text{ Hz}$

Q.4. The speed of an electromagnetic wave in a material medium is given by $v = \frac{1}{\sqrt{\mu\epsilon}}$, where μ is permeability and ϵ is permittivity of the medium. How does its frequency change?

Q.5. Why did Maxwell introduce displacement current in Ampere's circuital law?

Q.6. A variable frequency AC source is connected to a capacitor. How will the displacement current change with decrease in frequency?

Q.7. The charge on a parallel plate capacitor varies as $q = q_0 \cos 2\pi ft$. The plates are very large and close together. Find the displacement current through the capacitor?

Ans. $I_D = -2\pi q_0 f \sin 2\pi ft$

Q.8. A variable frequency ac source is connected to a capacitor. How will the displacement current change with decrease in frequency?

Ans. I_D will decrease.

Q.9. Suppose that the electric field of an electromagnetic wave in vacuum is $\vec{E} = 3.1 \cos(1.8y + 5.4 \times 10^6 t) \hat{i}$

(a) what is the direction of propagation

(b) what is the wavelength (λ)

(c) what is the frequency (ν)

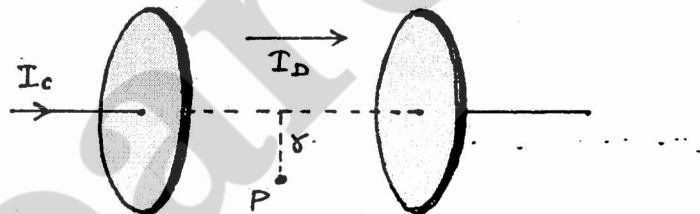
(d) what is the amplitude of the magnetic field?

(e) write an expression for the magnetic field.

Ans. (a) y-axis (b) $\lambda = 3.49 \text{ m}$ (c) $\nu = 8.6 \times 10^5 \text{ Hz}$

(d) $B_0 = 1.03 \times 10^{-8} \text{ T}$ (e) $B = 1.03 \times 10^{-8} \cos[1.8y + 5.4 \times 10^6 t] \hat{k}$

Q.10. Show that the magnetic field B at a point in between the plates of a parallel plate capacitor during charging is $B = \frac{\mu_0 \epsilon_0 r}{2} \frac{dE}{dt}$



Q.11. When an ideal capacitor is charged by a dc battery, no current flows. However when an a.c. source is used, the current flows continuously. How does one explain this, based on the concept of displacement current?

Q.12. A capacitor, made of two parallel plates each of plate area A and separation d , is being charged by an external ac source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor.

PHYSICS

Unit – 9. Ray Optics

Class – 12th

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9. Ray Optics

Ray & Ray Optics -

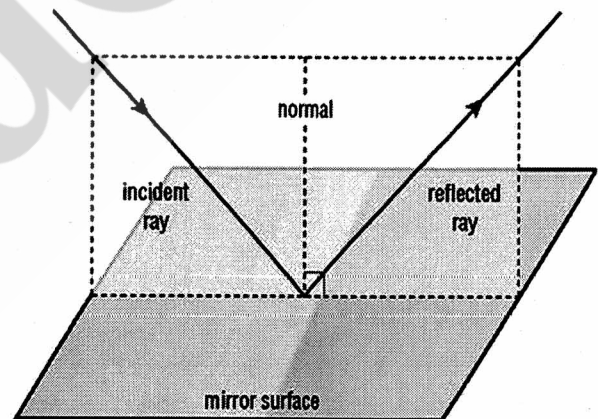
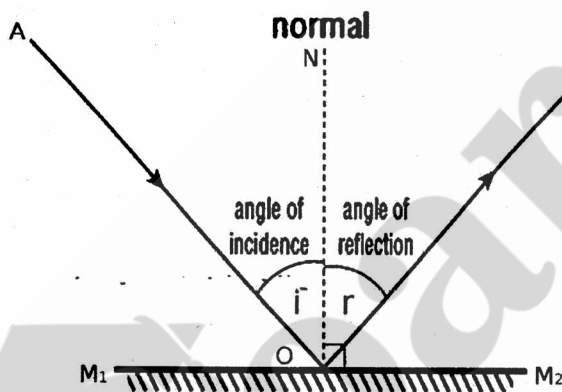
A ray of light is the straight line path followed by light in going from one point to another.

The ray optics therefore uses the geometry of straight lines to represent macroscopic phenomena like rectilinear propagation, reflection, refraction etc.

Reflection of Light -

Reflection is the phenomena of change in the path of light without any change in medium.

Mirror- A smooth and well polished surface which reflects regularly most of the light falling on it is called a mirror.



In figure M_1M_2 is a plane mirror. A ray of light falls on the mirror along AO at $\angle AON = i$ is called incident ray and i is called angle of incidence, where ON is normal to the mirror at O .

Incident ray is reflected along OB at $\angle NOB = r$, It is called reflected ray and r is known as angle of reflection.

Laws of reflection -

(i) Angle of incidence is equal to angle of reflection.

$$\angle i = \angle r$$

(ii) Incident ray (OA), reflected ray (OB) and normal (ON) to the mirror, all lie in the same plane.

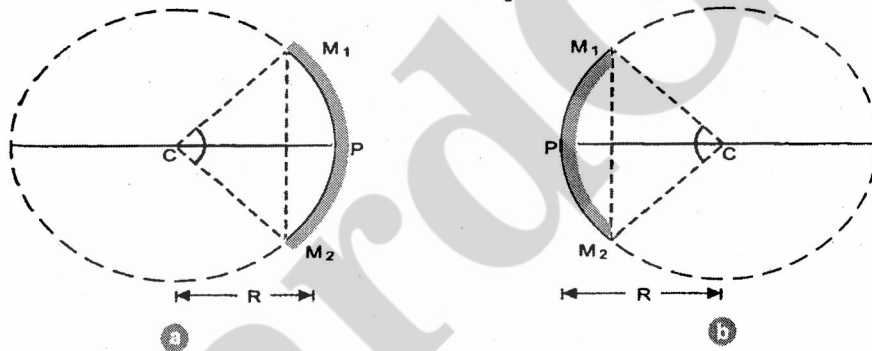
Spherical Mirrors -

A spherical mirror is a part of a hollow sphere, whose one side is reflecting and other side is opaque.

There are two types of spherical mirrors -

(1) Concave mirror -

whose reflecting surface is towards the centre of the sphere of which the mirror is a part. As shown in figure (a)



(2) Convex mirror -

whose reflecting surface is away from the centre of the sphere of which the mirror is a part. As shown in figure (b).

Some Important Terms -

(i) Centre of curvature (C) -

The centre of the sphere of which the mirror forms a part is called the centre of curvature. It is represented by 'C'.

(ii) Radius of curvature (R) -

The radius of the sphere of which the mirror forms a part is called the radius of curvature of the spherical mirror. It is represented by 'R'.

(iii) Vertex or Pole -

The middle point or centre of the spherical mirror is called vertex or pole of the mirror. It is represented by P.

(iv) Aperture of the mirror -

The diameter M_1M_2 of the spherical mirror is called aperture linear aperture of the mirror.

(v) Angular aperture -

The angle M_1CM_2 , subtended at C by the diameter of the spherical mirror is called angular aperture.

(vi) Principal axis -

The straight line joining the pole and the centre of curvature of spherical mirror extended on both sides is called principal axis of the mirror.

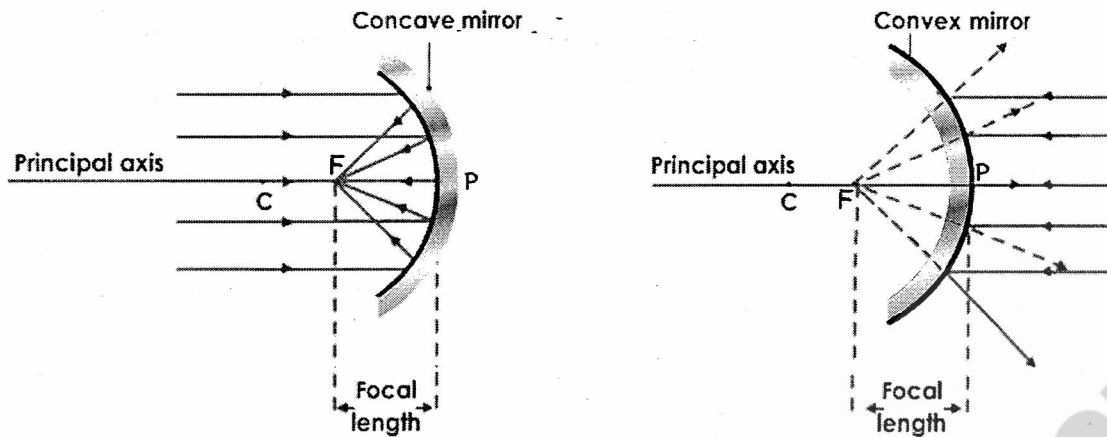
(vii) Principal section -

A section of the spherical mirror cut by a plane passing through pole and centre of curvature of the mirror is called principal section of the mirror.

(viii) Principal focus (F) -

It is a point on the principal axis of the mirror at which rays incident on the mirror in a direction parallel to the principal axis actually meet or appear to diverge after reflection from the mirror.

- * In case of a convex mirror, F is a virtual point, wherefrom the rays after reflection appear to diverge.
- * In case of a concave mirror F is a real point where the rays after reflection from the mirror actually meet.

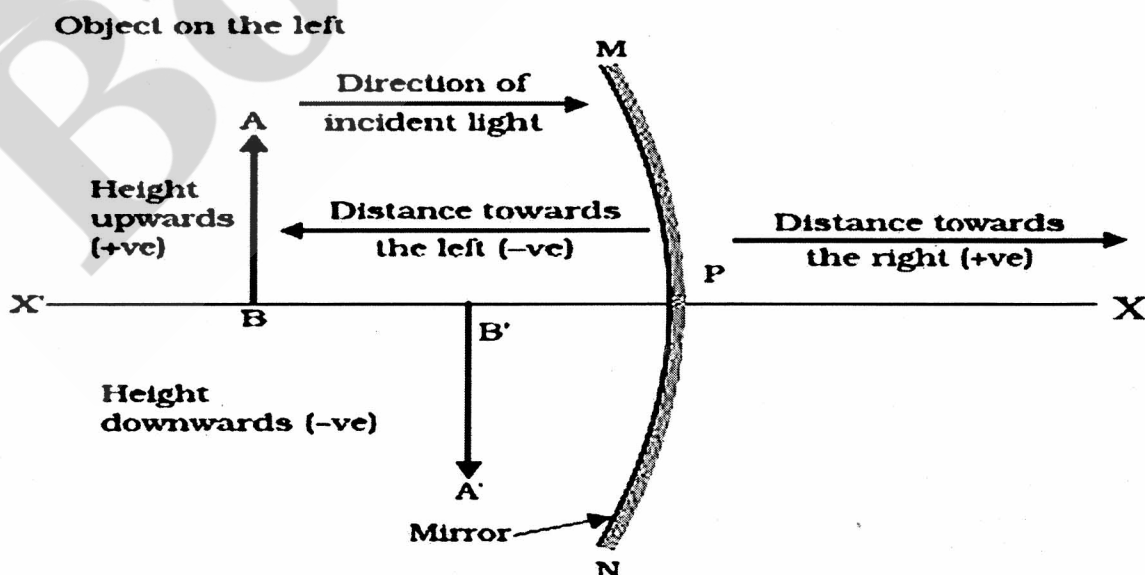


* The distance of principal focus F from the pole P of the spherical mirror is called focal length (f) of the mirror.

$$PF = f$$

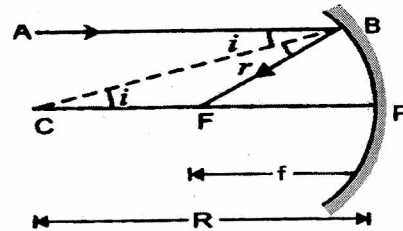
New Cartesian sign Conventions -

- (1) All the distances are measured from pole (P) of spherical mirror.
- (2) The distances measured in the direction of incidence of light are taken as positive and the distances measured in a direction opposite to the direction of incidence of light are taken as negative.
- (3) The heights measured upwards and perpendicular to the principal axis of the mirror are taken positive and the heights measured downwards are taken as negative.



Relation between f and R

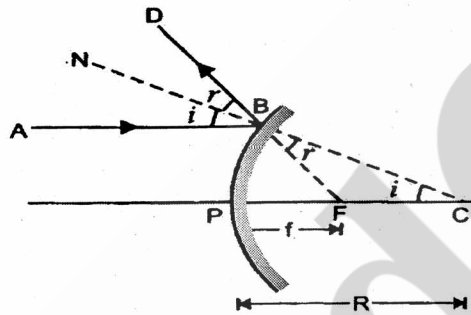
(a) Concave Mirror -



$$f = R/2$$

* The focal length of a concave mirror is equal to half the radius of curvature of the mirror.

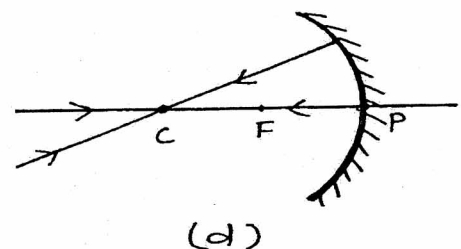
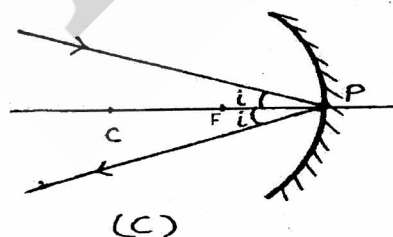
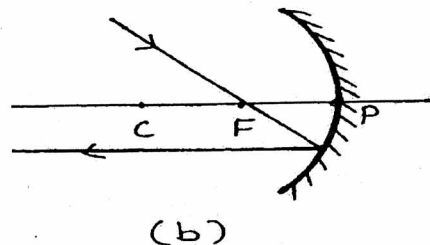
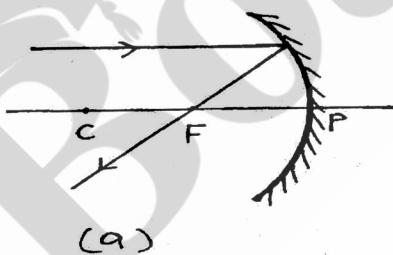
(b) Convex Mirror -



$$f = \frac{R}{2}$$

* Focal length of a convex mirror is equal to half of its radius of curvature.

Some Important Rules



Mirror Formula for Concave Mirror

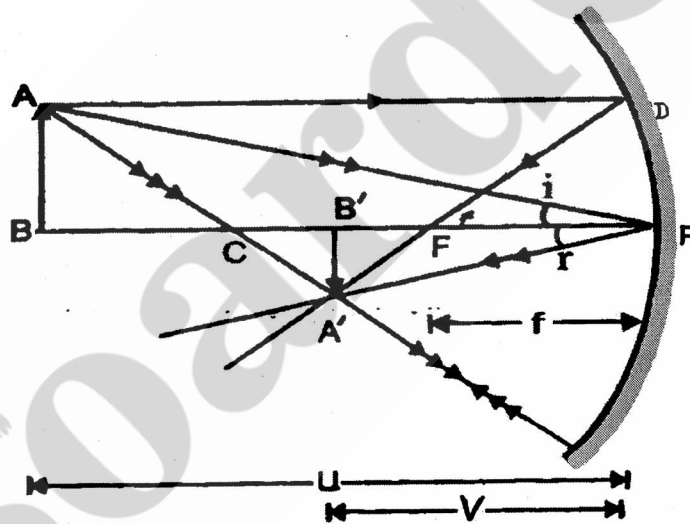
Mirror formula is a relation between focal length of the mirror and distances of object and image from the mirror.

Let P be the pole, F is the principal focus and C is the centre of curvature of a concave mirror of small aperture. Also $PF = f$ be the focal length and $PC = R$ be the radius of curvature of the mirror.

(a) Real Image -

When the object is held in front of a concave mirror beyond the principal focus F of the mirror, then image formed is real.

Let an object AB is held perpendicular to the principal axis of the mirror beyond C .



A ray of light starting from A and incident on the mirror along AD parallel to the principal axis, gets reflected from the mirror and passes through F .

Another ray of light incident along AP is reflected along PA' such that $\angle APB = \angle i = \angle BPA' = \angle r$

The two reflected rays actually meet at A' which is real image of the point A . A third ray starting from A and incident on the mirror along AC falls normally on the mirror and retrace its path; meeting the two reflected rays at A' .

From A' draw A'B' perpendicular on the principal axis. Therefore A'B' is real, inverted image of AB formed by reflection from the Concave mirror.

As $\triangle ABC$ and $\triangle A'B'C$ are similar, so that -

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \quad \text{--- (1)}$$

Again as $\triangle ABP$ and $\triangle A'B'P$ are similar, so that -

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \quad \text{--- (2)}$$

From eq. (1) and eq. (2) we get -

$$\frac{CB}{CB'} = \frac{PB}{PB'} \quad \text{--- (3)}$$

By measuring all distances from P, we have -

$$CB = PB - PC \text{ and } CB' = PC - PB'$$

$$\therefore \text{ from eq. (3) } \frac{PB - PC}{PC - PB'} = \frac{PB}{PB'} \quad \text{--- (4)}$$

Using new cartesian sign Conventions, we get -

$$PB = -u, PC = -R \text{ and } PB' = -v$$

so we get from eq. (4), that -

$$\frac{-u + R}{-R + v} = \frac{-u}{-v}$$

$$\text{or } uR - uv = uv - vR$$

$$\text{or } uR + vR = 2uv$$

by dividing both side by $u v R$, we get -

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$$

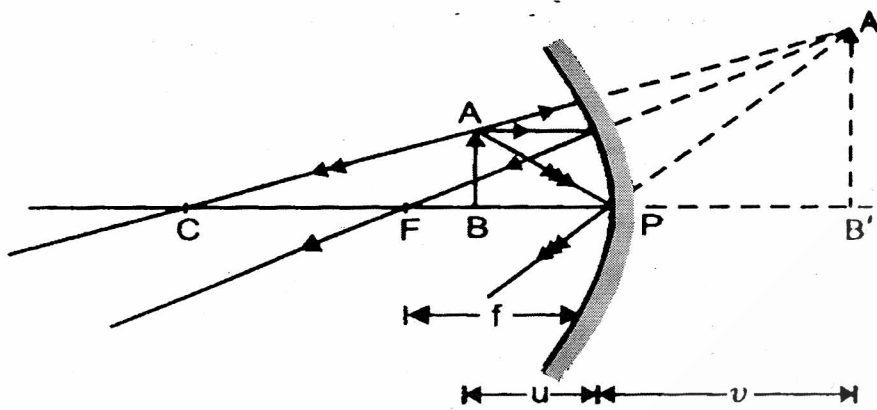
$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

$$\text{As } f = \frac{R}{2}$$

It is the required mirror formula.

(b) Virtual Image -

When the object is held in front of a Concave mirror between the pole P and principal focus F of the mirror, then the image formed is virtual, erect and magnified as shown in figure.



As $\triangle ABC$ and $\triangle A'B'C$ are similar, so that-

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \quad \text{--- (1)}$$

Again as $\triangle ABP$ and $\triangle A'B'P$ are similar, so that-

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \quad \text{--- (2)}$$

From eq. (1) and eq. (2) we get -

$$\frac{CB}{CB'} = \frac{PB}{PB'} \quad \text{--- (3)}$$

By measuring all distances from P, we get-

$$CB = PC - PB \text{ and } CB' = PC - PB'$$

$$\therefore \frac{PC - PB}{PC - PB'} = \frac{PB}{PB'}$$

Using new Cartesian sign conventions -

$$PB = -u, \quad PB' = +v, \quad PC = -R$$

$$\text{then } \frac{-R + u}{-R + v} = \frac{-u}{v}$$

$$\text{or } uR - uV = -VR + uV$$

$$uR + VR = 2uV$$

Dividing both sides by uVR , we get -

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$$

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

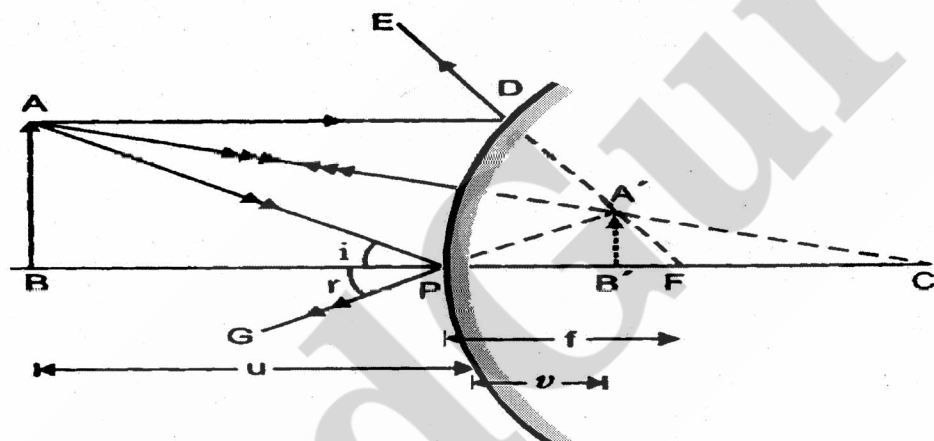
It is the required mirror formula.

Mirror Formula for Convex Mirror -

The image formed in a convex mirror is always virtual and erect, whatever be the position of the object.

Let P be the pole, F is the principal focus and C is the centre of curvature of a convex mirror of small aperture.

Let $PF = f$ is the focal length and $PC = R$, the radius of curvature of the mirror. AB is an object held in front of mirror perpendicular to its principal axis.



A ray of light starting from A and incident on the mirror along AD in a direction parallel to the principal axis is reflected along DE . On producing back, DE appears to come from F .

Another ray from A incident on the mirror along AP is reflected along PG such that

$$\angle APB = \angle i = \angle BPG = \angle r$$

On producing back, PG meets DF at A' . Therefore A' is virtual image of A .

A third ray incident on the mirror along AC falls normally on the mirror and retraces its path on reflection. This also appears to come from A' .

Draw $A'B'$ perpendicular to principal axis. Therefore, $A'B'$ is virtual image of the object AB formed after reflection at the convex mirror.

As $\triangle ABC$ and $\triangle A'B'C$ are similar, so that -

$$\frac{AB}{A'B'} = \frac{CB}{CB'} \quad \text{--- (1)}$$

Again as $\triangle ABP$ and $A'B'P$ are similar, so -

$$\frac{AB}{A'B'} = \frac{PB}{PB'} \quad \text{--- (2)}$$

from eq. (1) and (2) we get -

$$\frac{CB}{CB'} = \frac{PB}{PB'} \quad \text{--- (3)}$$

measuring the all distances from the pole P, we can write -

$$CB = PB + PC$$

$$CB' = PC - PB'$$

So by eq. (3) -
$$\frac{PB + PC}{PC - PB'} = \frac{PB}{PB'} \quad \text{--- (4)}$$

Using New Cartesian sign conventions -

$$PB = -u, \quad PB' = +v \quad \text{and} \quad PC = +R$$

so from eq. (4) we get -

$$\frac{-u + R}{R - v} = \frac{-u}{v}$$

$$-uv + vR = -uR + uv$$

or
$$vR + uR = 2uv$$

dividing both sides by uVR , we get -

$$\frac{VR}{uVR} + \frac{uR}{uVR} = \frac{2uv}{uVR}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

$$\boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

Concave Mirror					
Object		Image			Orientation
		Position	Size	Nature	
1.	at ∞	at Focus	Point	Real	Inverted
2.	between ∞ and C	between F and C	diminished	Real	Inverted
3.	at C	at C	Same	Real	Inverted
4.	between C and F	between C and ∞	Magnified	Real	Inverted
5.	at F	at ∞	Indefinetly Large	Real	Inverted
6.	between F and P	on other side of Mirror	Large	Virtual	Erect

Convex Mirror					
Object		Image			Orientation
		Position	Size	Nature	
1.	at ∞	at Focus	Point	Virtual	Erect
2.	between ∞ and C	between P and F	diminished	Virtual	Erect

Linear Magnification of a Spherical Mirror -

Linear magnification or simply magnification of a spherical mirror is the ratio of the size of the image formed by the mirror to the size of the object.

It is represented by m .

$$m = \frac{\text{Size of image (I)}}{\text{Size of object (O)}} = \frac{A'B'}{AB}$$

We assume that O is always positive. Magnification (m) will be +ve when I is +ve (i.e. image is erect) and m will be negative when I is negative (i.e. image is inverted).

For both Concave and Convex mirror we know that

$$\frac{A'B'}{AB} = \frac{PB'}{PB}$$

In case of Concave mirror, when image formed is real, as shown in figure.

Using new Cartesian sign Conventions -

$$A'B' = -I, AB = +O$$

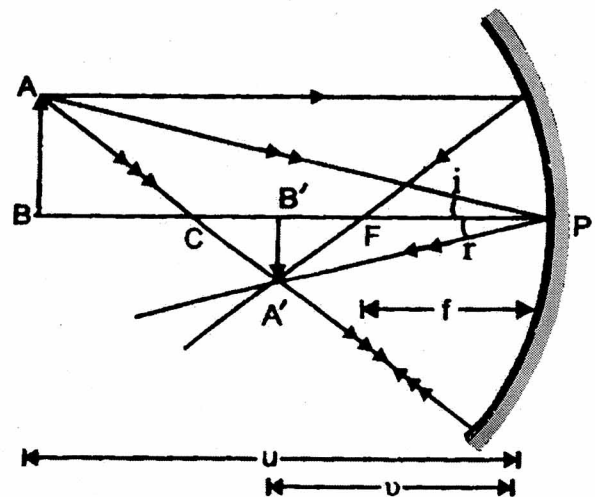
$$PB' = -V, PB = -u$$

$$\therefore -\frac{I}{O} = \frac{-V}{-u} = \frac{V}{u}$$

$$\therefore m = -\frac{I}{O} = \frac{V}{u}$$

So here m is negative.

$$m = \frac{I}{O} = -\frac{V}{u}$$



* The positive or negative sign of m is determined from the sign of I/O .

When image formed is virtual as shown in figure then, using new Cartesian sign Conventions.

$$A'B' = +I, \quad AB = +O$$

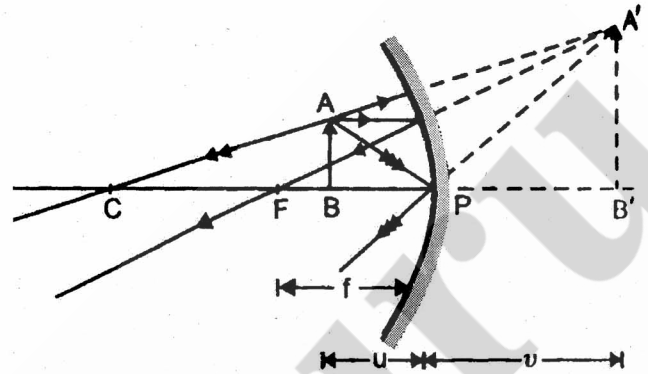
$$PB' = +V, \quad PB = -u$$

$$\therefore m = \frac{A'B'}{AB} = \frac{PB'}{PB}$$

we get -

$$m = \frac{I}{O} = \frac{V}{-u}$$

so m is Positive.



In case of Convex mirror as shown in figure, using new Cartesian sign Conventions -

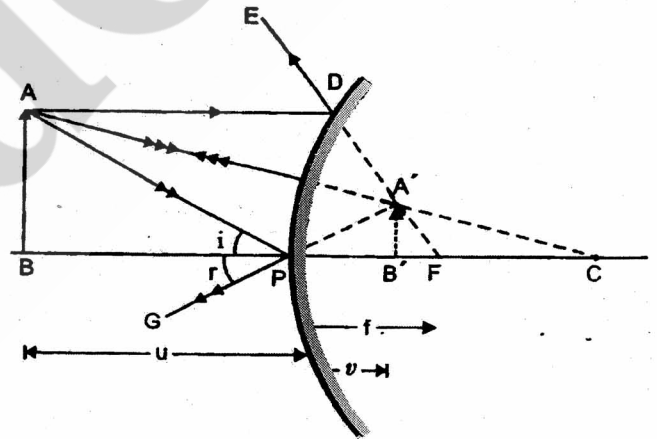
$$A'B' = +I, \quad AB = +O$$

$$PB' = +V, \quad PB = -u$$

$$\therefore m = \frac{I}{O} = \frac{+V}{-u}$$

$$\therefore m = \frac{I}{O} = -\frac{V}{u}$$

so m is positive.



Note -

- (i) When $m > 1$, image formed is enlarged.
- (ii) When $m < 1$, image formed is diminished.
- (iii) When m is positive, image must be erect (i.e. virtual)
- (iv) When m is negative, image must be inverted (i.e. real)
- (v) In case of a concave mirror, magnification may be positive or negative, but in case of convex mirror, magnification is positive only.

Practical Applications of Spherical Mirrors-

- (i) A convex mirror is used as a reflector in Street Lamps. As a result, the light from the Lamp diverges over a large area.
- (ii) A Convex mirror is used as a drivers mirror in all vehicles like cars and scooters etc., for looking at the traffic at the rear of the vehicle. Such a mirror has a much wider field of view as compared to a plane mirror or a concave mirror. The images formed are smaller and erect.
- (iii) A Concave mirror is used as a reflector in search lights, head lights of motor vehicles, telescope, solar cookers etc.
- (iv) A Concave mirror is also used as a shaving mirror or make up mirror as it can form erect and magnified image.

Q.

CBSE
2011

An object is placed 18 cm. in front of a mirror. If the image is formed at 4 cm. to the right of the mirror, calculate its focal length. Is the mirror convex or concave? What is the nature of the image? What is the radius of curvature of the mirror?

Sol. Here $u = -18 \text{ cm.}$, $v = 4 \text{ cm.}$

from $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{f} = \frac{1}{4} - \frac{1}{18} = \frac{7}{36}$$

so $f = \frac{36}{7} = 5.14 \text{ cm.}$

As focal length is positive, the mirror must be Convex.

The image is virtual, erect and smaller in size.

Also $R = 2f = 2 \times 5.14 = 10.28 \text{ cm.}$

Q. A small candle 2.5 cm in size is placed 27 cm. in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

sol. Size of candle (O) = 2.5 cm. , $u = -27$ cm.
 $R = -36$ cm , $f = R/2 = -18$ cm.

As $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ so $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$$\frac{1}{v} = \frac{1}{-18} + \frac{1}{27} = -\frac{1}{54} \quad \text{or} \quad v = -54 \text{ cm.}$$

so the screen should be placed at 54 cm. from the mirror on the same side as the object.

If I is size of image then as the image is real -

$$m = -\frac{I}{O} = \frac{-v}{-u} = \frac{v}{u}$$

$$-\frac{I}{2.5} = \frac{(-54)}{(-27)} = 2 \quad \Rightarrow \quad I = -5 \text{ cm.}$$

Minus sign indicates that image is inverted.

When the candle is moved closer to mirror, the screen has to be moved away from the mirror.

However when candle is at a distance less than 18 cm from the mirror, image formed would be virtual and screen is not required, as the virtual image can not be taken on the screen.

Q.
CBSE
2011

Use the mirror equation to show that -

- An object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.
- A convex mirror always produces a virtual image independent of the location of the object.
- An object placed between the pole and focus of a concave mirror produces a virtual & enlarged image.

Sol.(a) for a mirror $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ for concave mirror $f < 0$ and $u < 0$

Hence $-\frac{1}{|f|} = \frac{1}{v} - \frac{1}{|u|} \Rightarrow \frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$

where $|2f| > |u| > |f|$

$$\Rightarrow \frac{1}{v} < 0 \Rightarrow v < 0$$

Hence image forms on the same side as that of object i.e. real image forms.

Also $|2f| > |u| > |f|$ or $\frac{1}{|2f|} < \frac{1}{|u|} < \frac{1}{|f|}$

$$\frac{1}{|2f|} - \frac{1}{|f|} < \frac{1}{|u|} - \frac{1}{|f|} < 0$$

$$-\frac{1}{|2f|} < \frac{1}{|v|} < 0 \Rightarrow v > -|2f|$$

Hence image form beyond $2f$ on the same side of object.(b) for convex mirror $f > 0$ and $u < 0$

so that $\frac{1}{f} = \frac{1}{v} - \frac{1}{|u|} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{|u|}$

for f and $|u|$ to be positive

$$\frac{1}{v} > 0 \quad \text{Hence} \quad v > 0$$

Hence a virtual image of object is form.

(c) For Concave mirror $f < 0$, $u < 0$

Hence by mirror formula $-\frac{1}{|f|} = \frac{1}{v} - \frac{1}{|u|}$

$$\Rightarrow \frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$$

Here $|u| < |f|$ so that $\frac{1}{|u|} > \frac{1}{|f|}$

$$\Rightarrow \frac{1}{v} > 0 \quad \text{or} \quad v > 0$$

Here the image is formed on RHS of mirror i.e. virtual image.

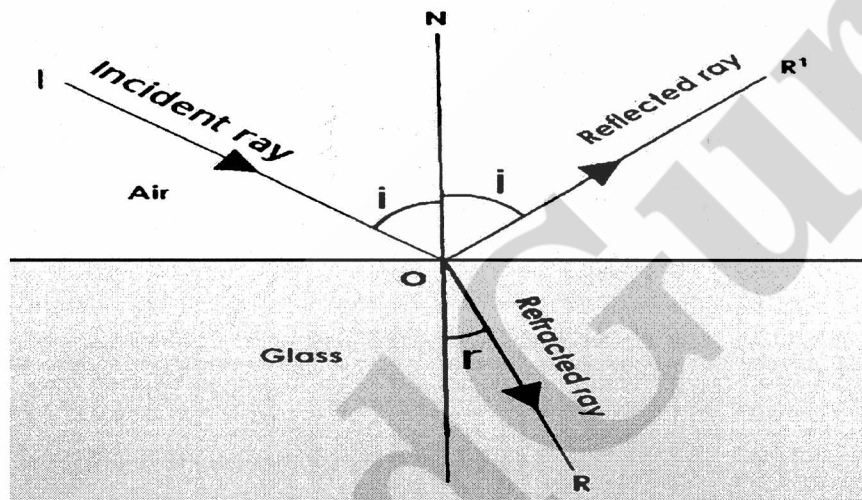
Also by $-\frac{1}{f} = \frac{1}{v} - \frac{1}{|u|} \Rightarrow \frac{1}{|v|} < \frac{1}{|u|}$

$$\Rightarrow \frac{|v|}{|u|} > 1 \Rightarrow m > 1$$

Hence image is enlarged.

REFRACTION

When a beam of light travelling in one transparent medium falls on another transparent medium, a part of light gets reflected back into the first medium and the rest of light enters into the other medium. The direction of propagation of an obliquely incident ray of light that enters the other medium changes at the interface of two media. This phenomenon is called refraction of light.



Refractive Index -

The ratio of speed of light in vacuum/air to the speed of light in the medium is called the refractive index (μ) of a medium.

$$\mu = \frac{\text{Speed of light in vacuum/air } (c)}{\text{speed of light in the medium } (v)}$$

When the light passes from one medium 1 to another medium 2, the refractive index of medium 2 with respect to medium 1 is written as ${}^1\mu_2$ and is called relative refractive index.

$${}^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2} = \frac{\text{Speed of light in medium 1}}{\text{speed of light in medium 2}}$$

★ A medium with higher refractive index is said to be optically denser compared to the medium with lower refractive index.

- * Refractive index is a characteristic property of the medium, whose value depends upon nature of material of the medium and the color or wavelength of light.

Laws of Refraction -

- (i) Whenever light goes from one medium to another, the frequency of light and phase of light do not change. However the velocity of light and the wavelength of light change.
- (ii) The incident ray, the refracted ray and normal to the interface at the point of incidence, all lie in the same plane.
- (iii) The product of refractive index and sine of angle of incidence / refraction at a point in a medium is constant.

$$\mu \sin i = \text{Constant} \quad , \therefore \mu_1 \sin i_1 = \mu_2 \sin i_2$$

If $i_1 = i$ and $i_2 = r$, then

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\text{or} \quad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = {}^1\mu_2$$

where ${}^1\mu_2$ is refractive index of medium 2 with respect to medium 1.

* This Law is also called Snell's Law.

Q.
CBSE
2012

For the same value of angle of incidence, the angle of refraction in three media A, B and C are 15° , 25° and 35° respectively. In which medium would the velocity of light be minimum?

sol. From Snell's Law - $\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$

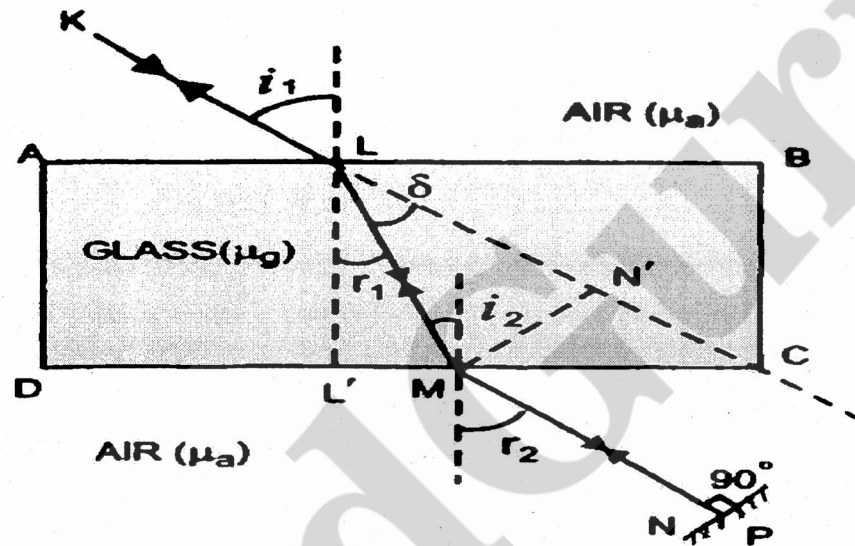
$$\Rightarrow v \propto \sin r \text{ for given } i$$

Hence velocity of light is minimum in medium A as angle of refraction is minimum (15°).

Refraction through a rectangular Glass Slab

Let in figure ABCD is a rectangular glass slab. A ray of light is incident along KL on the face AB of the slab at $\angle i_1$. It is refracted along LM at $\angle r_1$.

The refracted ray LM falls on face CD at $\angle i_2$ and emerges out along MN at $\angle r_2$.



In going from a rarer medium to a denser medium, the ray bends towards the normal and in going from a denser to a rarer medium, the ray bends away from the normal.

Applying Snell's Law at L -

$$\mu_a \sin i_1 = \mu_g \sin r_1$$

$$\boxed{\frac{\sin i_1}{\sin r_1} = \frac{\mu_g}{\mu_a} = {}^a\mu_g} \quad \text{--- (1)}$$

Again applying Snell's Law at M -

$$\mu_g \sin i_2 = \mu_a \sin r_2 \quad \Rightarrow \quad \boxed{\frac{\mu_a}{\mu_g} = \frac{\sin i_2}{\sin r_2} = {}^g\mu_a} \quad \text{--- (2)}$$

* According to the principle of reversibility of light when final path of a ray of light after any number of reflections and refractions is reversed, the ray retraces its entire path.

We know that

$${}^a\mu_g = \frac{1}{{}^g\mu_a}$$

Hence from eq. (1) and (2) we get -

$$\frac{\sin i_1}{\sin r_1} = \frac{\sin r_2}{\sin i_2} \quad \text{--- (3)}$$

From figure -

$$i_2 = r_1 \quad \text{so} \quad \sin i_2 = \sin r_1$$

Hence from eq (3) -

$$\sin r_2 = \sin i_1 \quad \text{or} \quad r_2 = i_1$$

* Hence the emergence ray MN is parallel to the incident ray KL.

Expression for Lateral displacement -

From M, draw MN' perpendicular to KL produced so the Lateral displacement of the ray on passing through the parallel slab = MN'

Let $\angle MLN' = \delta$ = deviation on first refraction.

$$\text{In } \triangle LMN' \quad \sin \delta = \frac{MN'}{LM}$$

$$\therefore MN' = LM \sin \delta \quad \text{--- (1)}$$

$$\text{In } \triangle LL'M \quad \cos r_1 = \frac{LL'}{LM}$$

$$\therefore LM = \frac{LL'}{\cos r_1} = \frac{t}{\cos r_1}$$

where $t = LL'$ = thickness of glass slab

$$\text{So from eq. (1) we get -} \quad MN' = \frac{t}{\cos r_1} \sin \delta$$

or

$$MN' = \frac{t \sin (i_1 - r_1)}{\cos r_1}$$

This is the expression for Lateral displacement.

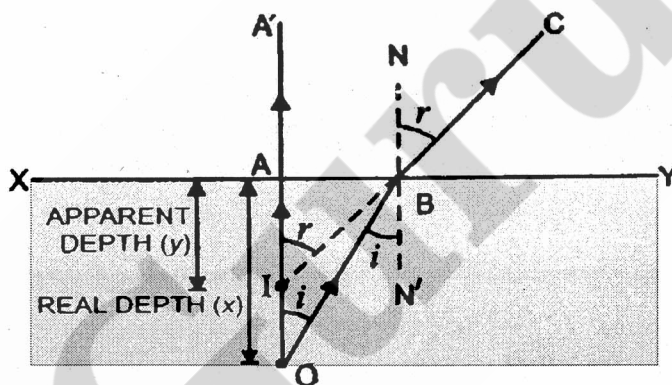
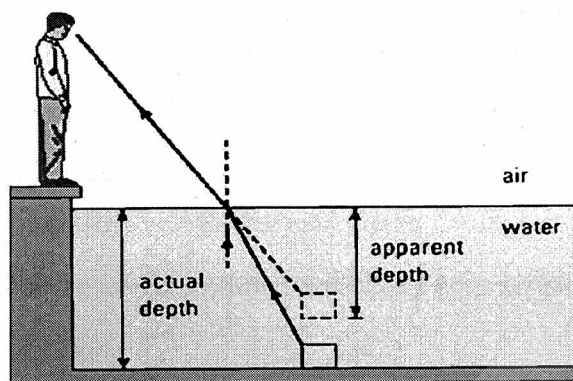
* Lateral displacement is proportional to the thickness (t) of the slab and angle of incidence i_1 .

Real Depth and apparent Depth of a Tank

A water tank appears less deep than what it actually is. This is due to refraction of light.

Proof -

Suppose O is a point object at an actual depth OA below the surface of water XY in a tank.



A ray of light incident on XY, normally along OA passes straight OAA'.

Another ray of light from O incident at $\angle i$ on XY, along OB deviates away from the normal. It is refracted at $\angle r$ along BC. On producing back, BC meets OA at I. Therefore I is the virtual image of O. (When seen through water, O appears at I).

Therefore apparent depth = AI (Less than OA)

$$\angle AOB = \angle OBN' = i \text{ and } \angle AIB = \angle NBC = r$$

$$\text{In } \triangle OAB \quad \sin i = \frac{AB}{OB}$$

$$\text{In } \triangle IAB \quad \sin r = \frac{AB}{IB}$$

As the light is travelling from denser medium (water) to rarer medium (air).

$$\therefore {}^a\mu_w = \frac{\sin r}{\sin i} = \frac{AB}{IB} \times \frac{OB}{AB} = \frac{OB}{IB}$$

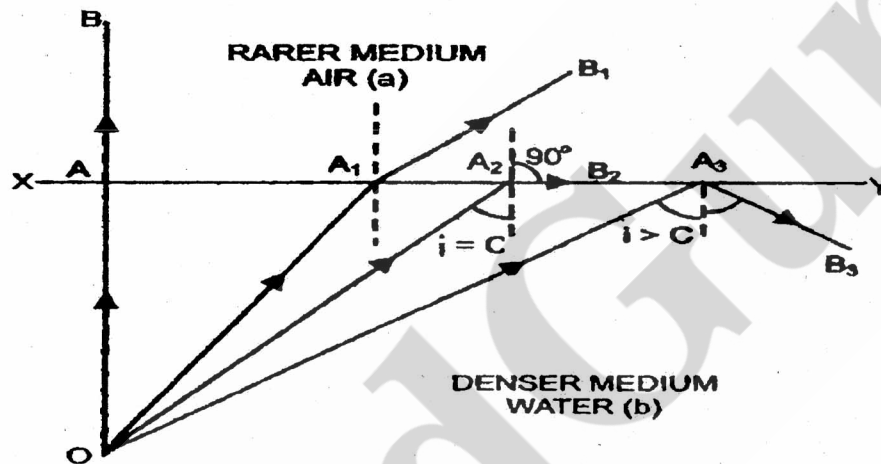
When angles are small, B is close to A. So

$${}^a\mu_w = \frac{OA}{IA} = \frac{\text{real depth}}{\text{apparent depth}}$$

* for air and water $\mu_{w} = 4/3$ so $IA = \frac{3}{4} OA$
 apperent depth is only $(\frac{3}{4})^{th}$ of the real depth
 of the tank of water.

Total Internal Reflection

Suppose an interface XY separates a rarer medium (a) air from a denser medium (b) say water.



O is a point object in the denser medium.
 A ray of light starting from O and incident normally along OA on XY passes straight along AB .

Another ray incident along OA_1 deviates away from normal and is refracted along A_1B_1 . Clearly angle of refraction is greater than the angle of incidence and it increases with increase in the angle of incidence.

For particular value of $i = c$, the critical angle, the incident ray OA_2 is refracted at $\angle r = 90^\circ$ and goes along the interface, along A_2B_2 .

When $i > c$ as for the incident ray OA_3 , the ray goes along A_3B_3 , as it is reflected from interface XY . This phenomenon is called Total Internal Reflection.

Total Internal Reflection -

It is the phenomenon of reflection of light into a denser medium from an interface of this denser medium and a rarer medium.

Essential Conditions for total Internal Reflection -

- (i) Light should travel from a denser medium to a rarer medium.
- (ii) Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact.

Critical Angle - (C) -

The angle of incidence in the denser medium corresponding to which angle of refraction in the rarer medium is 90° , is called critical angle for the pair of medium in contact.

It is represented by 'C' and its value depends on the nature of medium in contact.

Relation between Refractive index & Critical Angle

when $i = C$, $r = 90^\circ$

Applying Snell's Law at A_2 , $\mu_b \sin C = \mu_a \sin 90^\circ$

$$\frac{\mu_b}{\mu_a} = \frac{1}{\sin C}$$

$$\boxed{{}^a\mu_b = \frac{1}{\sin C}}$$

a = rarer

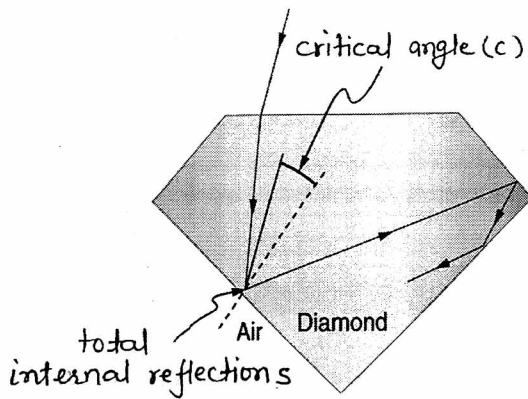
b = dense

As μ depends on wavelength, therefore critical angle for the same pair of medium in contact will be different for different colors.

Applications of Total Internal Reflection

(1) The brilliance of diamond -

The brilliance of diamond is due to total internal reflection of light. μ for diamond is 2.42, so that critical angle for diamond air interface as calculated from ${}^a\mu_b = 1/\sin C$ is 24.4°



The diamond is cut suitably so that light entering the diamond from any face falls at an angle greater than 24.4° . Therefore it suffers multiple total internal reflections at various faces and remain within the diamond and does not come out. Hence the diamond sparkles.

(2) Mirage -

Mirage is an optical illusion which occurs usually in desert on hot summer days. The objects such as a tree appears to be inverted as if the tree is on the bank of a pond of water.

On a hot summer day, temperature of air near the surface of earth is maximum. The upper layers of air have gradually decreasing temperature. Therefore density and refractive index of air goes on increasing slightly with height above the surface of earth.

A ray of light from the top of a tree goes from denser to rarer medium bending successively away from normal.

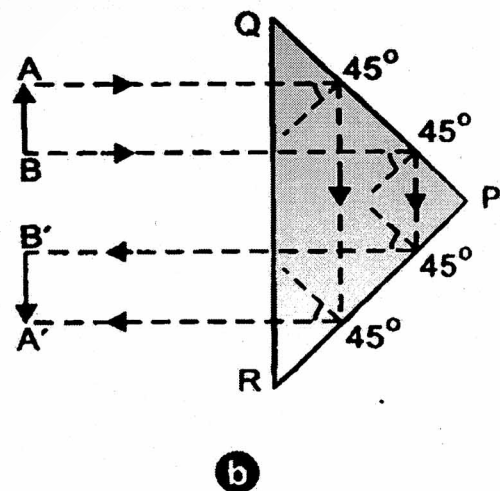
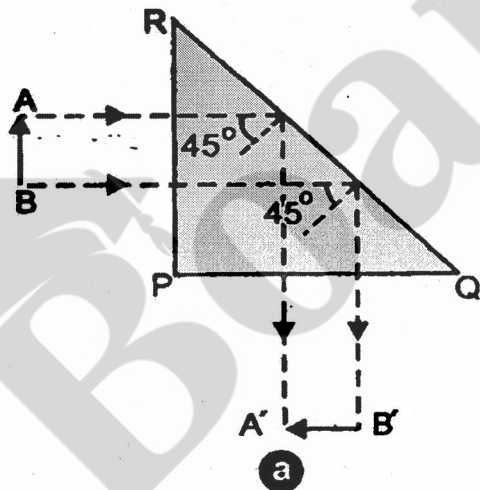
At a particular layer, when angle of incidence becomes greater than the critical angle, total internal reflection occurs and the totally reflected ray reaches the observer along AE, appearing to come from I, the mirror image of O.

Thus inverted image of tree creates the impression of reflection from a pond of water.

(3) Totally reflecting glass prisms -

These are right angled prisms which turn the light through 90° or 180° . They are based on the phenomenon of total internal reflection of light.

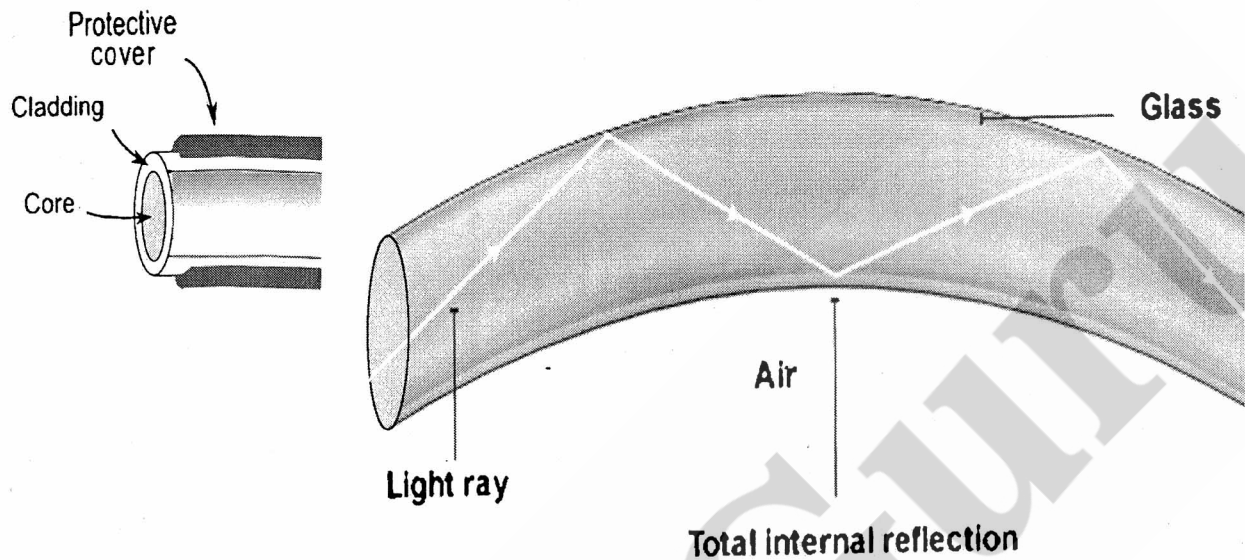
μ for glass is 1.5 so that critical angle for glass-air interface is 42° . In totally reflecting glass prisms, angle of incidence is made 45° which is greater than the critical angle of the glass-air interface. Hence the light suffers total internal reflection.



(4) Optical Fibers -

Optical fibers consist of several thousands of very long fine quality fibers of glass or quartz. The diameter of each fiber is of the order of 10^{-4} cm with refractive index of material of the order of 1.5.

The fibers are coated with a thin layer of material of lower refractive index of the order of 1.48.



Light incident on one end of the fiber at a small angle enters the fiber after refraction and undergoes repeated total internal reflections inside the fiber. It finally comes out of the fiber at the other end, even if the fiber is bent or twisted in any form. And there is almost no loss of light through the sides of the fiber.

Applications of Optical Fibers -

- (1) Optical fibers are used in transmission and reception of electrical signals by converting them first into light signals.
- (2) A bundle of optical fibers is called light pipe. This pipe can transmit an image. Hence it is used in medical and optical examinations of even the inaccessible parts of human body, as in endoscopy.
- (3) Optical fibers are used in telephone and other transmitting cables. Each fiber can carry up to 2000 telephone messages without much loss of intensity.

Spherical Refracting Surfaces -

A refracting surface which forms a part of a sphere of transparent refracting material is called a spherical refracting surface.

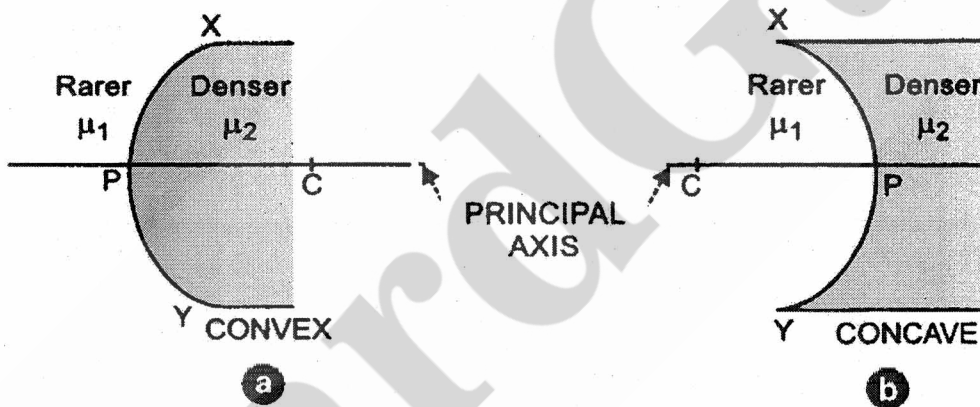
There are two types of spherical refracting surfaces -

(i) Convex Spherical refracting surface -

It is convex towards rarer medium side.

(ii) Concave spherical refracting surface -

It is concave towards the rarer medium side.



Important Terms -

(i) Pole (P)

The centre of spherical refracting surface is called pole (P).

(ii) Centre of curvature (C) -

The centre of the sphere of which the spherical refracting surface is a part, is called the centre of curvature (C).

(iii) Radius of Curvature (R) -

Radius of the sphere of which the spherical refracting surface is a part is called radius of curvature, of the surface. It is represented by R.

(iv) Principal axis -

The line passing through the pole and the centre of curvature of spherical refracting surface, extended on either side is called Principal axis.

New Cartesian Sign Conventions -

- (1) All distances are measured from the pole of the spherical refracting surface.
- (2) The distances measured in the direction of incidence of light are taken as positive and the distances measured in a direction opposite to the direction of incidence of light are taken as negative.

Assumptions -

- (i) The object consists only of a point laying on the principal axis of the spherical refracting surface.
- (ii) The aperture of spherical refracting surface is small.
- (iii) The incident and refracted rays make small angles with the principal axis of the surface.
So that -

and

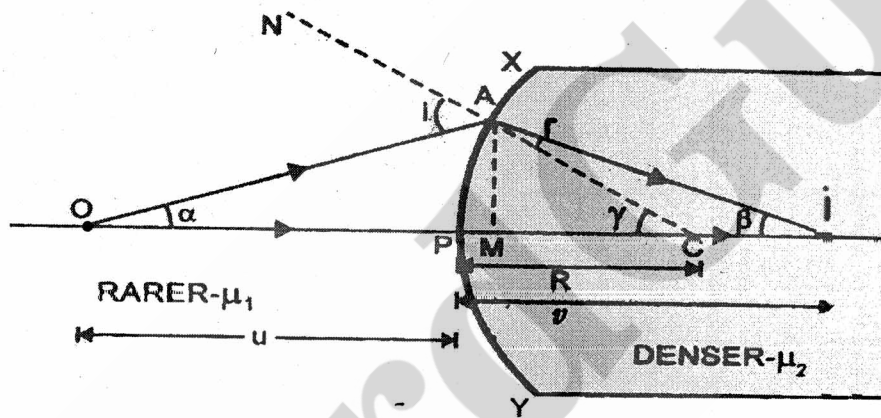
$\sin i \approx i$
$\sin r \approx r$

Refraction from Rarer to Denser Medium at a convex spherical Refracting Surface.

(1) When the image is Real.

Let a spherical refracting surface XY separate a rarer medium of refractive index μ_1 from a denser medium of refractive index μ_2 .

Suppose the surface is convex towards rarer medium side. Let P be the poles, C be the centre of curvature and $R = PC$ be the radius of curvature.



A ray of light starting from O and incident normally on the surface XY along OP passes straight. Another ray of light incident on XY along OA at $\angle i$ is refracted along AI at $\angle r$, bending towards the normal CAN . The two refracted rays actually meet at I , which is the real image of O .

From A , draw AM perpendicular to OI .

Let $\angle AOM = \alpha$, $\angle AIM = \beta$
and $\angle ACM = \gamma$

As external angle of a triangle is equal to sum of internal opposite angles, therefore in $\triangle IAC$

$$\left. \begin{array}{l} \gamma = r + \beta \\ \text{Similarly in } \triangle OAC \end{array} \right\} \begin{array}{l} \text{or } r = \gamma - \beta \\ i = \alpha + \gamma \end{array} \quad \text{--- (1)}$$

According to Snell's Law -

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\therefore \mu_1 i = \mu_2 r \quad (\because \text{angles are small})$$

Using eq. (1) we get -

$$\mu_1 (\alpha + r) = \mu_2 (r - \beta)$$

As angles α , β and r are small so using $\theta \approx \tan \theta$ we get -

$$\mu_1 \left(\frac{AM}{MO} + \frac{AM}{MC} \right) = \mu_2 \left(\frac{AM}{MC} - \frac{AM}{MI} \right)$$

As aperture of the spherical surface is small, M is close to P. Therefore -

$$MO \approx PO, \quad MI \approx PI, \quad MC \approx PC$$

$$\text{So} \quad \mu_1 \left(\frac{1}{PO} + \frac{1}{PC} \right) = \mu_2 \left(\frac{1}{PC} - \frac{1}{PI} \right)$$

$$\therefore \frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Using new Cartesian Sign Conventions, we get -

$$PO = -u, \quad PI = +v, \quad PC = R$$

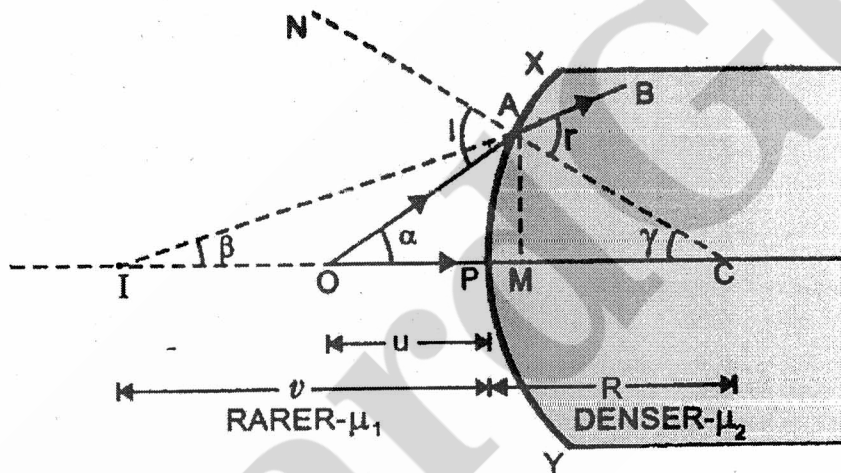
$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

This is the required relation governing refraction from rarer to denser medium at a convex spherical refracting surface.

(2) When the image is Virtual-

The point object O in this case lies on the principal axis close to the pole of the refracting surface. A ray of light OA starting from O suffers refraction at A , bends towards the normal CAN and is refracted along AB .

Another ray of light OP falling normally on the refracting surface travels along PC undeviated. The refracted rays AB and BC do not meet actually at any point but appear to come from a point I . Thus I is the virtual image of the point object O .



From Snell's Law -

$$\mu_1 \sin i = \mu_2 \sin r$$

or $\mu_1 i = \mu_2 r$ — (1)

Draw AM perpendicular to OI and Let-

$$\angle AOM = \alpha, \quad \angle AIM = \beta \quad \text{and} \quad \angle ACM = \gamma$$

In $\triangle OAC$ $i = \gamma + \alpha$ and in $\triangle IAC$ $r = \gamma + \beta$

by putting the values of i and r in eq. (1), we get-

$$\mu_1 [\gamma + \alpha] = \mu_2 [\gamma + \beta]$$

As angles are small, using $\theta \approx \tan \theta$, we get-

$$\mu_1 \left[\frac{AM}{MC} + \frac{AM}{MO} \right] = \mu_2 \left[\frac{AM}{MC} + \frac{AM}{MI} \right]$$

$$\text{or } \mu_1 \left[\frac{1}{MC} + \frac{1}{MO} \right] = \mu_2 \left[\frac{1}{MC} + \frac{1}{MI} \right]$$

As aperture is small, M is close to P

$$\therefore \mu_1 \left[\frac{1}{PC} + \frac{1}{PO} \right] = \mu_2 \left[\frac{1}{PC} + \frac{1}{PI} \right]$$

$$\text{or } \left[\frac{\mu_1}{PO} - \frac{\mu_2}{PI} \right] = \frac{\mu_2 - \mu_1}{PC}$$

Using new Cartesian Sign Conventions -

$$PO = -u, \quad PI = -v, \quad PC = +R$$

$$\frac{\mu_1}{-u} - \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{R}$$

$$\text{or } \boxed{-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

Note that this relation is the same as equation, when image formed is real.

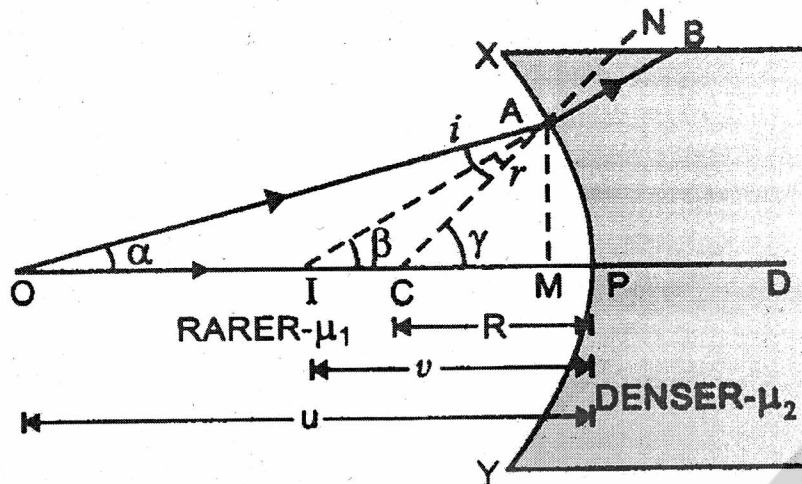
Refraction from Rarer to Denser medium at a concave spherical Refracting Surface -

Let O be a point object laying on the principal axis of a spherical refracting surface XY which is concave towards the rarer medium.

A ray of light OA starting from O suffers refraction at A, bends towards the normal CAN and is refracted along AB.

Another ray of light OP, falling normally on the refracting surface suffers no deviations and passes along PD.

The two refracted rays AB and PD do not meet at any point on the right side but appear to come from a point I on the left side. Thus I is virtual image of the point object O.



In $\triangle OAC$, $i = \gamma - \alpha$ and in $\triangle IAC$ $r = \gamma - \beta$
 Putting these values of i and r in the relation -
 $\mu_1 i = \mu_2 r$

or $\mu_1 (\gamma - \alpha) = \mu_2 (\gamma - \beta)$

As angles are small, using $\theta \approx \tan \theta$, we get -

$$\therefore \mu_1 \left[\frac{AM}{MC} - \frac{AM}{MO} \right] = \mu_2 \left[\frac{AM}{MC} - \frac{AM}{MI} \right]$$

or $\mu_1 \left[\frac{1}{MC} - \frac{1}{MO} \right] = \mu_2 \left[\frac{1}{MC} - \frac{1}{MI} \right]$

As M is close to P because of small aperture -

$$\therefore \mu_1 \left[\frac{1}{PC} - \frac{1}{PO} \right] = \mu_2 \left[\frac{1}{PC} - \frac{1}{PI} \right]$$

or $\frac{\mu_2}{PI} - \frac{\mu_1}{PO} = \frac{\mu_2 - \mu_1}{PC}$

Using new cartesian sign Conventions -

$$PO = -u, \quad PI = -v, \quad PC = -R$$

$$\frac{\mu_2}{-v} - \frac{\mu_1}{-u} = \frac{\mu_2 - \mu_1}{-R}$$

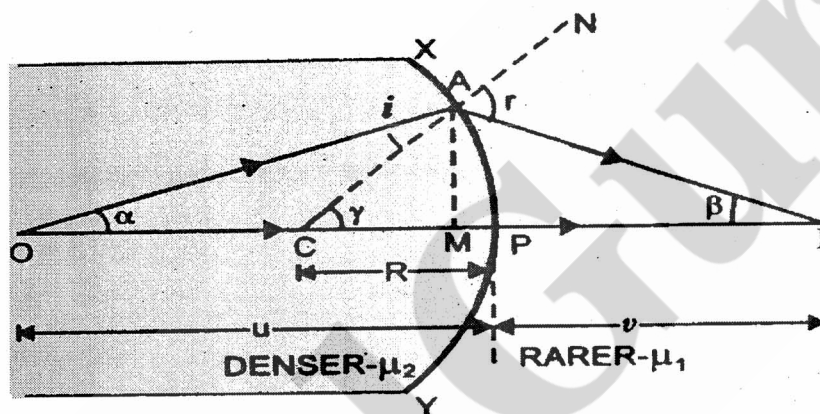
or $\boxed{-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$

It is the desired relation.

Refraction From Denser to Rarer Medium

(1) At a Convex Spherical Surface -

Let P be the pole and C be the centre of curvature of a spherical refracting surface XY . This surface is convex towards the rarer medium and separates a denser medium of refractive index μ_2 from a rarer medium of refractive index μ_1 . Clearly $\mu_2 > \mu_1$.



Let O be a point object lying on the principal axis of the spherical surface. A ray of light OA starting from O meets the refracting surface at A . On refraction it bends away from the normal CAN and moves along AI .

Another ray of light OP falling normally on the refracting surface goes undeviated along PI .

The two refracted rays AI and PI actually meet at I , which is therefore the real image of the point object O .

If i and r are the angles of incidence and refraction, then from Snell's law, we have -

$$\text{or } \mu_2 \sin i = \mu_1 \sin r$$

As i and r are small so $\sin i \approx i$ and $\sin r \approx r$

$$\text{so } \mu_2 i = \mu_1 r$$

$$\text{In } \triangle OAC, \quad i = \gamma - \alpha$$

$$\text{In } \triangle IAC, \quad r = \gamma + \beta$$

$$\text{or } \mu_2 \sin i = \mu_1 \sin r$$

As i and r are small so $\sin i \approx i$ and $\sin r \approx r$

$$\text{so } \mu_2 i = \mu_1 r$$

$$\text{In } \triangle OAC, \quad i = r - \alpha$$

$$\text{In } \triangle IAC, \quad r = i + \beta$$

$$\text{so that } \mu_2 (r - \alpha) = \mu_1 (i + \beta)$$

From A draw AM perpendicular to OI -

As angles are small, so using $\theta \approx \tan \theta$, we get -

$$\mu_2 \left[\frac{AM}{MC} - \frac{AM}{MO} \right] = \mu_1 \left[\frac{AM}{MC} + \frac{AM}{MI} \right]$$

$$\text{or } \mu_2 \left[\frac{1}{MC} - \frac{1}{MO} \right] = \mu_1 \left[\frac{1}{MC} + \frac{1}{MI} \right]$$

Since aperture of the refracting surface is small, M is close to P -

$$\therefore MC \approx PC, \quad MO \approx PO \quad \text{and} \quad MI \approx PI$$

$$\therefore \mu_2 \left[\frac{1}{PC} - \frac{1}{PO} \right] = \mu_1 \left[\frac{1}{PC} + \frac{1}{PI} \right]$$

Applying new Cartesian sign Conventions -

$$PO = -u, \quad PI = v, \quad PC = -R$$

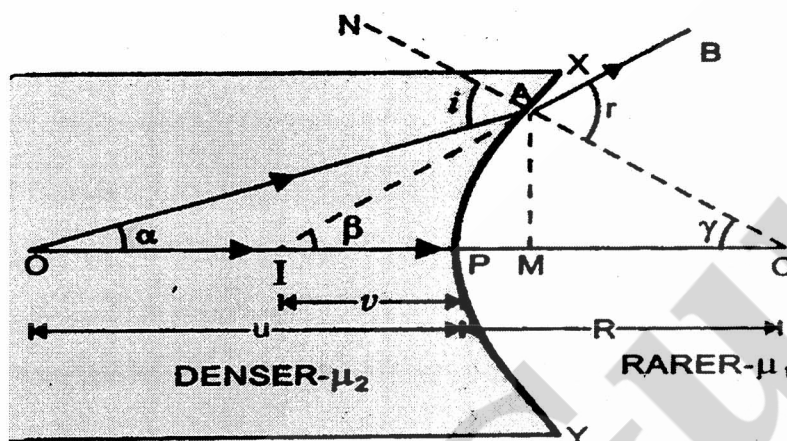
$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{-R}$$

$$\boxed{-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}}$$

This is the relation governing refraction from denser to rarer medium at a convex spherical refracting surface.

(2) At a concave spherical refracting surface

Let O be a point object lying on the principal axis of a spherical refracting surface XY, which is concave towards the rarer medium.



A ray of light OP falling normally on the surface goes undeviated along PC, whereas another ray OA, bends away from the normal and proceeds along AB after refraction.

The refracted rays PC and AB do not actually meet at any point, but appear to come from a point I. Thus I is the virtual image of the point object O.

$$\begin{array}{ll} \text{In } \triangle OAC & i = \gamma + \alpha \\ \text{and in } \triangle IAC & \gamma = \delta + \beta \end{array}$$

Putting these values of i and γ in the relation

$$\mu_2 i = \mu_1 \gamma$$

we get $\mu_2 (\gamma + \alpha) = \mu_1 (\delta + \beta)$

As angles are small, using $\theta \approx \tan \theta$, we get.

$$\mu_2 \left[\frac{AM}{MC} + \frac{AM}{MO} \right] = \mu_1 \left[\frac{AM}{MC} + \frac{AM}{MI} \right]$$

$$\text{or } \mu_2 \left[\frac{1}{MC} + \frac{1}{MO} \right] = \mu_1 \left[\frac{1}{MC} + \frac{1}{MI} \right]$$

Since aperture is small, M is close to P, so that—

$$\mu_2 \left[\frac{1}{PC} + \frac{1}{PO} \right] = \mu_1 \left[\frac{1}{PC} + \frac{1}{PI} \right]$$

$$\text{or } \frac{\mu_0}{PO} - \frac{\mu_1}{PI} = -\frac{\mu_2}{PC} + \frac{\mu_1}{PC} = -\frac{(\mu_2 - \mu_1)}{PC}$$

Using new Sign Conventions we get -

$$PO = -u, \quad PI = -v, \quad PC = R$$

$$\text{so } \frac{\mu_2}{-u} - \frac{\mu_1}{-v} = -\frac{(\mu_2 - \mu_1)}{R}$$

or

$$\boxed{-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}}$$

Lenses

A lense is a transparent refracting medium bound by two spherical surfaces or one spherical surface and the other plane surface.

Lenses are divided into two classes -

(i) Convex or Converging Lenses -

A convex lense is that, which is thicker in the middle and thin at the edges.

These are of three types -



Double-Convex
Lens



Plano-Convex
Lens

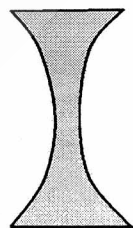


Concavo-Convex
Lens

(ii) Concave or Diverging Lenses -

A concave lense is thinner in the middle and thicker at the edges.

These Lenses are of three types -



Double-Concave
Lens



Plano-Concave
Lens



Convexo-Concave
Lens

Important terms related to Lenses -

(1) Principal axis -

It is defined as a straight line passing through the centres of curvature of two surfaces of a lens.

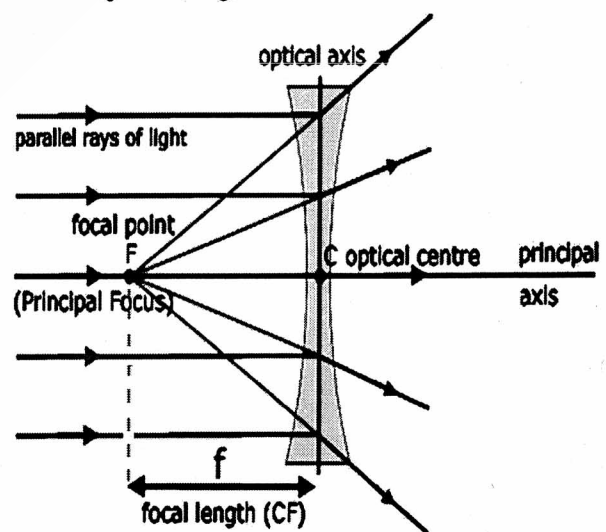
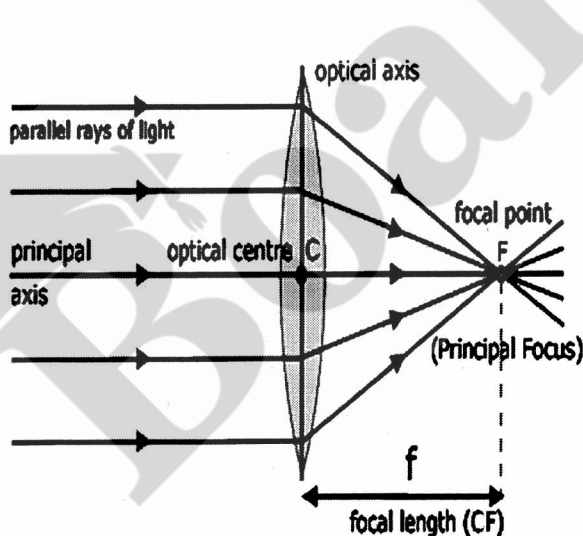
(2) Optical centre -

It is taken as a point lying on the principal axis so that a ray of light passing through it does not suffer any deviation from its path and it goes undeviated, without any lateral displacement.

(3) Principal Focus -

When a beam of light is incident on a lens in a direction parallel to the principal axis of the lens, the ray after refraction through the lens converge to (in case of convex lens) or appear to diverge (in case of concave lens) from a point on the principal axis. This point F is known as the principal focus of the lens.

$CF = f$ is principal focal length of the lens.



(4) Aperture -

Aperture of a lens is the effective diameter of its light transmitting area.

Lens Maker's Formula -

It is a relation that connects focal length of a lens to radius of curvature of the two surfaces of the lens and refractive index of the material of the lens.

The lens maker's formula is useful in designing lenses of desired focal length using suitable material and surfaces of suitable radius of curvature.

New Cartesian sign Conventions -

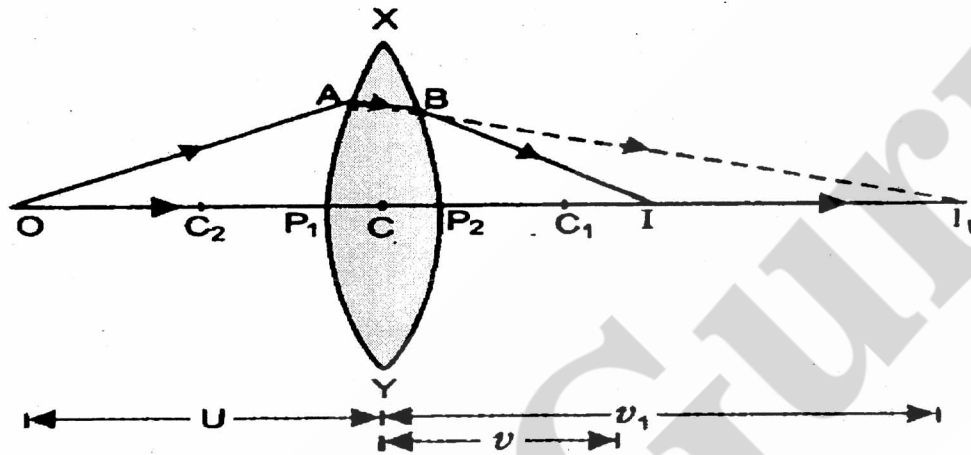
- (i) All distances are measured from the optical centre of the lens.
- (ii) All the distances measured in the direction of incidence of light are taken as positive, whereas all the distances measured in a direction opposite to the direction of incidence of light are taken as negative.
- (iii) For a convex lens, f is positive and for a concave lens, f is negative.

Assumptions -

- (i) The lens is thin so that distances measured from the poles of its surfaces can be taken as equal to the distances from the optical centre of the lens.
- (ii) The object consists only of a point lying on the principal axis of the lens.
- (iii) The aperture of the lens is small.
- (iv) The incident ray and refracted ray make small angles with the principal axis of the lens.

Derivation of Lens Maker's Formula for Convex Lens -

A convex lens is made up of two convex spherical refracting surfaces. The final image is formed after two refractions.



In figure P_1, P_2 are the poles, C_1, C_2 are the centre of curvature of two surfaces of a thin convex lens XY with optical centre at C .

Let μ_2 be the refractive index of the material of the lens and μ_1 be the refractive index of the rarer medium around the lens.

Consider a point lying on the principal axis of the lens. A ray of light starting from O and incident normally on the surface XP_1Y along OP_1 passes straight.

Another ray incident on XP_1Y along OA is refracted along AB . If the lens material were continuous and there were no second surface XP_2Y of the lens, the refracted ray AB would go straight meeting the first refracted ray at I_1 . Therefore I_1 would have been a real image of O formed after refraction at XP_1Y .

As the refraction occurs from rarer to denser medium so we can write -

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{R}} \quad \text{--- (1)}$$

(where $CI_1 \approx AI_1 = v_1$, $CC_1 \approx P_1C_1 = R_1$, $CO \approx P_1O = -u$)

Actually, the lense material is not continuous. Therefore the refracted ray AB suffers further refraction at B and emerges along BI, meeting actually the principal axis at I. Therefore I is the final real image of O, formed after refraction through the Convex lens.

For refraction at the second surface XP_2Y , we can regard I_1 as a virtual object, whose real image is formed at I.

Therefore $CI_1 \approx P_2I_1 = v_1$, Let $CI \approx P_2I = v$ and let R_2 be the radius of curvature of second surface of the lens.

As refraction is now taking place from denser to rarer medium, therefore -

$$\boxed{-\frac{\mu_2}{v_1} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R_2}} \quad \text{--- (2)}$$

Adding eq. (1) and (2) we get -

$$\frac{\mu_1}{-u} + \frac{\mu_1}{v} = (\mu_2 - \mu_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{or } \mu_1 \left[\frac{1}{v} - \frac{1}{u} \right] = (\mu_2 - \mu_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Put $\frac{\mu_2}{\mu_1} = {}^1\mu_2$ = refractive index of material with respect to surrounding material

When object on the left of lens is at infinity, image is formed at the principal focus of the lens.

$\therefore$ when $u = \infty$, $v = f$ = focal length of the lens.

$$\text{then } \frac{1}{f} - \frac{1}{\infty} = ({}^1\mu_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

or

$$\boxed{\frac{1}{f} = ({}^1\mu_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}$$

This is the lens maker's formula

Lens Maker's formula for Concave Lens -

It is the same as the lens maker's formula for convex lens -

$$\frac{1}{f} = (\mu_2 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Lens Formula -

It is a relation between focal length of a lens and distances of object and image from optical centre of the lens.

(A) Convex Lens -

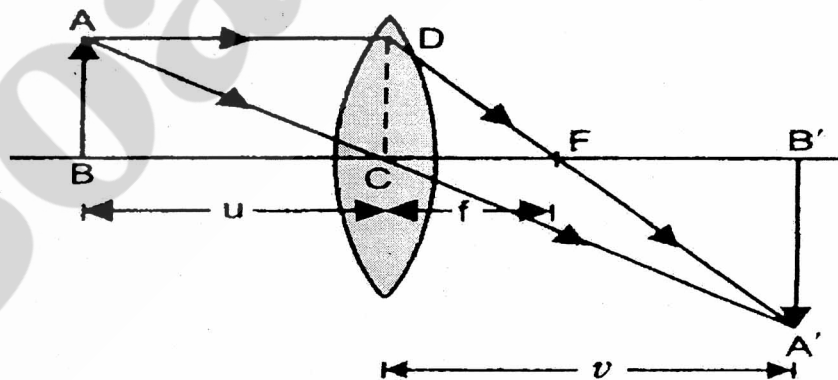
The image formed may be real or virtual depending on the position of the object.

(i) Real Image -

Let C be the optical centre and F be the principal focus of a convex lens of focal length $CF = f$.

AB is an object held perpendicular to the principal axis of the lens at a distance beyond focal length of the lens.

A real, magnified and inverted image $A'B'$ is formed.



As $\Delta A'B'C$ and ΔABC are similar, so

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \text{--- (1)}$$

again $\Delta A'B'F$ and ΔCDF are similar, so -

$$\frac{A'B'}{CD} = \frac{F'B'}{CF}$$

But $CD = AB$ so - $\frac{A'B'}{AB} = \frac{FB'}{CF}$ — (2)

From eq. (1) and (2) we get-

$$\frac{CB'}{CB} = \frac{FB'}{CF} = \frac{CB' - CF}{CF}$$

Using new Cartesian Sign Conventions -
Let $CB = -u$, $CB' = +v$, $CF = +f$

$$\therefore \frac{v}{-u} = \frac{v-f}{f}$$

$$vf = -uv + uf$$

$$\text{or } uv = uf - vf$$

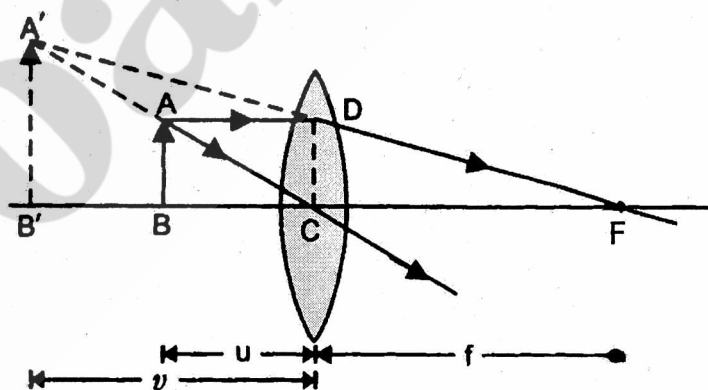
Divide both sides by uvf , we get -

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

This is the Lens formula.

(ii) Virtual Image -

When the object AB is held close to the lens, between C and F , the image $A'B'$ formed by convex lens is virtual, erect and magnified.



As $\triangle A'B'C$ and $\triangle ABC$ are similar, so that-

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \text{--- (1)}$$

Again as $\triangle A'B'F$ and $\triangle CDF$ are similar, so that-

$$\frac{A'B'}{CD} = \frac{B'F}{CF}$$

But $CD = AB$ So - $\frac{A'B'}{AB} = \frac{B'F}{CF}$ — (2)

from eq. (1) & (2) we get -

$$\frac{CB'}{CB} = \frac{B'F}{CF} = \frac{CB' + CF}{CF}$$

Using new sign Conventions, we put -
 $CB = -u$, $CB' = -v$, $CF = +f$

$$\therefore \frac{-v}{-u} = \frac{-v+f}{f} \quad \text{or} \quad uv = uf - vf$$

dividing both sides by uvf , we get -

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

* Lens formula is the same whether image is real or virtual.

Q. A convex lens of refractive index 1.5 has a focal length of 18 cm. in air. calculate the change in its focal length, when it is immersed in water of refractive index $4/3$.
CBSE 2008

Sol. Give that - ${}^a\mu_g = 1.5$, ${}^a\mu_w = 4/3$
 $f_1 = 18 \text{ cm.}$

$$\text{Now } {}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1.5}{4/3} = \frac{4.5}{4}$$

$$\text{As } \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\therefore \frac{f_2}{f_1} = \frac{({}^a\mu_g - 1)}{({}^w\mu_g - 1)} = \frac{(1.5 - 1)}{(\frac{4.5}{4} - 1)}$$

$$\frac{f_2}{f_1} = 4 \quad \Rightarrow \quad f_2 = 4f_1$$

$$f_2 = 4 \times 18 = 72 \text{ cm.}$$

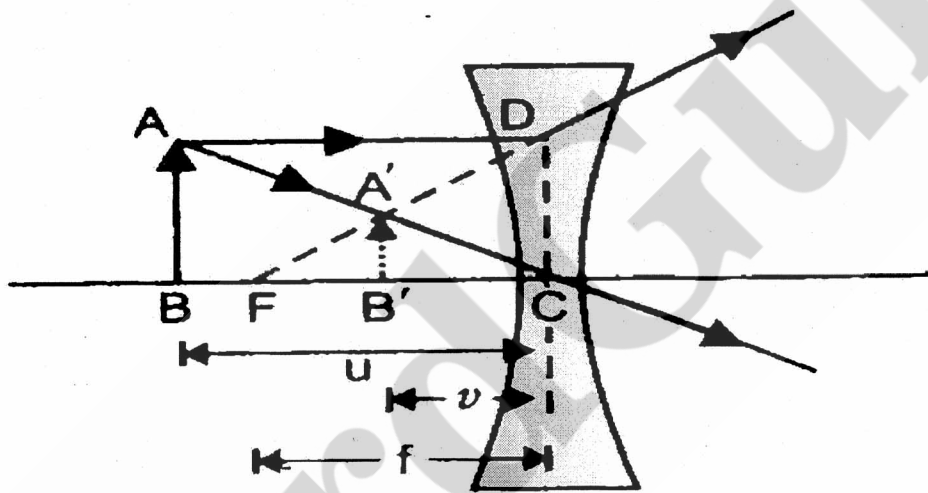
$$\text{Change in focal length} = 72 - 18 = 54 \text{ cm.}$$

(B) Concave Lens -

In this case, the image formed is always virtual, whatever be the position of the object.

Let C be the optical centre and F be the principal focus of a concave lens of focal length f .

AB is an object held perpendicular to the principal axis of the lens. A virtual, erect and smaller image $A'B'$ is formed due to refraction through concave lens.



As $\triangle A'B'C$ and $\triangle ABC$ are similar, so that-

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \quad \text{--- (1)}$$

Again as $\triangle A'B'F$ and $\triangle CDF$ are similar-

$$\frac{A'B'}{CD} = \frac{B'F}{CF}$$

But $CD = AB$, so that $\frac{A'B'}{AB} = \frac{B'F}{CF} \quad \text{--- (2)}$

from equation (1) and (2)

$$\frac{CB'}{CB} = \frac{B'F}{CF} = \frac{CF - CB'}{CF}$$

Using New Cartesian Sign Conventions, we get -
 $CB = -u$, $CB' = -v$, $CF = -f$

$$-\frac{v}{u} = \frac{-f + v}{-f}$$

$$vf = uf - uv$$

$$uv = uf - vf$$

divide both sides by uvf , we get

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

It is the required Lens formula

* Note that lens formula is same whether the lens is Convex or Concave

Q.
CBSE
2007

A double convex lens of glass of refractive index 1.5 has its both surfaces of equal radius of curvature of 30 cm each. An object of height 5 cm. is placed at a distance of 12.5 cm. from the lens. Calculate the size of the image formed.

Sol. Given that - $R_1 = 30 \text{ cm}$, $R_2 = -30 \text{ cm}$, $\mu = 1.5$
Using Lens maker's formula

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = (1.5 - 1) \left(\frac{1}{30} + \frac{1}{30} \right)$$

$$\Rightarrow \frac{1}{f} = \frac{0.5}{15} \Rightarrow f = 30 \text{ cm.}$$

Given, $u = -12.5 \text{ cm}$

$\therefore$ Using lens equation

$$\Rightarrow \frac{1}{30} = \frac{1}{v} + \frac{1}{12.5}$$

$$\Rightarrow \frac{1}{v} = \frac{12.5 - 30}{12.5 \times 30}$$

$$\Rightarrow v = -\frac{150}{7} \text{ cm.}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{12.5}$$

$$= -\frac{7}{150}$$

But magnification (m) = $\frac{I}{O} = \frac{v}{u}$

$$\text{or } I = \frac{v}{u} \times O = \frac{(-150/7)}{-12.5} \times 5$$

$$I = \frac{60}{7} \approx 8.55 \text{ cm.}$$

Linear Magnification Produced by a Lens (m)-

The linear magnification produced by a lens is defined as the ratio of the size of the image (I) to the size of the object (O).

It is represented by m.

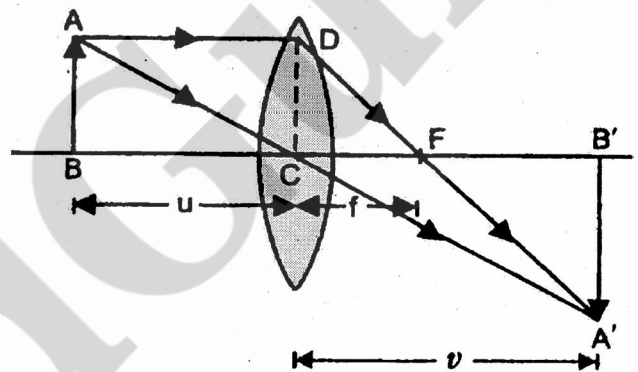
$$m = \frac{\text{Size of Image (A'B')}}{\text{Size of object (AB)}} = \frac{I}{O}$$

In a convex lens, when image formed is real, as shown in figure, then-

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

$$m = -\frac{I}{O} = \frac{+v}{-u}$$

or $m = \frac{I}{O} = \frac{v}{u}$

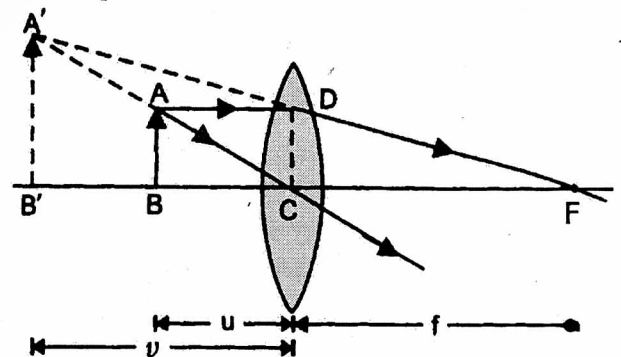


When image formed is virtual as shown in figure, then

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

or $\frac{I}{O} = \frac{-v}{-u}$

or $m = \frac{I}{O} = \frac{v}{u}$



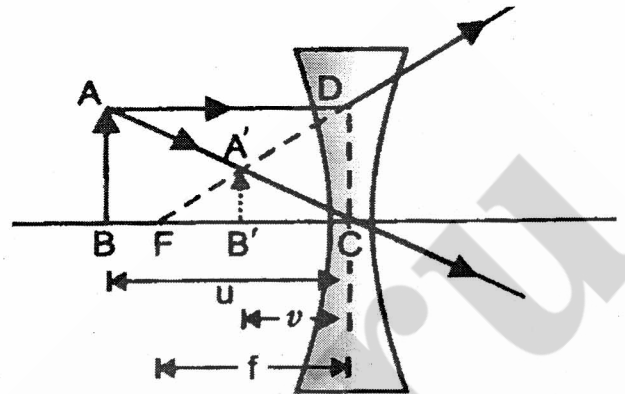
Thus in a convex lens, linear magnification is positive, when image formed is virtual and linear magnification is negative, when image formed is real.

In a concave lens, image formed is always virtual, as shown in figure.

$$\text{So } \frac{A'B'}{AB} = \frac{CB'}{CB}$$

$$\frac{I}{O} = \frac{-V}{-u}$$

$$\text{or } \boxed{m = \frac{I}{O} = \frac{V}{u}}$$



* Hence the linear magnification in case of a concave lens is always positive.

Power of a Lens - (P)

The ability of the lens to converge a beam of light falling on the lens is called power of lens.

It is measured as the reciprocal of focal length of the lens.

$$\boxed{P = \frac{1}{f}}$$

According to lens maker's formula -

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

or

$$\boxed{P = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}$$

For a converging lens, power is positive and for a diverging lens, power is negative.

* The SI unit of power is dioptre (D).

1 dioptre -

It is the power of a lens of focal length one meter.

* when f is in cm.

$$\boxed{P = \frac{1}{f/100} = \frac{100}{f}}$$

Q. Find the radius of curvature of convex surface of a plano convex lens, whose focal length is 0.3m and $\mu = 1.5$.

Sol. $R_1 = \infty$, $R_2 = ?$, $f = 0.3\text{m}$, $\mu = 1.5$

$$\text{from } \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{0.3} = (1.5 - 1) \left(\frac{1}{\infty} - \frac{1}{R_2} \right)$$

$$-\frac{1}{R_2} = \frac{1}{0.3 \times 0.5} = \frac{100}{15}$$

$$R_2 = -0.15\text{m}.$$

Q. A convex lens made up of glass of refractive index 1.5 is dipped in turn, in

(a) a medium of refractive index 1.65

(b) a medium of refractive index 1.33

will it behave as a converging or a diverging lens in the two cases?

Sol. From Lens maker's formula -

$$\frac{1}{f} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{or } \frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

As $\mu_g < \mu_m$ for the first medium of refractive index 1.65.

And $\mu_g > \mu_m$ for the second medium of refractive index 1.33.

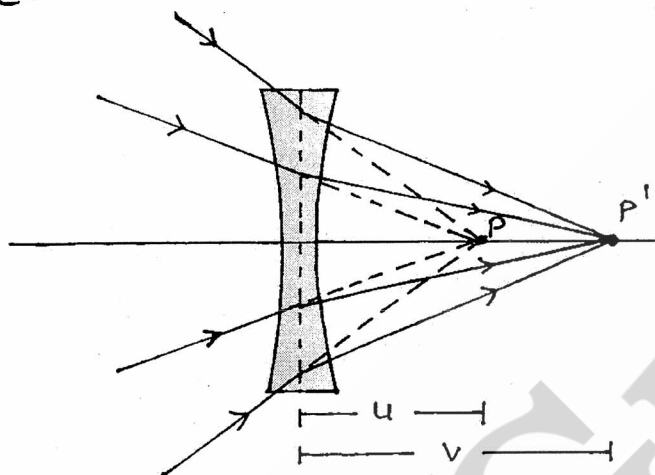
Hence, the value of focal length f will be negative in the first medium, so the convex lens behaves as the diverging lens for the first medium.

The value of focal length f will be positive in the second medium, so the convex lens behaves as the converging lens for second medium.

Q.
CBSE
2011

A beam of light converges at a point P. A Concave lens of focal length 16 cm. is placed in the path of this beam 12 cm. from P. Draw a ray diagram and find the location of the point at which the beam would now converge.

Sol.



$$\begin{aligned} u &= +12 \text{ cm} \\ v &= +48 \text{ cm.} \\ f &= -16 \text{ cm.} \end{aligned}$$

Using lens formula -

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$-\frac{1}{16} = \frac{1}{v} - \frac{1}{12} \Rightarrow \frac{1}{v} = \frac{1}{12} - \frac{1}{16}$$

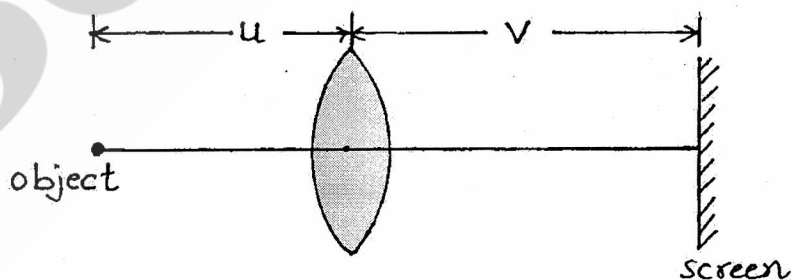
$$\frac{1}{v} = \frac{1}{48} \Rightarrow v = +48 \text{ cm.}$$

The image of virtual object at P forms at P' which is at a distance 48 cm from the lens.

Q.
CBSE
2008

A screen is placed 100 cm. from an object. The image of the object on the screen is formed by a convex lens at two different locations, separated by 20 cm. Calculate the focal length of the lens used

Sol.



$\therefore$ object and screen are 100 cm. from each other.

$$\therefore u + v = 100 \text{ cm.} \quad \text{--- (1)}$$

$$\therefore \text{Due to reversible image of light } v - u = 20 \text{ cm.} \quad \text{--- (2)}$$

From equation ① and ② we get -

$$v = 60 \text{ cm.}, \quad u = 40 \text{ cm.}$$

Now using lens formula -

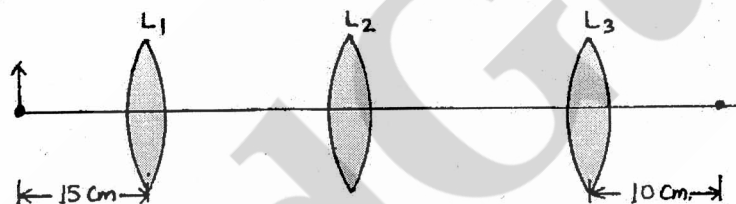
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{60} - \left(\frac{1}{-40}\right)$$

$$\frac{1}{f} = \frac{1}{60} + \frac{1}{40} \Rightarrow \frac{1}{f} = \frac{1}{24}$$

$$\Rightarrow f = 24 \text{ cm.}$$

Q.
CBSE
2012

You are given three Lenses L_1 , L_2 and L_3 each of focal length 10 cm. An object is kept at 15 cm in front of L_1 as shown. The final real image is formed at the focus of L_3 . Find the separation between L_1 , L_2 and L_3 .



Sol. For Lens L_1 $\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$

Here $u = -15 \text{ cm}$ and $f = +10 \text{ cm}$ so that

$$\frac{1}{10} = \frac{1}{v_1} + \frac{1}{15} \Rightarrow v_1 = 30 \text{ cm}$$

Hence Distance of image from L_1 is. $v_1 = 30 \text{ cm}$.

For lens L_3 $\frac{1}{f_3} = \frac{1}{v_3} - \frac{1}{u_3}$

Here $v_3 = +10 \text{ cm}$ and $f_3 = +10 \text{ cm}$.

so that $\frac{1}{10} = \frac{1}{10} - \frac{1}{u_3}$

$$\frac{1}{u_3} = 0 \Rightarrow u_3 = \infty$$

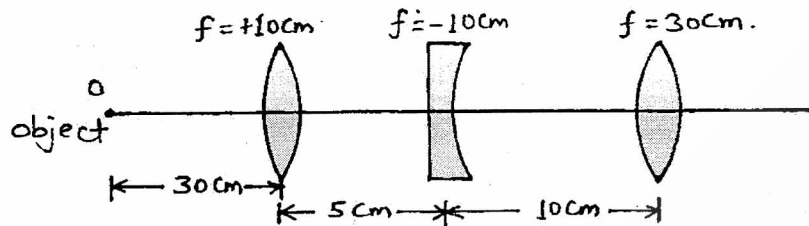
The refracted rays from Lens L_2 becomes parallel to principal axis. It is possible only when image formed by L_1 lies at first focus of L_2 i.e. at a distance of 10 cm from L_2 .

$\therefore$ separation between L_1 and $L_2 = 30 + 10 = 40 \text{ cm}$.

The distance between L_2 and L_3 may take any value.

Q.
CBSE
2011

Find the position of the image formed of the object O by the lens combination given in the figure.



Sol. For lens of focal length 10 cm - $f = +10\text{cm}$, $u = -30\text{cm}$.
Using lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad \frac{1}{10} = \frac{1}{v} - \frac{1}{(-30)}$$

$$\frac{1}{v} = \frac{1}{15} \quad \Rightarrow \quad v = 15\text{cm}$$

The image formed by first lens acts as an virtual object for plano-concave lens.

For plano-concave lens -

$$u = +10\text{cm}, \quad f = -10\text{cm}$$

Hence by using lens formula -

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad -\frac{1}{10} = \frac{1}{v} - \frac{1}{10}$$

$$\frac{1}{v} = 0 \quad \Rightarrow \quad v = \infty$$

The refracted ray becomes parallel to principal axis for convex lens of focal length 30 cm.

For the second Convex lens -

$$u = -\infty, \quad f = 30\text{cm}.$$

by using the lens formula -

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad \frac{1}{30} = \frac{1}{v} - \frac{1}{(-\infty)}$$

$$\frac{1}{v} = \frac{1}{30} \quad \Rightarrow \quad v = 30\text{cm}.$$

So the final image forms at a distance of 30cm. from the second Convex lens on the other side of it.

Combination of Lenses in contact -

In various optical instruments, two or more lenses are combined to

- (i) Increase the magnification of the image.
- (ii) make the final image erect w.r.t. the object.
- (iii) reduce certain aberrations (i.e. defects of images by single lens)

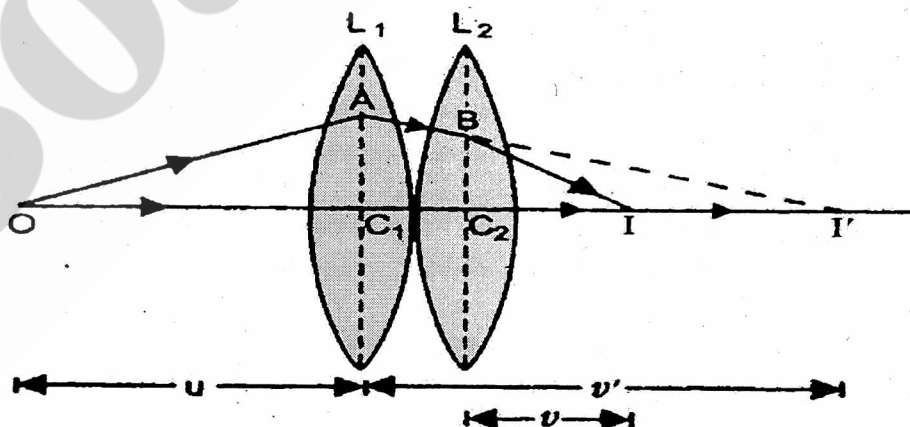
To obtain the position, size, and nature of the final image produced by a combination of lenses, we first find the image of the object formed by the first lens.

This image acts as an object for the second lens and we locate its image. The image formed by the second lens serves as the object for the third lens and so on.

Focal length of Equivalent Lens -

(a) Both Lenses are Convex -

Let C_1, C_2 be the optical centres of two thin convex lenses L_1 and L_2 held co-axially in contact with each other in air. Suppose f_1 and f_2 are their respective focal lengths.



Let a point object O be placed on the common principal axis at a distance $OC_1 = u$. The lens L_1 alone would form its image at I' where $C_1I' = v'$.

From the lens formula -

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \quad \text{--- (1)}$$

I' would serve as a virtual object for lens L_2 , which forms a final image I at distance $C_2I = v$.

As the lenses are thin, therefore, for the lens L_2

$$u = C_2I' \approx C_1I' = v'$$

from the lens formula for L_2 -

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \quad \text{--- (2)}$$

Adding eq. (1) and (2), we get -

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{--- (3)}$$

Let the two lenses be replaced by a single lens of focal length F , which forms image I at distance v of an object at distance u from the lens.
For this lens -

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} \quad \text{--- (4)}$$

from eq. (3) and (4), we get -

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or

$$F = \frac{f_1 f_2}{f_1 + f_2}$$

(b) One lens is convex and the other is concave

If f_1 is the focal length of convex lens and f_2 is focal length of concave lens, then their equivalent focal length F would be given as -

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{-f_2} = \frac{1}{f_1} - \frac{1}{f_2}$$

or

$$F = \frac{f_1 f_2}{f_2 - f_1}$$

There are three cases arise -

Case 1 :

If $f_1 = f_2$, then $F = \infty$.

The combination would behave like a plane glass plate.

Case 2 :

If $f_1 > f_2$, then F is negative.

Therefore the combination behave as a concave lens, when focal length of convex lens is larger.

Case 3 :

If $f_1 < f_2$ then F is positive.

Therefore the combination would behave as a convex lens, when focal length of convex lens is smaller.

Note -

- (i) If the lenses of focal lengths f_1 and f_2 are separated by a finite distance d , the focal length F of the equivalent lense is given by -

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

- (ii) Total Power of any number of Lenses in contact is equal to the algebraic sum of the powers of individual lenses.

$$P = P_1 + P_2 + \dots + P_n$$

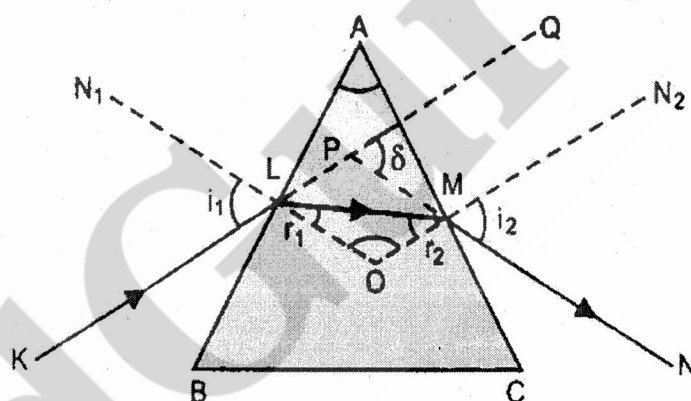
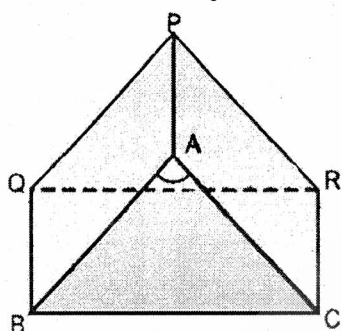
- * If the sum of the terms on right side is positive, the system of lenses would behaves like convex.
- * If the sum of the terms on right side is negative, the system of lenses would behave as concave.

Optical Instruments

Prism -

A prism is a portion of a transparent medium bounded by two plane faces inclined to each other at a suitable angle.

In figure ABQP and ACRP are the two refracting faces. Angle A between them is the refracting angle or angle of prism.



The line AP where the two refracting faces meet is called the refracting edge of the prism.

A section ABC of the prism made by a plane at right angles to the refracting edge of the prism is called Principal section of the prism.

Refraction through a Prism -

A ray of light suffers two refractions on passing through a prism and hence deviates through a certain angle from its original path.

(i) Calculation of angle of deviation -

In figure ABC is principal section of a prism with angle of prism = A .

A ray of light KL is incident on the face AB of the prism at L at i_1 . It is refracted along LM at r_1 , bending towards the normal N_1O .

The refracted ray LM is incident at $\angle r_2$ on face AC of the prism. It bends away from normal N_2O and emerges along MN at $\angle i_2$.

In passing through the prism ray KL suffers two refractions and has turned through an $\angle QPN = \delta$, which is the angle of deviation.

$$\begin{aligned} \text{In } \triangle PLM, \quad \delta &= \angle PLM + \angle PML \\ \delta &= (i_1 - r_1) + (i_2 - r_2) \\ \delta &= (i_1 + i_2) - (r_1 + r_2) \quad \text{--- (1)} \end{aligned}$$

$$\text{In } \triangle OLM, \quad \angle O + r_1 + r_2 = 180^\circ \quad \text{--- (2)}$$

$$\begin{aligned} \text{In quadrilateral ALOM,} \\ \text{As } \angle L + \angle M = 180^\circ \quad (\because \text{each angle is } 90^\circ) \\ \therefore \quad A + \angle O = 180^\circ \end{aligned}$$

$$\text{Using equation (2), } \angle O + r_1 + r_2 = A + \angle O$$

$$\boxed{r_1 + r_2 = A} \quad \text{--- (3)}$$

$$\text{Put in eq. (1)} \quad \boxed{\delta = (i_1 + i_2) - A} \quad \text{--- (4)}$$

If μ is refractive index of the material of the prism, w.r.t. air then according to Snell's Law -

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{i_1}{r_1} \quad (\text{When angles are small})$$

$$\therefore \quad i_1 = \mu r_1 \quad \text{Similarly} \quad i_2 = \mu r_2$$

Putting in equation (4) we get -

$$\delta = (\mu r_1 + \mu r_2) - A$$

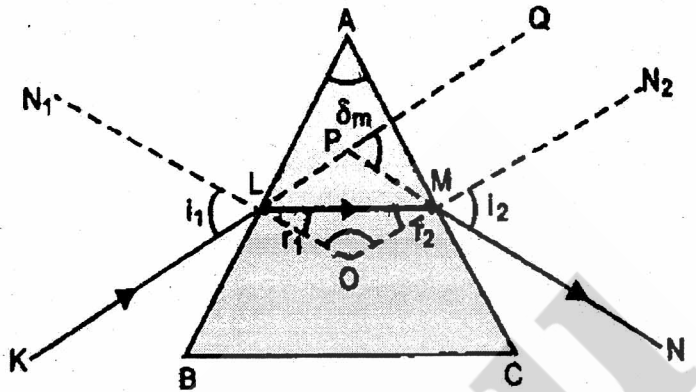
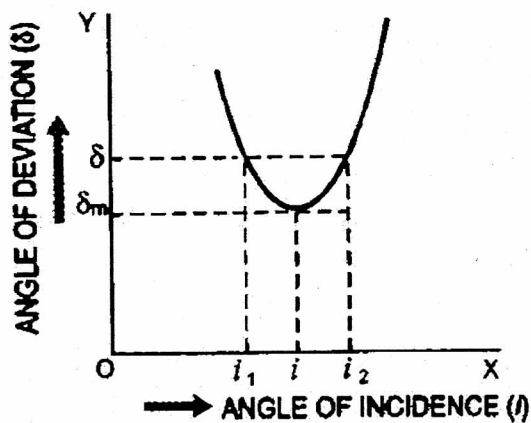
$$\delta = \mu(r_1 + r_2) - A$$

by using eq. (3) we get -

$$\delta = \mu A - A$$

$$\boxed{\delta = (\mu - 1)A} \quad \text{--- (5)}$$

This is the angle through which a ray deviates on passing through a thin prism of small refracting angle A



In the minimum deviation position, $i_1 = i_2 = i$

As $i_1 = i_2$, therefore $r_1 = r_2 = r$

then from equation (3),

$$r + r = A$$

$\therefore$

$$r = \frac{A}{2}$$

From eq. (4),

$$\delta_m = i + i - A$$

$$\text{or } A + \delta_m = 2i \quad \text{or}$$

$$i = \frac{A + \delta_m}{2}$$

If μ is refractive index of the prism w.r.t. air then by Snell's Law,

$$\mu = \frac{\sin i}{\sin r}$$

$\Rightarrow$

$$\mu = \frac{\sin \left(\frac{A + \delta_m}{2} \right)}{\sin A/2}$$

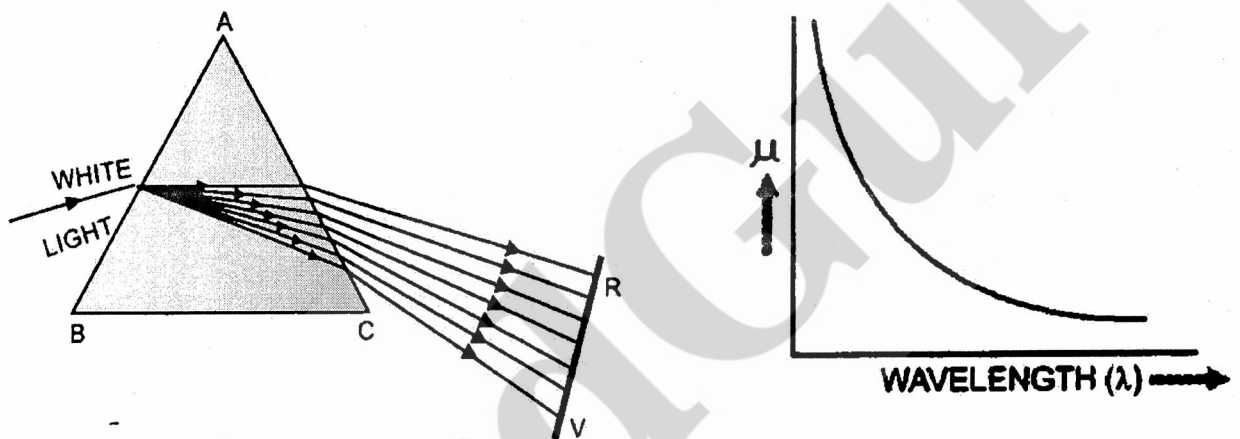
This relation is called Prism formula.

* It is used for accurate determination of refractive index of a transparent medium of which the prism is made.

Dispersion of Light -

It is the phenomenon of splitting of a beam of white light into its constituent colours on passing through a prism. The band of seven colours so obtained is called the (visible) spectrum.

The order of colours from the lower end of the spectrum is violet (V), Indigo (I), Blue (B), Green (G), Yellow (Y), Orange (O) and Red (R) and can be given by the word VIBGYOR.



Cause of Dispersion -

Each colour has its own wavelength (λ). According to the Cauchy's formula, refractive index (μ) of a material depends on wavelength (λ) of light as

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \dots \quad (\text{where } A, B, C, \dots \text{ are constants.})$$

$\therefore \mu$ of material of prism is different for different colours / wavelengths.

As Angle of deviation is

$$\delta = (\mu - 1)A$$

As μ of prism is different for different colours, therefore different colours deviate through different angles on passing through the prism.

This is the cause of dispersion.

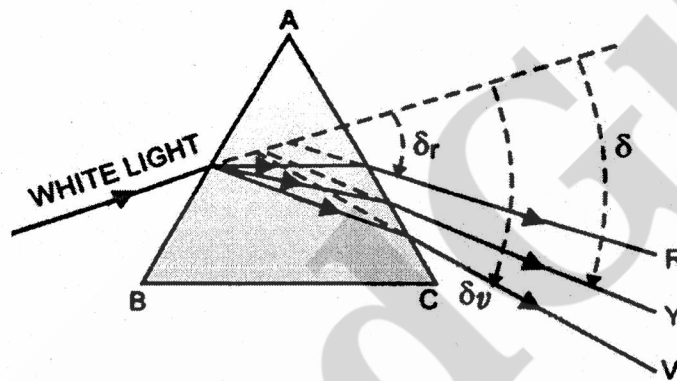
As $\lambda_{\text{violet}} < \lambda_{\text{red}}$ therefore $\mu_{\text{violet}} > \mu_{\text{red}}$

Hence $\delta_{\text{violet}} > \delta_{\text{red}}$

i.e. maximum deviation for violet colour and minimum deviation is for red colour.

Angular Dispersion -

Angular dispersion produced by a prism for white light is the difference in the angle of deviation of two extreme colours i.e. violet and red colours.



$$\text{Angular dispersion} = \delta_v - \delta_r$$

As $\delta_v = (\mu_v - 1)A$
and $\delta_r = (\mu_r - 1)A$

Hence $\delta_v - \delta_r = (\mu_v - 1)A - (\mu_r - 1)A$

or

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

Dispersive Power (ω) -

It is defined as the ratio of angular dispersion to the mean deviation produced by the prism. It is represented by ω .

$$\text{dispersive Power} = \frac{\text{Angular dispersion}}{\text{mean deviation}}$$

$$\omega = \frac{\delta_V - \delta_R}{\delta}$$

$$\omega = \frac{(\mu_V - \mu_R)A}{(\mu - 1)A} = \frac{\mu_V - \mu_R}{\mu - 1}$$

Q.
Delhi
2003

A ray of light passing through an equilateral triangular glass from air undergoes minimum deviation when angle of incidence is $3/4$ of the angle of prism. Calculate the speed of light in the prism.

Sol.

When prism is adjusted at angle of minimum deviation then-

$$\mu = \frac{\sin i}{\sin r}$$

Where $r = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$

and $i = \frac{3}{4} A = \frac{3}{4} \times 60 = 45^\circ$

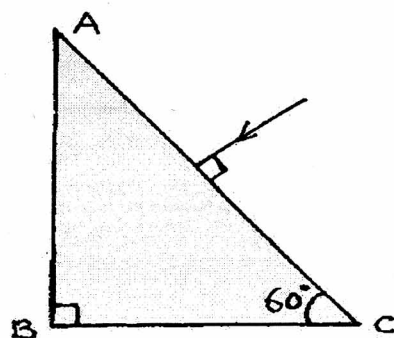
$$\Rightarrow \mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$

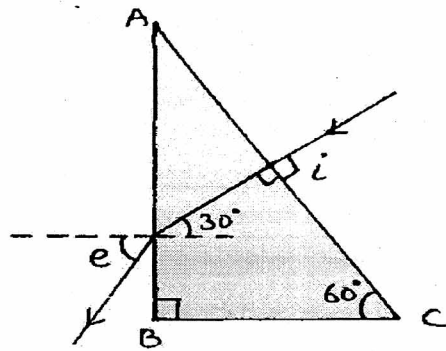
But refractive index $\mu = \frac{c}{v}$

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{\sqrt{2}} = 2.1 \times 10^8 \text{ m/s}$$

Q.
CBSE
2012

Trace the path of a ray of light passing through a glass prism ABC as shown in figure. If the refractive index of the glass is $\sqrt{3}$. Find out the value of the angle of emergence from the prism.



Sol.

Given that -
 $\mu_g = \sqrt{3}$

Since $i = 0$, At the interface AC, we have according to Snell's Law -

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a}$$

$$\sin r = \frac{\mu_a}{\mu_g} \sin i = 0 \quad (\text{as } i = 0)$$

Hence $r = 0$

This ray passes unrefracted at AC interface and reaches AB interface. By the diagram we can see angle of incidence becomes 30° .

Thus applying Snell's Law -

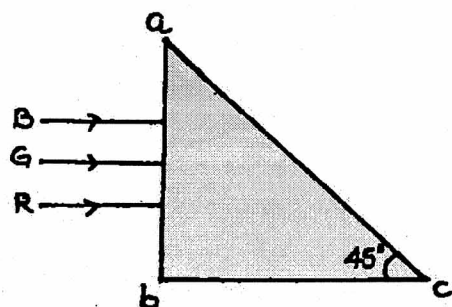
$$\frac{\sin 30^\circ}{\sin e} = \frac{\mu_a}{\mu_g} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sin e = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}$$

Thus $e = 60^\circ$ (angle of emergence)

Q.
 Delhi
 2009.

Three light rays red (R), green (G) and blue (B) are incident on a right angled prism abc at face ab. The refractive indices of the material of the prism for red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. Out of the three which colour ray will emerge out of face ac? Trace the path of these rays after passing through face ab.



Sol.

By geometry, angle of incidence (i) of all three rays is 45° at face ac .

Light suffer total internal reflection for which this angle of incidence is greater than critical angle.

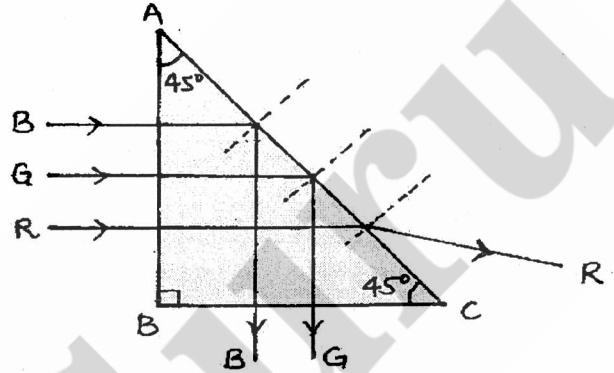
$$i > i_c$$

$$\Rightarrow \sin i > \sin i_c$$

$$\sin 45^\circ > \sin i_c$$

$$\frac{1}{\sin 45^\circ} < \frac{1}{\sin i_c}$$

$$\text{Hence } \sqrt{2} < \mu$$



Total internal reflection takes place on ac for rays $\mu > \sqrt{2} = 1.414$.

Hence green and blue suffer total internal reflection whereas red undergo refraction, and red colour ray will emerge out of face ac .

Scattering of Light.

When sunlight travels through the atmosphere, it gets scattered by the large number of molecules present in the atmosphere, this change in direction of the light is called scattering.

According to the Rayleigh, when size of scatterer is much smaller than wavelength of light, then intensity of scattered light (I_s) is inversely proportional to the fourth power of the wavelength of incident light (λ).

$$I_s \propto \frac{1}{\lambda^4}$$

* Rayleigh scattering is valid only when size of scatterer is much smaller than wavelength of light

But when size of scatterer is much greater than wavelength of light then Rayleigh scattering is not valid, and all wavelengths are scattered nearly equally.

Applications of Scattering of Light-

(1) Blue Colour of Sky-

Light from the sun when, while travelling through earth's atmosphere, gets scattered by large number of molecules in the earth's atmosphere.

For upper layers of atmosphere $x \ll \lambda$, hence Rayleigh scattering is valid. The intensity of scattered light varies inversely as the fourth power of wavelength of light. A blue colour has a shorter wavelength than red, therefore blue colour is scattered much more and sky looks blue.

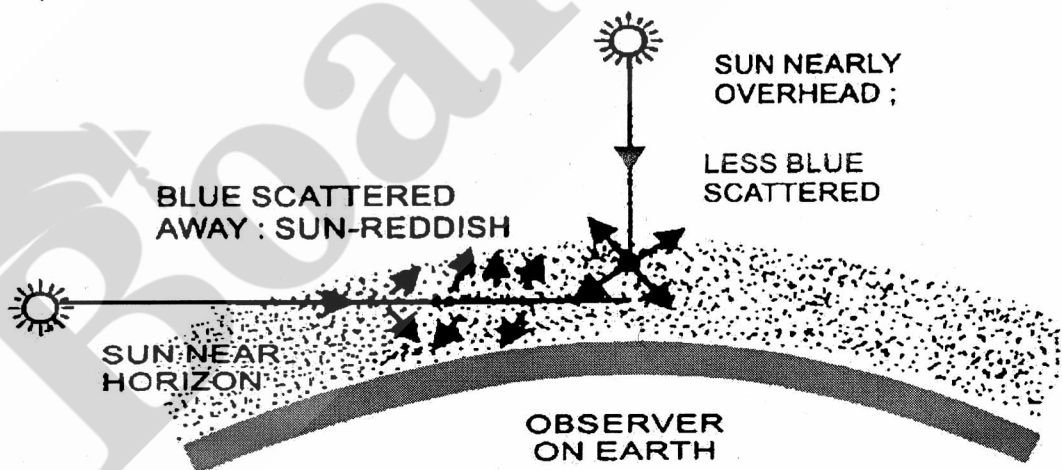
(2) White Colour of clouds-

The clouds are at much lower height. The lower part of atmosphere contains large dust particles, water droplets, ice particles etc. In this case size of scatterer $x \gg \lambda$. Therefore all wavelengths are scattered nearly equally. All colours scattered equally merge to give us the sensation of white. Hence clouds generally appears white.

(3) The sun looks reddish at the time of sun rise and sun set -

At the time of sun rise and sun set, the sun is near the horizon. The rays from the sun have to travel a large part of the atmosphere.

As $\lambda_b \ll \lambda_r$ and intensity of scattered light $I_s \propto \frac{1}{\lambda^4}$, therefore most of the blue light is scattered away. Only red colour which is least scattered enters our eyes and appears to come from the sun. Hence sun looks red both the time of sun rise and sun set.



(4) Danger signals are red -

As wavelength of red colour is large and $I_s \propto \frac{1}{\lambda^4}$, therefore, red colour is least scattered and can be seen from large distance.

Rainbow -

A rainbow is a spectrum of sun's light in nature. It occurs in the form of concentric coloured circular arcs in the sky, when the sun shines on rain drops during or after a shower.

There are two kinds of rainbow:

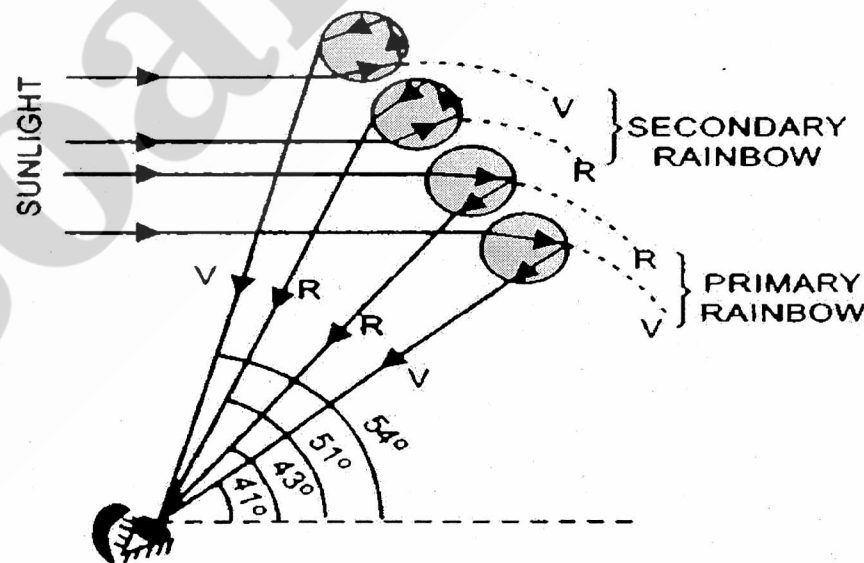
- (i) Primary Rainbow (ii) Secondary Rainbow

(i) Primary Rainbow -

The inner of the two bows seen is the primary rainbow. It is brighter of the two and subtend an angle of 42° at the eye of the observer

It shows all the colours of the solar spectrum with red colour on the outside (at an angle of 43°) and violet colour on the inside (at an angle of 41°)

The primary rainbow is formed by the rays which undergo one internal reflection (not total internal reflection) and two refractions before emerging finally from the water droplets.



(ii) Secondary Rainbow -

The outer of the two bows is called secondary rainbow. It is not as bright as the primary rainbow.

It subtends an angle of about 52.5° at the eye of the observer. It also shows all the colours of the solar spectrum, but in reverse order.

Hence it shows red on the inner edge making an angle of 51° and violet on the outer edge making an angle of 54° .

Secondary rainbow is formed by the rays which have undergone two internal reflections and two refraction before emerging finally from the water droplets.

The Eye -

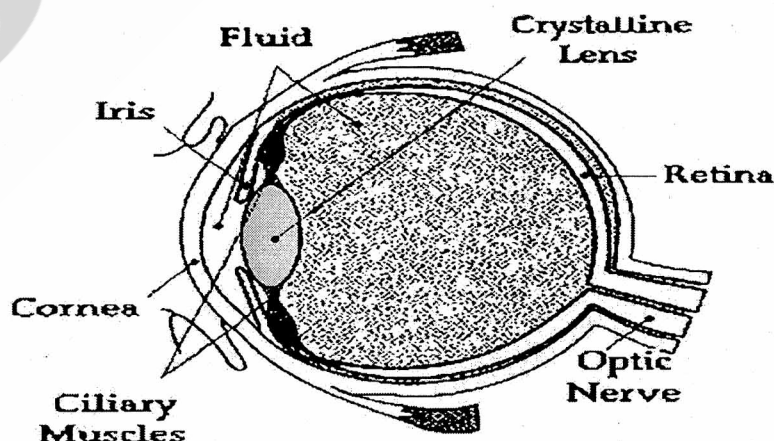
The essential parts of the eye are:

Lens - There is a crystalline lens in the eye. This is held in position by ciliary muscles.

Iris and pupil - In front of the crystalline lens there is a contractable diaphragm with a circular aperture, near the centre. This diaphragm is called Iris and the circular aperture is called pupil of the eye.

The function of Iris is to adjust and allow suitable quantity of light into the eye.

Cornea - The front transparent part of the hard outer surface of the eye ball is called Cornea. It protects the inner delicate parts of the eye.



Retina - The image of an external object is formed by the crystalline lens on the retina. The retina is a film of nerve fibers covering the curved back surface of the eye.

The retina contains rods and cones, which sense light intensity and colour respectively. They transmit electrical signal via the optic nerve to the brain, which finally process this information.

Aqueous humour and Vitreous humour - The eye lens divides the interior of the eye into two chambers. The anterior chamber between cornea and the lens contains a watery fluid called aqueous humour, and the posterior chamber between lens and the retina contains a transparent jelly called vitreous humour.

Important Points -

- (i) The shape i.e. the curvature and the focal length of the crystalline lens can be modified by the ciliary muscles. The average refractive index μ of crystalline lens is 1.43 and average refractive index of aqueous humour and vitreous humour is 1.336.
- (ii) The least sensitive point on the retina is called the blind spot. There is another spot at about the centre of the retina, which is most sensitive to light. This is called yellow spot.
- (iii) The ability of the eye to observe distinctly the objects situated at widely different distance from the eye is called accommodation of the eye. It becomes possible by changing the curvature of lens and hence its focal length by a change in tension in the ciliary muscles holding the lens.

(iv) The most distant point which an eye can observe clearly is called Far Point (F) of the eye. For a normal eye, F lies at infinity.

(v) If the object is too close to the eye, the lens cannot curve enough to focus the image on the retina. Therefore the image is blurred.

The closest distance for which lens can focus light on the retina is called the least distance of distinct vision of the distance of Near point (N) of the eye. For a normal eye, the least distance of distinct vision is $d = 25 \text{ cm}$.

Defects of Vision-

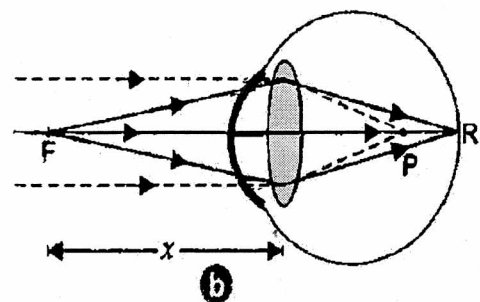
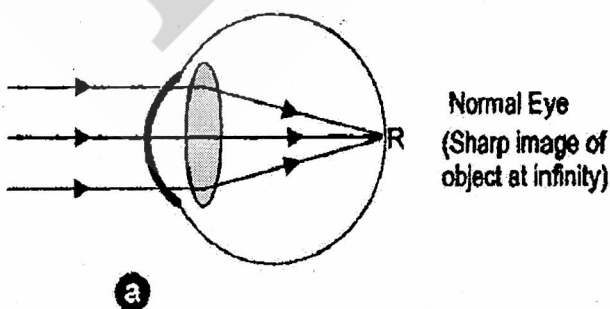
Some common optical defect of the eye are:

- (1) Myopia or Short sightedness
- (2) Hypermetropia or long sightedness
- (3) Presbyopia
- (4) Astigmatism

(1) Myopia or Short Sightedness-

Myopia or short sightedness is that defect of human eye by which the eye can see clearly the objects lying near it, but the far off objects cannot be seen distinctly.

Hence for a myopic eye, the far point shift towards the eye. It is no longer at infinity. It is at F



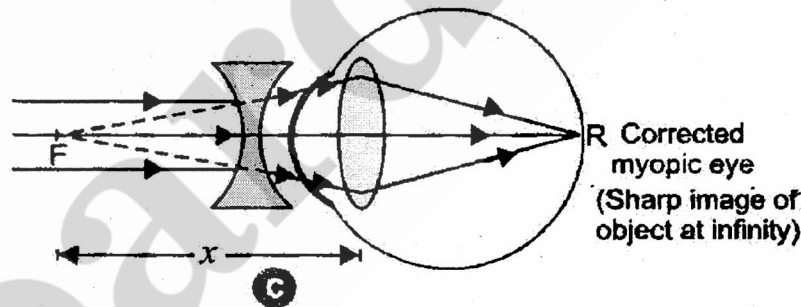
The myopic eye focusses the parallel rays from infinity at P in front of the retina. However, the myopic eye focusses rays from a point F on the retina. Thus F is the far point of the myopic eye. It cannot see clearly beyond F.

The two possible causes of this defect are:

- (i) Increase in size of eye ball, i.e. distance of retina from the eye lens increases.
- (ii) Decrease in focal length of the eye lens, when the eye is fully relaxed.

Correction-

To correct a myopic eye the person has to use spectacles with a Concave lens of suitable focal length. Hence the parallel rays of light from infinity after refraction through the Concave lens, should appear to come from F (the far point of the myopic eye).



Let x be the distance of far point of the myopic eye and f be the focal length of concave lens to be used.

Now for concave lens $u = \infty$, $v = -x$

As $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ or $\frac{1}{f} = -\frac{1}{x} - \frac{1}{\infty}$

or

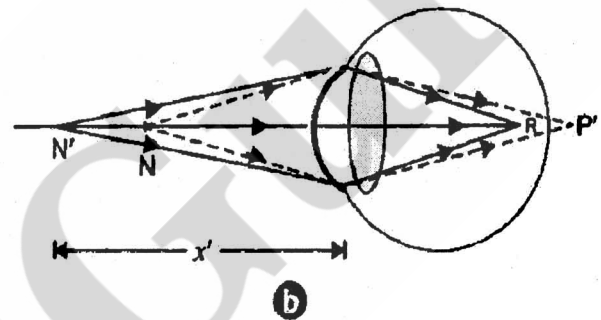
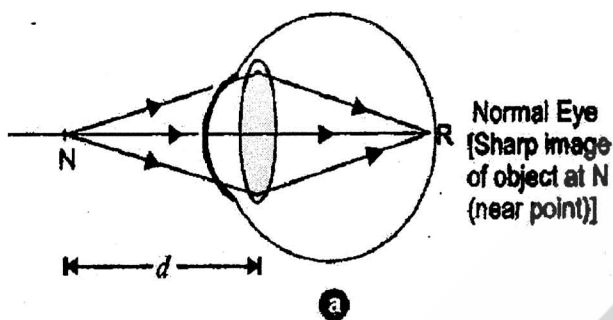
$$\boxed{f = -x}$$

Hence focal length of Concave lens put in front of the eye should be equal to distance of far point of the defective (myopic) eye.

(2) Hypermetropia or Long sightedness-

Hypermetropia or long sightedness is that defect of human eye by which the eye can see clearly the far off objects, but the nearby objects cannot be seen clearly.

Hence for a hypermetropic eye the near point shifts away from the eye. It is no longer at 25 cm, (the least distance of distinct vision).

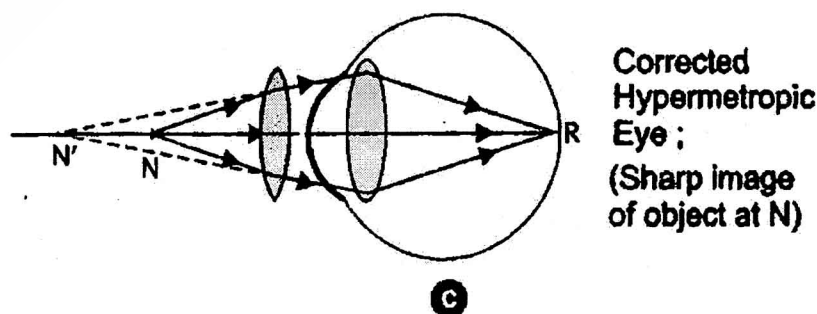


The hypermetropic eye focusses the rays from N at P' which is a point beyond the retina. However the defective eye focusses rays from another point N' on the retina. Thus the near point of hypermetropic eye has shifted away from N to N'.

The two possible causes of this defect are:

- (i) Contraction in the size of the eye ball i.e. distance of retina from the eye lens has decreased
- (ii) Increase in the focal length of the eye lens, when the eye is fully relaxed.

Correction -



To correct a hypermetropic eye the person has to use spectacles with a convex lens of suitable focal length. The rays starting from N, after refraction through convex lens, should appear to come from N', so that they are focussed on the retina.

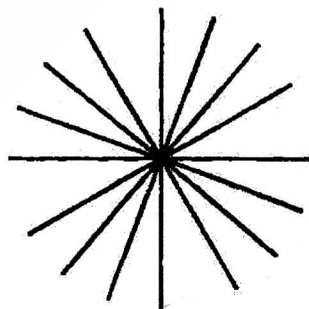
(3) Presbyopia (old sight) -

With increasing age the ciliary muscles holding the eye lens weaken and the lens loses some of its elasticity. Therefore power of accommodation of the eye decreases with age. This defect is called presbyopia.

Correction- To remove this defect, bifocal spectacle are used.

(4) Astigmatism -

To the normal eye all the lines in figure look equally black. But an astigmatic eye will find variation in the intensity of different lines. This defect arises when cornea has different curvature in different directions. This defect can occur along with myopia or hypermetropia.



Correction- Astigmatism can be corrected by using a cylindrical lens of suitable radius of curvature and suitable axis.

Q.
Delhi
2009

The far point of a myopic person is 80 cm. in front of the eye. What is the power of the lens required to enable him to see distant objects clearly? In what way does the corrective lens help the above person? Does the lens magnify very distant objects? Explain.

Sol. Here $u = -\infty$ and $v = -80 \text{ cm}$

$\therefore$ by lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{f} = \frac{1}{-80} - \frac{1}{-\infty} \quad \Rightarrow \quad f = -80 \text{ cm.}$$

$$\text{Power of the lens } P = \frac{1}{f} = -\frac{1}{80 \times 10^{-2}}$$

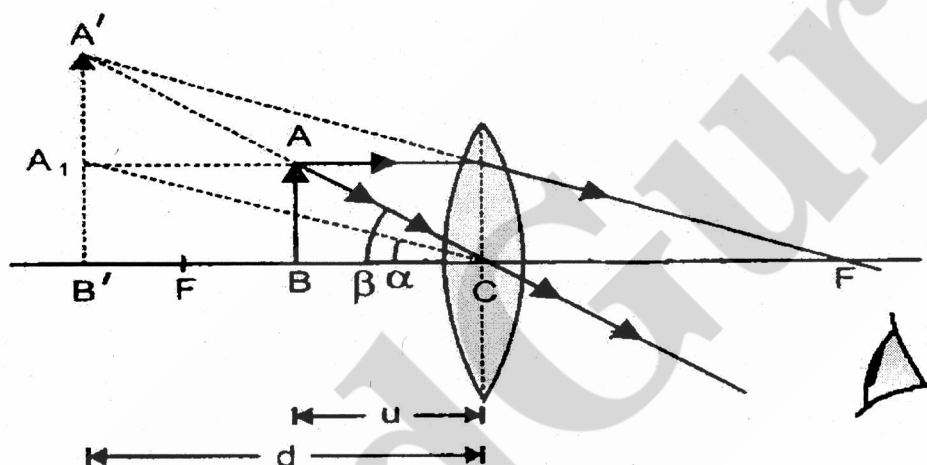
$$P = -1.25 \text{ D}$$

The corrective concave lens diverges the parallel rays from distant object as if they are coming from far point F.

No, lens does not magnify the distant objects.

Simple Microscope or Magnifying Glass -

A simple microscope consists of a converging lens of small focal length. A virtual, erect and magnified image of the object is formed at the least distance of distinct vision from the eye held close to the lens. That is why the simple microscope is also called a magnifying glass.



Let an object AB is held between optical centre C and principal focus F of the lens perpendicular to the principal axis. A virtual, erect and magnified image $A'B'$ is formed as traced in fig. The eye is held close to the lens, and $CB' = d$ ($= 25\text{ cm}$).

Magnifying power of a simple microscope is defined as the ratio of the angles subtended by the image and the object on the eye, when both are at the least distance of distinct vision from eye.

Let $\angle A'CB' = \beta$ and $\angle ACB' = \alpha$.

By definition, Magnifying Power (m) = $\frac{\beta}{\alpha}$

for small angles $\tan \theta \approx \theta$

Hence $\tan \alpha = \alpha$ and $\tan \beta = \beta$

$$\therefore m = \frac{\tan \beta}{\tan \alpha}$$

In $\triangle ABC$ $\tan \beta = \frac{AB}{CB}$

In $\triangle A_1B_1C$ $\tan \alpha = \frac{A_1B_1}{CB_1} = \frac{AB}{CB'}$

Hence $m = \frac{AB}{CB} \times \frac{CB'}{AB} = \frac{CB'}{CB} = \frac{-v}{-u} = \frac{v}{u}$

From lens formula - $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Multiply both sides by v - $\frac{v}{f} = 1 - \frac{v}{u}$

or $\frac{v}{f} = 1 - m$ or $m = 1 - \frac{v}{f}$

but $v = -d$ $\therefore$ $m = \left(1 + \frac{d}{f}\right)$

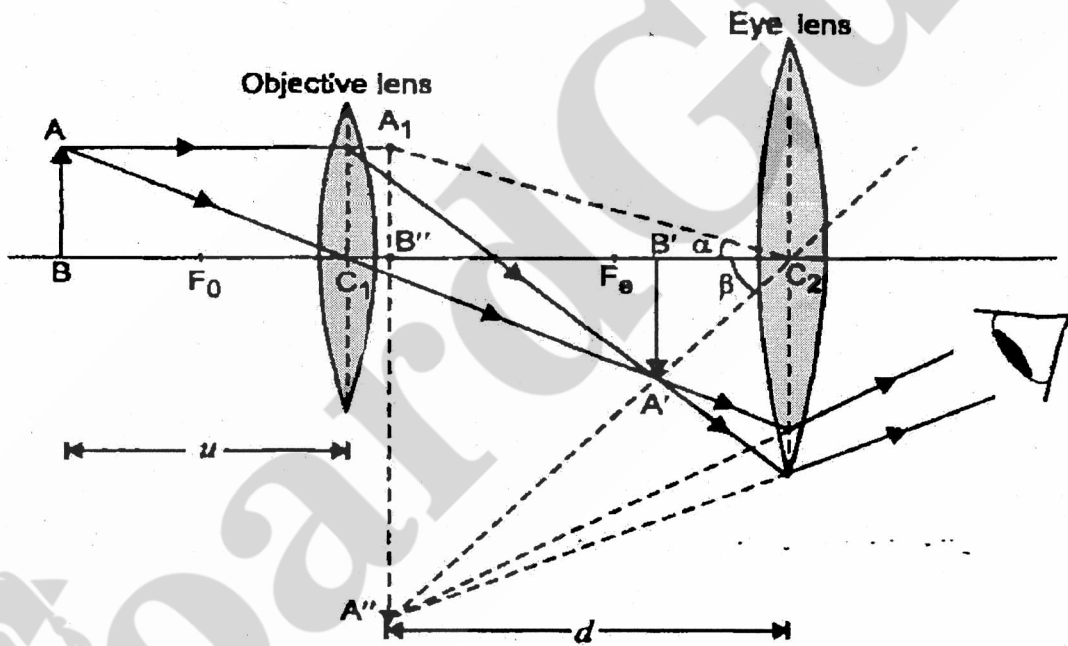
This is the expression for magnifying power of a simple microscope (magnifying glass).

* As f decreases, m increases i.e. smaller the focal length of the lens, greater is its magnifying power.

Compound Microscope-

A Compound microscope consists of two converging lenses: an objective lens O of very small focal length and short aperture, and an eye piece E of moderate focal length and large aperture.

The two lenses are held co-axially at the free ends of a tube at a suitable fixed distance from each other. The distance of the objective lens from the object can be adjusted by rack and pinion arrangement.



Let AB is a tiny object held perpendicular to the principal axis in front of the objective lens beyond its principal focus F_0 . A real, inverted and enlarged image $A'B'$ of this object is formed by the objective lens.

Now $A'B'$ acts as an object for the eye lens, whose position is so adjusted that $A'B'$ lies between optical centre C_2 of eye lens and its principal focus F_e . A virtual and magnified image $A''B''$ is formed by the eye lens.

This image is erect w.r.t. $A'B'$ but inverted w.r.t. AB . The final image $A''B''$ is seen by the eye held close to eye lens. The adjustments are so made that $A''B''$ is at the least distance of distinct vision (d) from the eye. i.e. $C_2B'' = d$.

Magnifying power of a compound microscope is defined as the ratio of the angle subtended at the eye by the final image to the angle subtended at the eye by the object, when both the final image and the object are situated at the least distance of distinct vision from the eye.

In fig. $C_2B'' = d$. Imagine the object AB to be shifted to A_1B_1 so that it is at a distance d from the eye. If $\angle A''C_2B'' = \beta$ and $\angle A_1C_2B'' = \alpha$ then

$$\text{Magnifying power } (m) = \frac{\beta}{\alpha}$$

For small angles, $\alpha = \tan \alpha$ and $\beta = \tan \beta$

$$\therefore m = \frac{\tan \beta}{\tan \alpha}$$

In $\Delta A''B''C_2$,

$$\tan \beta = \frac{A''B''}{C_2B''}$$

In $\Delta A_1B_1C_2$,

$$\tan \alpha = \frac{A_1B_1}{C_2B''} = \frac{AB}{C_2B''}$$

$$\text{Hence } m = \frac{A''B''}{C_2B''} \times \frac{C_2B''}{AB} = \frac{A''B''}{AB}$$

$$\text{or we can write - } m = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

$$m = m_e \times m_o$$

where $m_e = \frac{A''B''}{A'B'}$ = magnification by eye lens.

and $m_o = \frac{A'B'}{AB}$ = magnification by objective lens.

Now we know that - $m_e = \left(1 + \frac{d}{f_e}\right)$

where d is C_2B'' = least distance of distinct vision and f_e is the focal length of eye lens.

$$\text{Also } m_o = \frac{A'B'}{AB} = \frac{C_1B'}{C_1B} = \frac{V_o}{-u_o}$$

Hence Magnifying power $m = -\frac{V_o}{u_o} \left(1 + \frac{d}{f_e}\right)$

length of microscope tube $L = V_o + u_e$

If the object AB lies very close to F_o , therefore -
 $u_o = C_1B \approx C_1F_o = f_o$

As $A'B'$ is formed very close to eye lens, therefore
 $V_o = C_1B' \approx C_1C_2 = L = \text{length of microscope tube}$

Hence we get - $m = \frac{L}{-f_o} \left(1 + \frac{d}{f_e}\right) = \frac{L}{|f_o|} \left(1 + \frac{d}{f_e}\right)$

Note -

- (i) As magnifying power (m) is negative, the image seen in a microscope is always inverted.
- (ii) For large magnifying power f_o and f_e both have to be small. Also f_o is taken to be smaller than f_e so that field of view may be increased.

Q.

CBSE
2011

A compound microscope uses an objective lens of focal length 4 cm. and eyepiece lens of focal length 10 cm. An object is placed at 6 cm. from the objective lens. Calculate the magnifying power of the Compound microscope. Also calculate the length of the microscope.

Sol. Given that -

$$f_o = 4 \text{ cm.} \quad f_e = 10 \text{ cm.}, \quad u_o = -6 \text{ cm.}, \\ v_e = D = 25 \text{ cm.}$$

For objective lens - $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

$$\Rightarrow \frac{1}{4} = \frac{1}{v_o} + \frac{1}{6} \quad \Rightarrow v_o = 12 \text{ cm.}$$

$$\text{Magnifying power (m)} = -\frac{v_o}{u_o} \left(1 + \frac{D}{f_e}\right)$$

$$m = -\frac{12}{6} \left(1 + \frac{25}{10}\right) = -7$$

$$\text{Length of microscope} = |v_o| + |u_e|$$

for eye lens $\Rightarrow v_e = -25 \text{ cm.}, f_e = 10 \text{ cm.}$

by lens formula

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} \quad \Rightarrow \frac{1}{u_e} = \frac{1}{-25} - \frac{1}{10}$$

$$u_e = -\frac{50}{7} = -7.14 \text{ cm.}$$

$$\therefore \text{Length of microscope} = 12 + 7.14$$

$$L = 19.14 \text{ cm}$$

Astronomical Telescope -

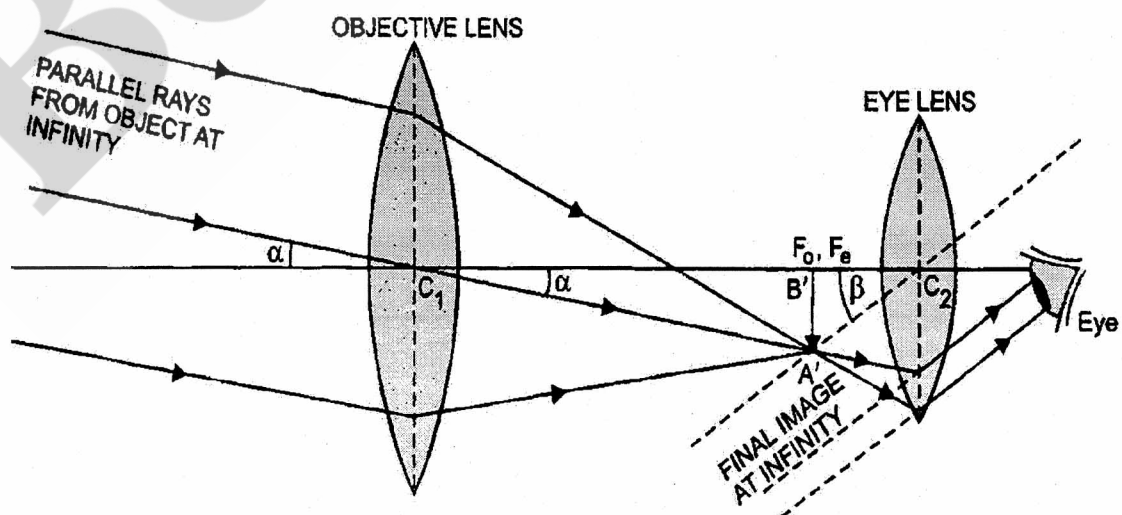
An astronomical telescope is an optical instrument which is used for observing distinct images of heavenly bodies like stars, planets etc.

It consists of two lenses, the objective lens which is of large focal length and large aperture and the eye lens, which has a small focal length and small aperture.

The two lenses are mounted co-axially at the free ends of the two tubes. The distance between these lenses can be adjusted using a rack and pinion arrangement.

(i) Normal Adjustment -

Let a parallel beam of light from an astronomical object (at infinity) is made to fall on the objective lens of the telescope. It forms a real, inverted and diminished image $A'B'$ of the object. The eye piece is so adjusted that $A'B'$ lies just at the focus of the eye piece. Therefore a final highly magnified image is formed at infinity. The final image is erect w.r.t. $A'B'$ and is inverted w.r.t. the object.



Magnifying Power of an astronomical telescope in normal adjustment is defined as the ratio of the angle subtended at the eye by the final image to the angle subtended at the eye, by the object directly, when the final image and the object both lie at infinite distance from the eye.

As the object lies at very huge distance, therefore angle subtended by the object at C_2 is almost the same as the angle subtended by the object at C_1 (because C_1 is close to C_2). Let it be α i.e. $\angle A'C_1B' = \alpha$.

Rays coming from the final image at infinity make $\angle A'C_2B' = \beta$ on the eye. Therefore-

$$\text{Magnifying power } m = \frac{\beta}{\alpha}$$

As angles are small - $\alpha = \tan \alpha$, $\beta = \tan \beta$

$$m = \frac{\tan \beta}{\tan \alpha}$$

$$\text{In } \Delta A'B'C_2, \quad \tan \beta = A'B'/C_2B'$$

$$\text{In } \Delta A'B'C_1, \quad \tan \alpha = A'B'/C_1B'$$

$$\text{Hence } m = \frac{A'B'}{C_2B'} \times \frac{C_1B'}{A'B'} = \frac{C_1B'}{C_2B'}$$

or

$$\text{Magnifying Power (m)} = \frac{f_o}{-f_e}$$

* Negative sign of m indicates that final image is inverted.

length of the telescope

$$L = f_o + f_e$$

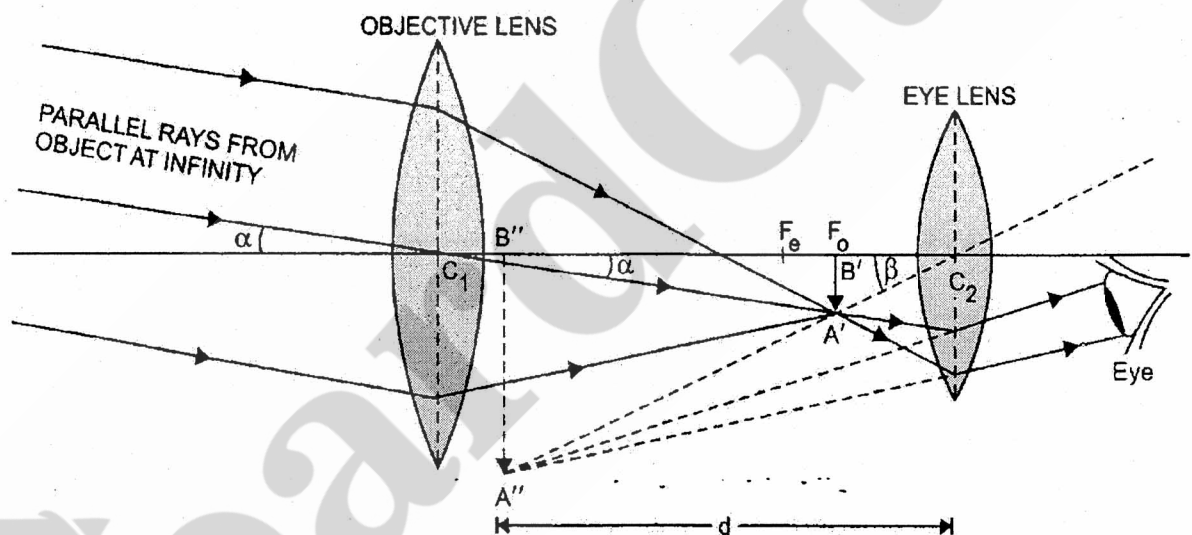
* Thus to increase magnifying power of an astronomical telescope in normal adjustment, focal length of objective lens should be large and focal length of eye lens should be small.

Note-

In normal adjustment of telescope, distance between the objective lens and eye lens = $f_o + f_e$.

(ii) When the final Image is formed at the least distance of distinct vision (d) from the eye-

Magnifying Power of an astronomical telescope is defined as the ratio of the angle subtended at the eye by the final image at the least distance of distinct vision to the angle subtended at the eye by the object at infinity, when seen directly.



As the object is very far off and C_1, C_2 are close by, angle subtended by the object on the eye at C_2 is almost the same as the angle subtended by it at C_1 . Let it be α .

$$\therefore \angle A'C_1B' = \alpha$$

and let $\angle A''C_2B'' = \beta$, where $C_2B'' = d$

$$\therefore \text{Magnifying Power (m)} = \frac{\beta}{\alpha}$$

As angles α and β are small, therefore -

$$m = \frac{\tan \beta}{\tan \alpha}$$

$$\text{In } \triangle A'B'C_2, \quad \tan \beta = \frac{A'B'}{C_2B'}$$

$$\text{In } \triangle A'B'C_1, \quad \tan \alpha = \frac{A'B'}{C_1B'}$$

$$\text{So that - } m = \frac{A'B'}{C_2B'} \times \frac{C_1B'}{A'B'} = \frac{C_1B'}{C_2B'} = \frac{f_o}{-u_e}$$

where f_o = focal length of objective lens and u_e = distance of $A'B'$, acting as the object for eye lens.

$$\text{For eye lens } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\text{or } \frac{1}{-d} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\text{or } \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{d} = \frac{1}{f_e} \left(1 + \frac{f_e}{d} \right)$$

Hence

$$m = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{d} \right)$$

* Negative sign of magnifying power indicates that final image in an telescope is inverted.

length of the telescope

$$L = f_o + u_e$$

* For large magnifying power, f_o must be as large as possible and f_e must be as small as possible. Hence in telescope aperture of objective lens is made large to increase magnifying power and resolving power of the telescope.

Hence

$$m_{\min} = -\frac{f_o}{f_e}$$

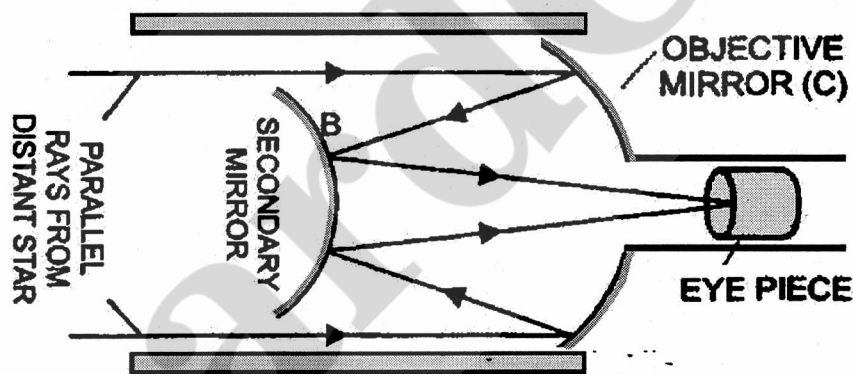
$$m_{\max} = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{d} \right)$$

Reflecting type Telescope (Cassegrainian Telescope)-

It is an improvement over the refracting type telescope, objective lens is replaced by a concave parabolic mirror of large aperture, which is free from chromatic and spherical aberrations.

The image formed is much brighter and the reflecting type telescope has much higher resolving power compared to the refracting type telescope. Such a telescope is known as Cassegrainian telescope on the name of its inventor.

A reflecting type telescope was designed initially by Newton and modified by Cassegrainian.



In diagram C is a parabolic concave reflector of about 200 inch aperture with a narrow hole at the centre. Parallel rays from a distant star entering the telescope in a direction parallel to principal axis of the mirror tend to collect at the focus of the mirror. But these reflected rays encounter a secondary convex mirror B before meeting at the focus. The convex mirror reflects them onto the eye piece, and the final image is seen through the eye piece.

The final image as seen by the eye piece is inverted w.r.t. the object.

In normal adjustment, magnifying power of a reflecting type telescope is given by-

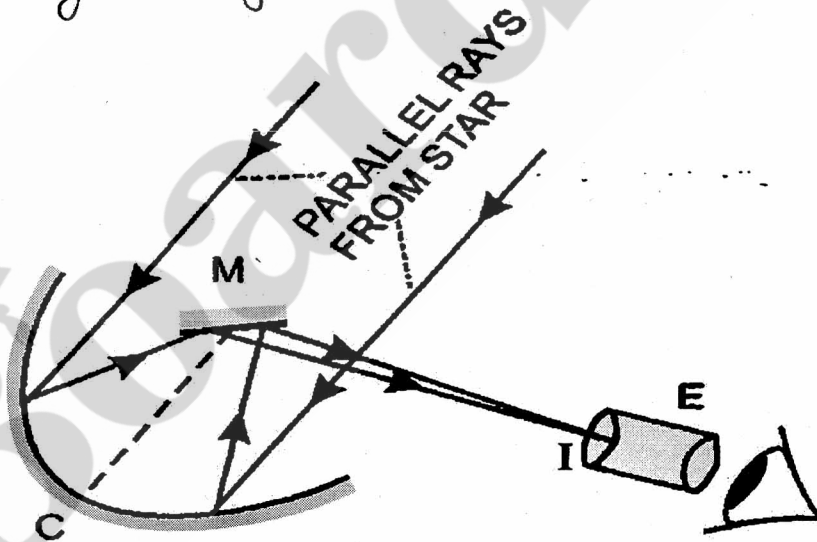
$$m = \frac{f_o}{f_e} = \frac{R/2}{f_e}$$

where R is radius of curvature of Concave reflector.

Newtonian reflecting type Telescope-

The parallel beam of light coming from the distant star is reflected by large parabolic concave reflector C on to a plane mirror M . This mirror is inclined at an angle of 45° to the axis of C . The plane mirror reflects the beam forming a real image I in front of an eye piece E .

The eye piece acts as a magnifier and the final virtual and magnified image of the star is seen distinctly by the eye.



Advantages-

- (i) There is no chromatic and spherical aberration due to use of mirror objective in place of lenses.
- (ii) Image is brighter as compared to refracting type telescope and high resolution is achieved by using a mirror of large aperture.

Resolving Power -

Resolving power of an optical instrument is the power or ability of the instrument to produce distinctly separate images of two close objects.

The minimum distance between two objects which can just be seen as separate by the optical instrument is called the limit of resolution of the instrument. Smaller the limit of resolution of the optical instrument, greater is its resolving power and vice-versa.

Resolving Power of Microscope -

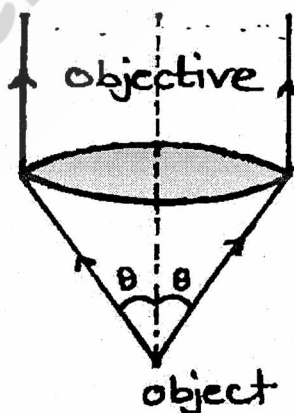
$$\text{R.P. of microscope} = \frac{1}{d} = \frac{2\mu \sin \theta}{\lambda}$$

where -

d = minimum distance between two objects whose image can be seen through microscope vivid and clear.

$\mu \sin \theta$ is called numerical aperture of the microscope

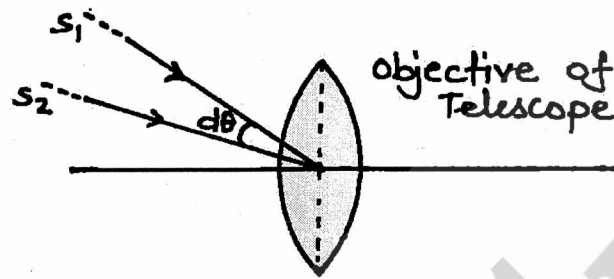
θ = semi-vertical angle of the cone formed by the object at objective



* To increase the resolving power of microscope, we decrease λ i.e. use blue light of smaller wavelength. We can not increase θ because in that case aperture of the lens would increase.

Resolving Power of telescope -

The limit of resolution of telescope is measured by the angle ($d\theta$) subtended at its objective, by those two distant objects (say stars) whose images through the telescope are just seen as separate.



$$\text{R.P. of Telescope} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

Where D = diameter of the objective of the telescope
 λ = wavelength of light used.

* A telescope is used to observe distant objects, which are generally seen in sunlight. Therefore there is no control on λ . Hence to increase the resolving power of telescope, we must use the objective lens of large aperture (D).

An additional use of large objective is that it will collect greater amount of light and images seen will be much brighter.

Q.
Delhi
2011

The objective of an astronomical telescope has a diameter of 150 mm and a focal length of 4 m. The eye piece has a focal length of 25 mm. Find the magnifying and resolving power of telescope ($\lambda = 6000 \text{ \AA}$ for yellow colour).

Sol. Given that - $f_o = 4 \text{ m.}$, $f_e = 25 \times 10^{-3} \text{ m.}$
 $D = 25 \text{ cm} = 0.25 \text{ m.}$

$$\text{Magnifying power (m)} = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right)$$

$$m = -\frac{4}{25 \times 10^{-3}} \left(1 + \frac{25 \times 10^{-3}}{0.25}\right) = -\frac{4000}{25} \times \frac{11}{10}$$

$$m = -176$$

$$\begin{aligned} \text{Resolving Power} &= \frac{1}{d\theta} = \frac{D}{1.22\lambda} \\ &= \frac{150 \times 10^{-3}}{1.22 \times 6 \times 10^{-7}} = 2.05 \times 10^4 \end{aligned}$$

$$\text{Resolving Power} = 2.05 \times 10^4$$

Q.
CBSE
2012

A small telescope has an objective lens of focal length 150 cm. and an eye piece of focal length 5 cm. If this telescope is used to view a 100 m. high tower 3 km. away, find the height of the final image when it is formed 25 cm. away from the eye piece.

Sol. Given that -
 $f_o = 150 \text{ cm.}$, $f_e = 5 \text{ cm.}$, $D = 25 \text{ cm.}$

$$\text{Magnification } m = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right)$$

$$m = -\frac{150}{5} \left(1 + \frac{5}{25}\right) = -36$$

Let height of final image is $h \text{ cm.}$

$$\therefore \tan \beta = \frac{h}{25} \quad \left(\beta = \text{Visual angle formed by final image at eye.} \right)$$

and $\tan \alpha = \frac{100}{3000} = \frac{1}{30}$ $\left(\alpha = \text{Visual angle subtended by object at objective} \right)$

But Magnification $m = \frac{\tan \beta}{\tan \alpha}$

$$-36 = \frac{(h/25)}{(1/30)} = \frac{30h}{25}$$

$$h = \frac{-36 \times 25}{30} = -30 \text{ cm.}$$

Here negative sign indicates the inverted image.

Very Important Questions

1 Mark Questions-

- Q.1. How does focal length of a lens change when red light incident on it is replaced by violet light?
- Q.2. Name the physical quantity which remains same for microwaves of wavelength 1mm and UV radiation of $1600 \text{ \AA}$ in vacuum. (CBSE 2012)
- Q.3. Under what condition does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid? (Delhi 2012)
Answer - $\mu_g = \mu_l$
- Q.4. When monochromatic light travels from one medium to another, its wavelength changes but frequency remains the same. Explain. (Delhi 2011)
- Q.5. The refractive index of diamond is much greater than glass. How does a diamond cutter make use of this fact? (CBSE 2011)
- Q.6. If a ray of light propagates from a rarer to a denser medium, how does its frequency change? (CBSE 2011)
- Q.7. State the criteria for the phenomenon of total internal reflection of light to take place. (Delhi 2011, CBSE 2018)
- Q.8. A lens behaves as a converging lens in air but diverging lens in water ($\mu = 4/3$). What will be the condition on the value of refractive index of the material of the lens? (CBSE 2010)
Answer - $\mu_w > \mu_{\text{lens}}$
- Q.9. Why does the sky appear blue? (CBSE 2010)

Q.10. When light travels from a rarer to a denser medium, the speed decreases. Does this decrease in speed imply a decrease in the energy carried by the light wave?

(RBSE 2005, CBSE 2010)

Q.11 A glass lens of refractive index 1.45 disappears when immersed in a liquid. What is the value of refractive index of the liquid?

(RBSE 2006, Delhi 2010)

Q.12. Calculate the speed of light in a medium whose critical angle is 30° .

(Delhi 2006)

Q.13. Under what condition does the formation of rainbow occur?

(RBSE 2007, 10)

Q.14. Two thin lenses of power +6D and -2D are in contact. What is the focal length of the combination?

(CBSE 2005, 10)

Q.15. Why are convex mirrors used as side view mirrors in cars?

(Delhi 2009)

Q.16. A converging lens of refractive index 1.5 is kept in a liquid medium having same refractive index. What would be the focal length of the lens in this medium?

(Delhi 2008)

Answer - $f = \infty$

Q.17. How does power of the convex lens vary, if the incident red light is replaced by violet light?

(CBSE 2008)

Q.18. Explain with reason why the sun looks reddish at sunrise or sunset?

(RBSE 2008)

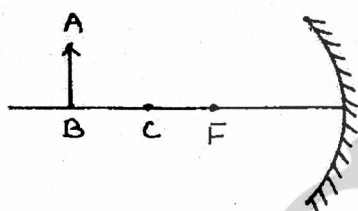
2 Marks Questions -

Q.1. A ray of light, incident on an equilateral glass prism ($\mu_g = \sqrt{3}$) moves parallel to the base line of the prism inside it. Find the angle of incidence for this ray.

(Delhi 2012)

Answer - $i = 60^\circ$

Q.2. An object AB is kept in front of a Concave mirror as shown in figure-



(a) Complete the ray diagram showing the image formation of the object.

(b) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black?

(CBSE 2012, RBSE 06)

Answer- (b) The position of the image remains same but intensity of image decreases

Q.3. Draw a diagram showing the formation of primary rainbow and explain at what angles the primary rainbow is visible.

(Delhi 2010)

Q.4. The radius of curvature of the faces of a double convex lens are 10 cm. and 15 cm. If focal length of the lens is 12 cm. find the refractive index of the material of the lens.

(CBSE 2005, 09)

Answer - $\mu = 3/2$.

Q.5. A biconvex lens has a focal length $2/3$ times the radius of curvature of either surface. Calculate the refractive index of lens material.

Answer - $\mu = \frac{7}{4}$

(Delhi 2004, 10)

Q.6. Find the radius of curvature of a plano-convex lens, whose focal length is 0.3m and the refractive index of the material of the lens is 1.5.

Answer- $R = 15 \text{ cm}$.

(Delhi 2004, 10)

Q.7. (a) Violet color is seen at the bottom of the spectrum when white light is dispersed by a prism. Why?

(b) Give the formula that can be used to determine refractive index of material of a prism in minimum deviation condition.

(RBSE 2005, 10)

Q.8. The given table shows the values of angle of deviation, for different values of the angle of incidence for a triangular prism:

Angle of Incidence	33°	38°	42°	52°	60°	71°
Angle of deviation	60°	50°	46°	40°	43°	50°

(a) For what value of the angle of incidence, is the angle of emergence likely to be equal to the angle of incidence itself?

(b) Draw a ray diagram, showing the passing of a ray of light through this prism when the angle of incidence has the above value.

(Delhi 2005, 10)

Q.9. State the principle on which the working of an optical fiber is based. What are the necessary conditions for this phenomenon to occur?

(CBSE 2009)

Q.10. What is the relation between critical angle and refractive index of a material? Does critical angle depend on the color of light?

(RBSE 2009)

Q.11. Calculate the distance of an object of height h from a Concave mirror of focal length 10cm , so as to obtain a real image of magnification 2 .

Answer - $u = -5\text{cm}$.

(Delhi 2002, 08)

Q.12. Draw a ray diagram to show the formation of the image in a far-sighted (hypermetropic) eye. Show with the help of a ray diagram how this defect is corrected?

(CBSE 2002, 10)

Q.13. Draw a labelled ray diagram of an astronomical telescope in the near point position. Write the expression for its magnifying power.

(CBSE 2008)

Q.14. Draw a ray diagram of an astronomical telescope in the normal adjustment position. State two drawback of this type of telescope.

(Delhi 2003, 08)

Q.15. Draw a ray diagram of compound microscope. Write the expression for its magnifying power.

(RBSE 2008)

Q.16. Draw a ray diagram of a reflecting type telescope. State two advantages of this telescope over a reflecting telescope.

(Delhi 2004, 08)

Q.17. Draw a labelled ray diagram of a reflecting type telescope. Write its any one advantage over refracting type telescope.

(Delhi 2006)

3 Marks Questions -

- Q.1. Define power of a lens. Write its unit. Deduce the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ for two thin lenses kept in contact coaxially. (CBSE 2012)
- Q.2. State the necessary conditions for producing total internal reflection of light. Draw ray diagrams to show how specially designed prisms make use of total internal reflection to obtain inverted image of the object by deviation rays.
(a) Through 90° and (b) Through 180° (Delhi 2011)
- Q.3. With the help of suitable ray diagram, derive a relation between the object distance (u), image distance (v) and radius of curvature R for the convex spherical surface, when a ray of light travels from rarer to denser medium. (RBSE 2015, 10)
- Q.4. A ray of light is incident on one face of a glass prism and emerges out from the other face. Trace the path of the ray and derive an expression for refractive index of the glass prism. (CBSE 2011)
- Q.5. An object of 2cm height is placed at a distance of 30 cm. from a Convex mirror of focal length 15cm. Find the nature, position and size of the image formed. (CBSE 2010)
- Q.6. With the help of a suitable diagram, derive the mirror formula for a concave mirror. (CBSE 2009)

Q.7. A tank is filled with water to a height of 12.4 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.3 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.6 upto the same height, by what distance would the microscope have to be moved to focus on the needle again? (Delhi 2002, 09)

Answer- 2.18 cm.

Q.8. The velocity of a certain monochromatic light in a given transparent medium is 2.225×10^8 m/s. What is the (a) Critical angle of incidence and (b) polarizing angle for this medium? (CBSE 2009)

Answer- (a) $i_c = \sin^{-1}(0.7416)$

(b) $i_p = \tan^{-1}(1.34)$

Q.9. When a ray of light passes through a prism, find out the relation for the total deviation δ in terms of the angle of incidence i and angle of emergence e .

Plot a graph showing the variation of angle of deviation with the angle of incidence and obtain the condition for the angle of minimum deviation.

(RBSE 2009, 03)

Q.10. How does the frequency of a beam of ultraviolet light get affected when it goes from air into glass? A ray of light incident on an equilateral glass prism shows minimum deviation of 30° . Calculate the speed of light through the glass prism.

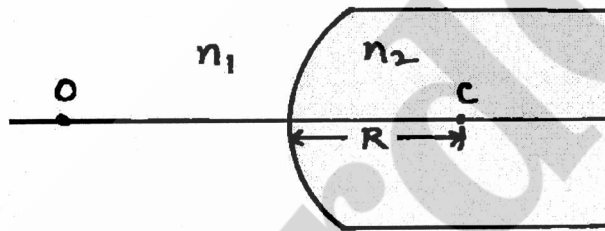
Answer- $v = 2.12 \times 10^8$ m/s

(CBSE 2007)

5 Marks Questions -

Q.1. (a) Draw a ray diagram to show that image formation by a concave mirror when the object is kept between its focus and the pole. Using this diagram, derive the magnification formula for the image formed.

(b) Figure shows a convex spherical surface with centre of curvature C , separating the two media of refractive index n_1 and n_2 . Draw a ray diagram showing the formation of the image of a point object O lying on the principle axis. Derive the relationship between the object and image distance in terms of refractive indices of the media and the radius of curvature R of the surface.



(CBSE 2010, 11)

Q.2. Derive the mirror equation using the ray diagram for the formation of a real image by a concave mirror.

(b) Define the resolving power of a telescope. Write any two advantages of a reflecting telescope over a refracting telescope.

(Delhi 2005, 10)

Q.3. Draw a ray diagram for formation of image of a point object by a thin double convex lens having radius of curvatures R_1 and R_2 and hence derive lens maker's formula.

(b) If a convex lens of focal length 50 cm is placed in contact coaxially with a concave lens of focal length 20 cm. What is the power of the combination?

Answer - $P = -3D$

(CBSE 2002, 09)

Q.4. (a) Derive the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a Concave lens, using necessary ray diagram.

(b) Two lens of power 10D and -5D are placed in contact- (i) Calculate the power of new lens.

(ii) where should an object be held from the lens so as to obtain a virtual image of magnification 2?

Answer- (i) $P = 5D$ (ii) $u = -10cm$.

(CBSE 2003, 08)

Q.5. For a ray of light travelling from a denser medium of refractive index n_1 to a rarer medium of refractive index n_2 , prove that $\frac{n_2}{n_1} = \sin i_c$, where i_c is the critical angle of incidence for the media.

(b) Explain with diagram, how the above principle is used for transmission of video signal using optical fibers.

(Delhi 2008)

Q.6. Derive the relation between the focal length of a Convex lens in terms of the radius of curvature of the two surfaces and refractive index of its material write the sign convention and two assumptions used in the derivation of this relation.

(CBSE 2007)

Q.7. Derive the expression for the refractive index of the material of the prism in terms of the angle of the prism, and angle of minimum deviation.

Use this formula to calculate the angle of minimum deviation for the equilateral triangular prism of a refractive index $\sqrt{3}$.

(Delhi 2006)

Answer - $\delta_m = \pi/3$

- Q.1. A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image we are obviously bringing it on to the screen (i.e. retina) of our eye. Is there a contradiction?
- Q.2. A fish under water is viewing obliquely at a fisherman standing on the bank of a lake, does the man look taller or shorter than what he actually is?
- Q.3. Why must both the objective and eye-piece of a compound microscope have short focal lengths?
- Q.4. The refractive index of a material of a concave lens is μ_1 . It is immersed in a medium of refractive index μ_2 . A parallel beam of light is incident on the lens. Trace the path of emergent ray when (i) $\mu_2 = \mu_1$, (ii) $\mu_2 > \mu_1$, (iii) $\mu_2 < \mu_1$.
- Q.5. Will the focal length of a lens for red light be more, same or less than that for blue light?
- Q.6. What are two important limitations of a refracting telescope over a reflecting type telescope?
- Q.7. Draw a plot showing the variation of power of a lens with the wavelength of the incident light.
- Q.8. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index for water is $\frac{4}{3}$.
- Ans. $A = 2.6 \text{ m}^2$

Q.9. A convex lens of focal length 25 cm. is placed coaxially in contact with a concave lens of focal length 20 cm. Determine the power of the combination. Will the system be converging or diverging in nature?

Ans. $P = -1D$

Q.10. A convex lens of focal length 20 cm, has a point object placed on its principal axis at distance of 40 cm. from it. A plane mirror is placed 30 cm. behind the convex lens. Locate the position of image formed by this combination. Is the image real or virtual?

Ans. 10 cm. in front of plane mirror

Q.11. The focal lengths of the objective and eyepiece of a microscope are 1.25 cm. and 5 cm. respectively. Find the position of the object relative to the objective in order to obtain an angular magnification of 30 in normal adjustment.

Ans. $u_o = -1.5 \text{ cm.}$

Q.12. A convex lens of glass ($\mu_g = 1.5$) is placed in a transparent liquid such that it becomes invisible. What is the refractive index of liquid?

Ans. $\mu_l = 1.5$

Q.13. Can a convex lens behave as a diverging lens? If yes, state the condition.

Q.14. Which has larger critical angle for total internal reflection: blue colour, or yellow colour or red colour?

Q.15. Write the relationship between angle of incidence ' i ', angle of prism ' A ', and angle of minimum deviation ' δ_m ' for a triangular prism.

PHYSICS

Unit - 10. Wave Optics

Class - 12th

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10.	Wave Optics
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Wavefront -

A wavefront is defined as the continuous locus of those particles of a medium, which are vibrating in the same phase.

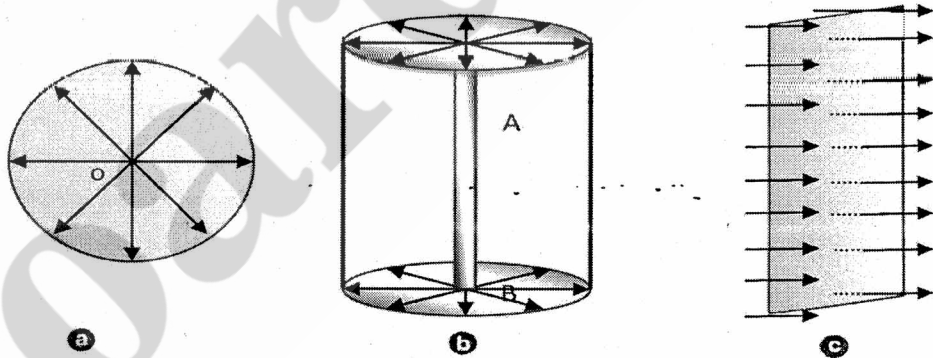
Depending on the shape of the source of light, wavefront can be of three types:

(i) Spherical Wavefront -

When the source of light is a point source the wavefront is a sphere with centre at the source.

(ii) Cylindrical Wavefront -

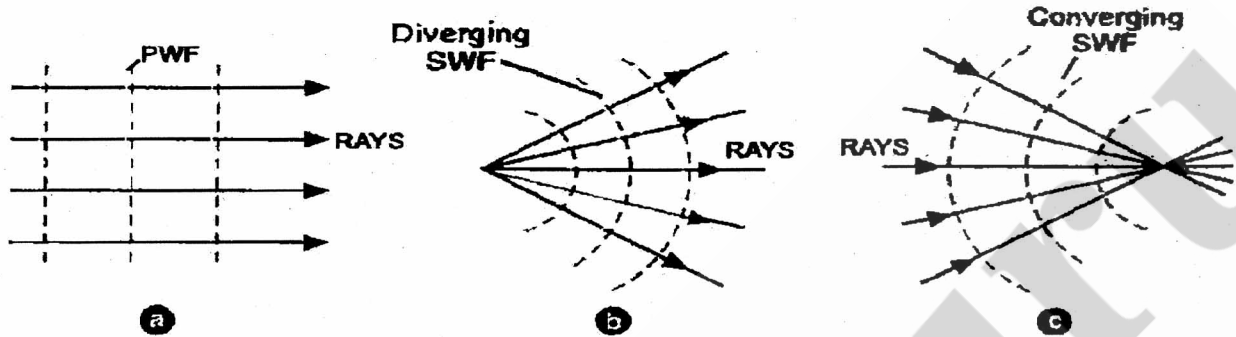
When the source of light is linear (a slit), then all the points equidistant from the source lie on a cylinder. Therefore the wavefront is cylindrical.

**(iii) Plane Wavefront -**

When the point source or linear source of light is at very large distance, a small portion of spherical or cylindrical wave front appears to be plan. Such a wavefront is called a plane wavefront.

* If we draw a set of straight lines which are perpendicular to the wavefront then these lines are called the ray of light.

The given figures (a), (b) and (c) represents the wavefront and rays of light corresponding to plane wavefront (PWF), diverging spherical wavefront (DSWF) and converging spherical wavefront (CSWF) respectively.

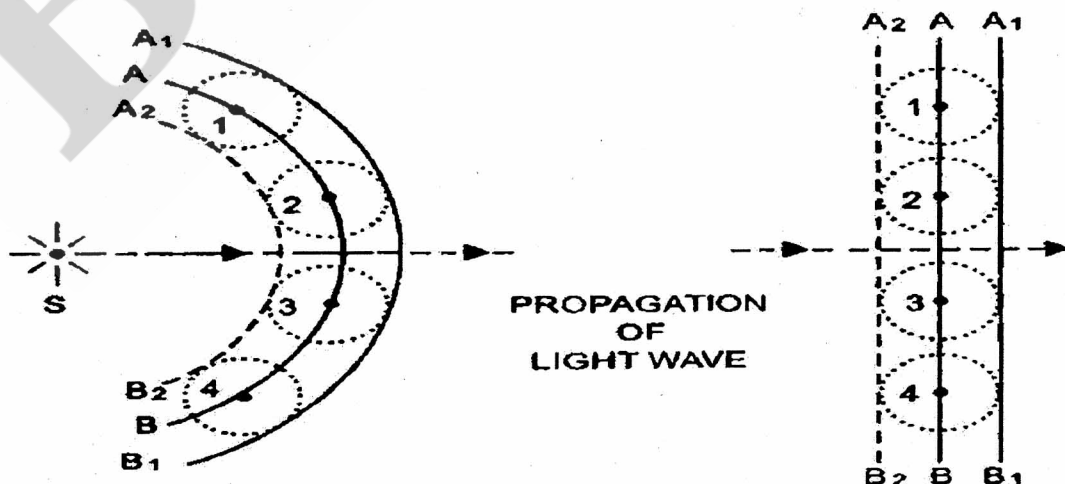


Huygen's Principle-

The Huygen's principle tells the way in which the wavefront is propagated further in a medium.

According to Huygen's -

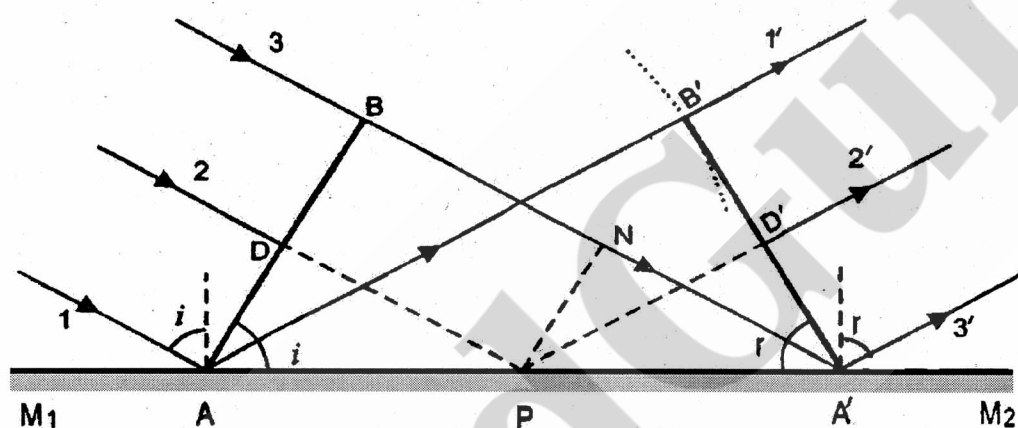
- (i) Every point on the given wavefront (called primary wavefront) acts as a fresh source of new disturbance, called secondary wavelets, which travel in all directions with the velocity of light in the medium.
- (ii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wave front.



Reflection on the basis of Wave Theory -

Huygen's principle can be used to explain the phenomena of reflection and refraction of light on the basis of wave theory.

In the given figure AB is a plane wavefront incident on a plane mirror M_1M_2 at $\angle BAA' = \angle i$. 1, 2, 3 are the corresponding incident rays, which are perpendicular to AB .



According to Huygens principle, every point on AB is a source of secondary wavelets. Let the secondary wavelets from B strike M_1M_2 at A' in t seconds.

$$\therefore BA' = cxt \quad \text{--- (1)}$$

The secondary wavelets from A will travel the same distance cxt in the same time. Therefore, with A as centre and cxt as radius, draw an arc, so that -

$$AB' = cxt \quad \text{--- (2)}$$

From A' draw a tangent plane $A'B'$ touching the spherical arc tangentially at B' . Therefore $A'B'$ is the secondary wavefront after t seconds. This would advance in the direction of rays $1'$, $2'$ and $3'$, which are the corresponding reflected rays perpendicular to $A'B'$.

Hence angle of incidence $i = \angle BAA'$
and angle of reflection $r = \angle B'A'A$

In $\triangle AA'B$ and $\triangle AA'B'$,

AA' is common, $BA' = AB' = c \times t$
and $\angle B = \angle B' = 90^\circ$

$\therefore \triangle AA'B$ and $\triangle AA'B'$ are congruent.

$\therefore \angle BAA' = \angle B'A'A$ i.e. $\angle i = \angle r$

which is the first law of reflection.

Further the incident wavefront AB , the reflecting surface M_1M_2 and the reflected wavefront $A'B'$ are all perpendicular to the plane of the paper.

Therefore incident ray, normal to the mirror M_1M_2 and reflected ray all lie in the plane of the paper. This is second law of reflection.

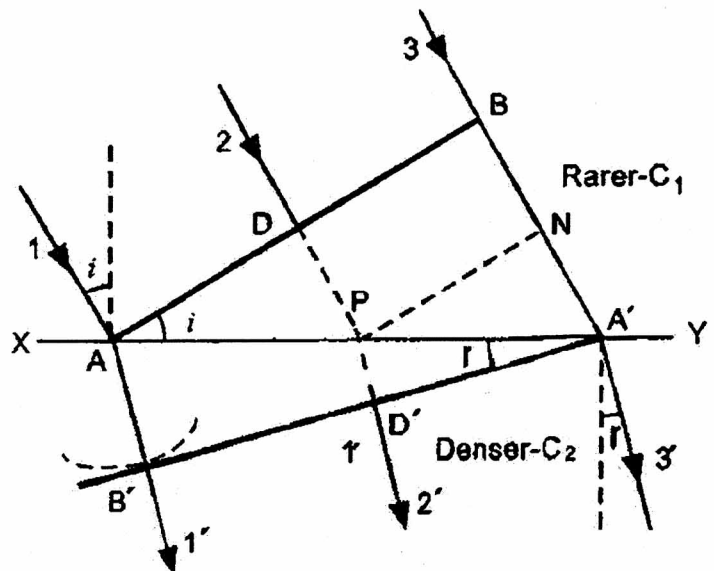
Refraction on the basis of Wave Theory-

In the given figure XY is a plane surface that separates a denser medium of refractive index μ from a rarer medium.

If v_1 is velocity of light in rarer medium and v_2 is velocity of light in denser medium then -

$$\text{as } \mu_1 = \frac{c}{v_1} \text{ and } \mu_2 = \frac{c}{v_2}$$

$$\text{or } \boxed{\mu_2 = \frac{v_1}{v_2}} \quad \text{--- (1)}$$



Here AB is a plane wavefront incident on XY at $\angle BAA' = \angle i$. 1, 2, and 3 are the corresponding incident rays normal to AB.

According to Huygen's principle, every point on AB is a source of secondary wavelets. Let the secondary wavelets from B strike XY at A' in t seconds.

$$\therefore BA' = V_1 \times t \quad \text{--- (2)}$$

The secondary wavelets from A travel in the denser medium with a velocity V_2 and would cover a distance $(V_2 \times t)$ in t seconds. Therefore with A as centre and radius equal to $(V_2 \times t)$ draw an arc B'.

From A', draw a tangent plane touching the spherical arc tangentially at B'. Therefore A'B' is the secondary wavefront after t sec. This would advance in the direction of rays 1', 2' and 3', which are the corresponding refracted rays, perpendicular to A'B'.

Let r be the angle of refraction. As angle of refraction is equal to the angle which the refracted plane wavefront A'B' makes with the refracting surface AA', therefore $\angle AAB' = \angle r$

$$\text{In } \triangle AA'B, \quad \sin i = \frac{BA'}{AA'} = \frac{V_1 \times t}{AA'}$$

$$\text{In } \triangle AA'B', \quad \sin r = \frac{AB'}{AA'} = \frac{V_2 \times t}{AA'}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{V_1}{V_2} = \mu_2 \quad (\text{by eq. (1)})$$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{Or} \quad \boxed{\mu_1 \sin i = \mu_2 \sin r}$$

which Proves Snell's Law of refraction.

It is clear from figure that the incident rays, normal to the interface XY and refracted rays, all lie in the same plane. This is second law of refraction.

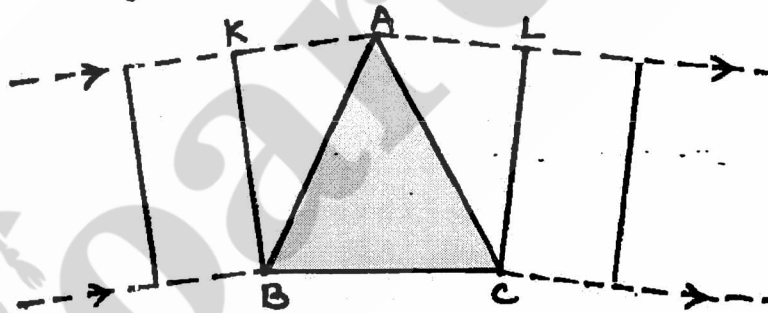
Behaviour of Prism towards Plane Wavefront -

In figure ABC is a prism with small angle A, on which a plane wavefront KB is incident.

Different parts of the wavefront travel different thickness of glass prism, maximum at the bottom and minimum at the top.

As speed of light in material of prism is less than the speed of light in air, therefore emerging plane wavefront is CL where -

$$\frac{KA + AL}{\text{speed of light in air}} = \frac{BC}{\text{speed of light in prism}}$$

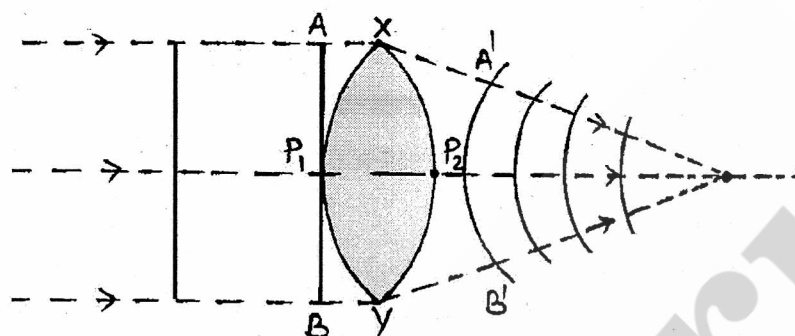


It is clear from the figure that on passing through a prism, the rays bend towards the base of the prism.

Behaviour of a Lens towards Plane Wavefront -

When a plane wavefront AB is incident on a convex lens XY, then different portion of this wavefront have to travel through different

thicknesses of the lens before emerging out, being maximum at the centre and minimum at the edges.

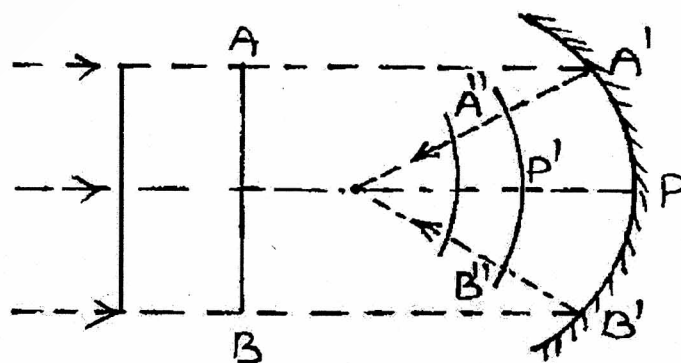


As light travels slower in lens material than in air, therefore refracted wavefront is converging spherical wavefront $A'P_2B'$. Hence on passing through a Convex lens, the parallel rays tend to converge.

Behaviour of a Spherical Mirror towards Plane Wavefront

When a plane wavefront AB is incident on a Concave mirror with pole at P . The centre of the wavefront has to travel the largest distance to P before it gets reflected, and the peripheral portion of wavefront has to travel the smallest distance before getting reflected.

Therefore the reflected wavefront is converging spherical wavefront $A''P'B''$.



Q.
CBSE
2012

When monochromatic light is incident on a surface separating two media, why does the refracted light have the same frequency as that of the incident light?

Sol. The frequency of an EM wave depends only on the source which produces it. The frequency is independent of the medium through which it travels, hence it does not change due to medium.

But the speed and wavelength depends on the medium through the wave travels.

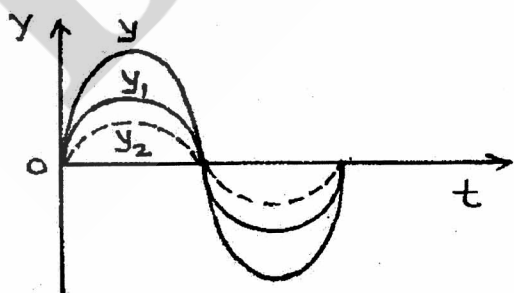
Q.
CBSE
2011

When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply reduction in its energy? Explain.

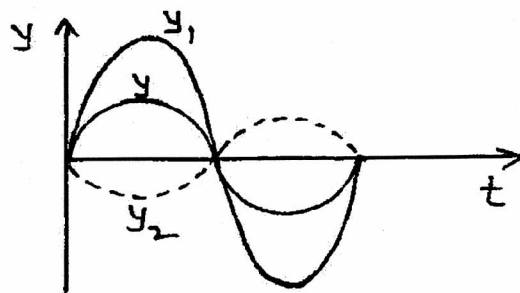
Sol. No, the energy carried by the wave does not depend on its speed. Instead it depends on the amplitude of wave.

Superposition Principle -

According to superposition principle, when two or more waves travelling through a medium superimpose one other, a new wave is formed in which resultant displacement ($\vec{y}$) at any instant is equal to the vector sum of the displacements due to individual waves ($\vec{y}_1, \vec{y}_2, \dots$) at that instant



$$y = y_1 + y_2$$



$$y = y_1 - y_2$$

Coherent Sources-

The sources of light, which emit continuous light waves of the same wavelength, same frequency and in same phase or having a constant phase difference are called coherent sources.

***Two independent sources of light cannot be coherent.**

This is because light is emitted from individual atoms, when they return to ground state after being excited by heat. Even the smallest source of light contains billions and billions of such atoms which obviously cannot emit light waves in the same phase.

Hence two independent sources of light whose light waves do not possess a constant phase difference are called non-coherent sources.

Coherent sources of light can be obtained by deriving two sources from a source by -

- (i) Division of wavefront
- (ii.) Division of amplitude

Interference of Light-

Interference of light is the phenomena of redistribution of light energy in a medium on account of superposition of light waves from two coherent sources.

* At a point, where the resultant intensity of light is maximum, interference is said to be constructive.

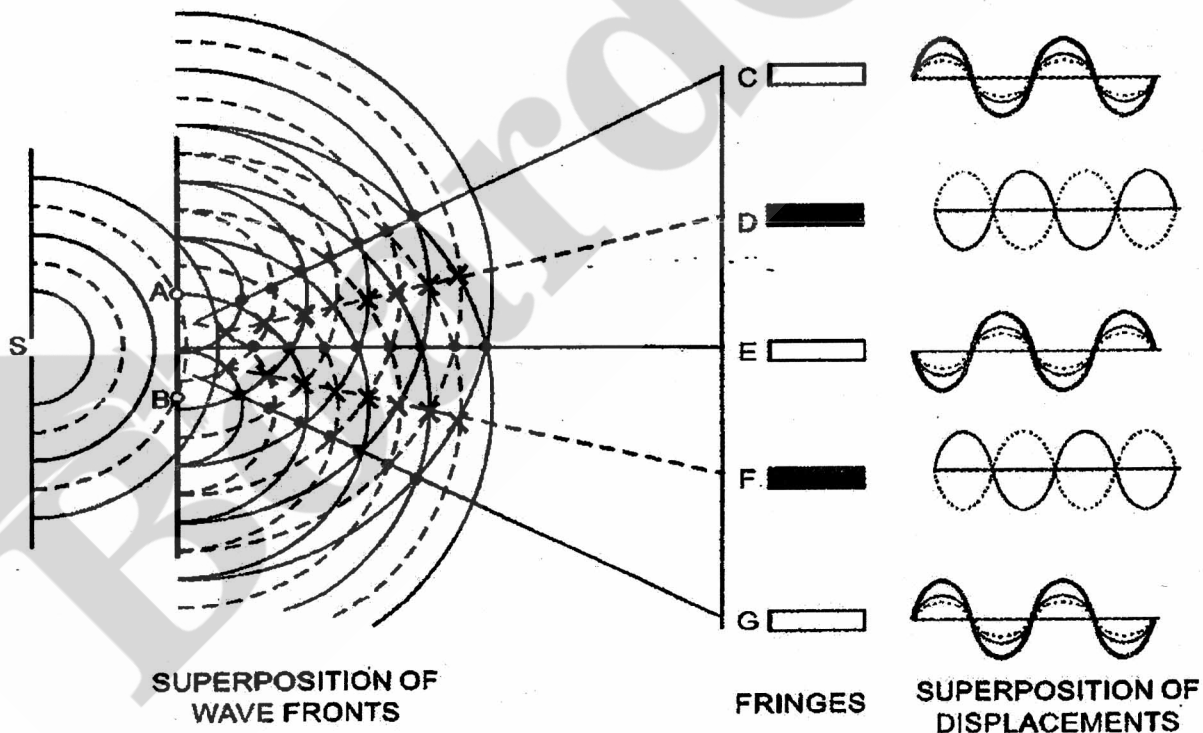
* At the points where the resultant intensity of light is minimum, the interference is said to be destructive.

* For observing interference the two sources of light must be coherent.

Young's double slit experiment -

In this experiment S is a narrow slit (of width about 1 mm) illuminated by a monochromatic source of light. At a suitable distance (about 10 cm) from S , there are two fine slits A and B about 0.5 mm apart.

When a screen is placed at a large distance (about 2 m) from the slits A and B , alternate bright and dark fringes running parallel to the lengths of slits appear on the screen. These are the interference fringes.



Explanation -

According to Huygen's principle, the monochromatic source of light illuminating the slit S sends out spherical wavefronts.

Let the solid arcs represent the crests and the dotted arcs represents the troughs. These wavefront reach the slit A and B, which in turn becomes the sources of secondary wavelets. Thus the two waves of same amplitude and same frequency with zero phase difference are given out by A and B. These waves on superposition produce interference

The dots (•) represent the positions of constructive interference, where crest of one wave falls on crest of the other and trough falls on trough. The resultant amplitude and hence intensity of light is maximum at these positions.

Similarly the crosses (x) represent the positions of destructive interference, where crest of one wave falls on trough of the other and vice-versa. The resultant amplitude and hence intensity of light is minimum at these positions.

Thus we have bright fringes at C, E and G and dark fringes at D and F. These bright and dark fringes are placed alternately and they are equally spaced.

Q Show that the superposition of the waves originating from the two coherent sources S_1 and S_2 having displacement $Y_1 = a \cos \omega t$ and $Y_2 = a \cos (\omega t + \phi)$ at a point produce a resultant intensity -

CBSE
2010

$$I = 4a^2 \cos^2 \phi/2$$

Hence, write the conditions for the appearance of dark and bright fringes.

Sol. Given that, the displacement of two coherent sources is -

$$Y_1 = a \cos \omega t$$

$$Y_2 = a \cos (\omega t + \phi)$$

By Principle of superposition -

$$Y = Y_1 + Y_2 = a \cos \omega t + a \cos(\omega t + \phi)$$

$$Y = a \cos \omega t + a \cos \omega t \cos \phi - a \sin \omega t \sin \phi$$

$$Y = a(1 + \cos \phi) \cdot \cos \omega t + (-a \sin \phi) \sin \omega t$$

$$\text{Let } a(1 + \cos \phi) = A \cos \theta \quad \text{--- (1)}$$

$$\text{and } a \sin \phi = A \sin \theta \quad \text{--- (2)}$$

$$\therefore Y = A \cos \theta \cos \omega t - A \sin \theta \sin \omega t$$

$$Y = A \cos(\omega t + \theta)$$

Squaring and adding eq. (1) and (2), we get-

$$(A \cos \theta)^2 + (A \sin \theta)^2 = a^2(1 + \cos \phi)^2 + (a \sin \phi)^2$$

$$A^2(\sin^2 \theta + \cos^2 \theta) = a^2[1 + \cos^2 \phi + 2 \cos \phi + \sin^2 \phi]$$

$$A^2 = a^2 + a^2 + 2a^2 \cos \phi$$

$$A^2 = 2a^2(1 + \cos \phi)$$

$$A^2 = 2a^2 \left(2 \cos^2 \frac{\phi}{2} \right) = 4a^2 \cos^2 \frac{\phi}{2}$$

If I is the resultant intensity, then-

$$\text{As } I = A^2$$

$$\boxed{I = 4a^2 \cos^2 \frac{\phi}{2}}$$

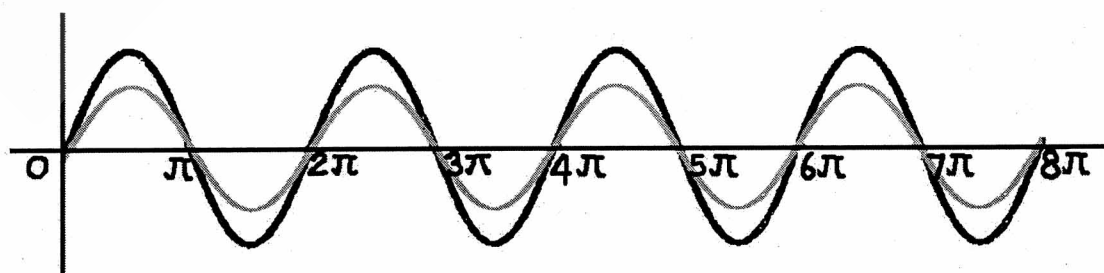
Condition for bright fringe or Constructive Interference

$$\text{When } \cos\left(\frac{\phi}{2}\right) = \pm 1 \quad \text{then } I = I_0 = 4a^2$$

$$\text{Hence } \frac{\phi}{2} = n\pi$$

or

$$\boxed{\phi = 2n\pi}$$



Also

$$\frac{\text{path difference}}{\lambda} = \frac{\text{Phase difference}}{2\pi}$$

or $x = \frac{\lambda}{2\pi} \times 2n\pi$

 $\therefore$

$$\text{Path difference } x = n\lambda \quad n = 0, 1, 2, \dots$$

Hence the bright fringe is obtained when path difference of interfering waves is $n\lambda$ and phase difference is $2n\pi$.

Condition for dark fringe or destructive Interference

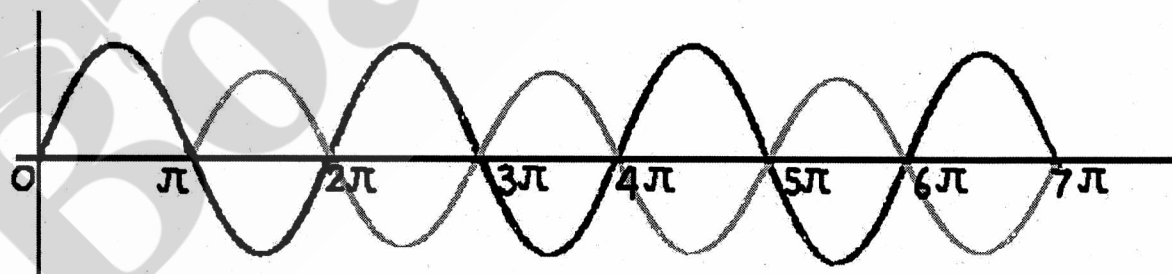
When $\cos \frac{\phi}{2} = 0$ then $I = 0$

Hence $\frac{\phi}{2} = (2n-1)\frac{\pi}{2}$

or $\phi = (2n-1)\pi$, where $n = 1, 2, \dots$

Path difference $x = \frac{\lambda}{2\pi} \times \phi$

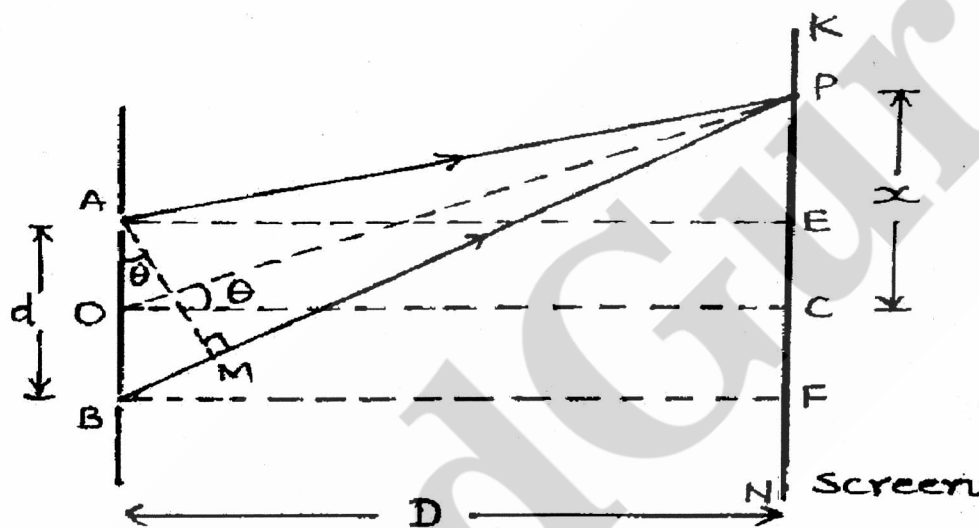
or $\text{Path difference } x = (2n-1)\frac{\lambda}{2}$



Hence dark fringes are obtained, when interfering wave have path difference is odd multiple of $\lambda/2$ and phase difference is odd multiple of π .

Expression for Fringe width in Interference.

Suppose A and B are two fine slits and separated by a distance d . The slits are illuminated by a monochromatic light of wavelength λ . KN is a screen at a distance D from the slits. The two waves starting from A and B superimpose on each other, resulting in interference pattern on the screen.



O is the centre of distance d between the slits A and B. Draw AE, BF and OC perpendicular to KN. ...

Consider a point P on the screen, at a distance x from C.

The path difference between interfering waves is given by $= BP - AP$

$$\begin{aligned} \text{path difference} &= BP - AP = BM \\ &= d \sin \theta \end{aligned}$$

Here $AM \perp BP$

and $\angle BAM = \angle POC$ (By geometry)

If θ is small, then $\sin \theta \approx \theta \approx \tan \theta$

$$\begin{aligned} \therefore \text{path difference} &= BM = d \sin \theta \approx d \tan \theta \\ &= \frac{dx}{D} \quad \left(\because \tan \theta = \frac{x}{D} \right) \end{aligned}$$

For Constructive Interference -

path difference = $n\lambda$, where $n = 0, 1, 2, \dots$

Hence $\frac{dx}{D} = n\lambda$ or

$$x_n = \frac{n\lambda D}{d}$$

$\therefore$ Fringe width of dark fringe $\beta = x_{n+1} - x_n$

$$\beta = \frac{D\lambda}{d}(n+1) - \frac{D\lambda n}{d} = \frac{D\lambda}{d}$$

$$\text{Fringe width of dark fringe } \beta = \frac{D\lambda}{d}$$

For destructive Interference -

Path difference = $(2n-1)\frac{\lambda}{2}$, ($n = 1, 2, \dots$)

Hence $(2n-1)\frac{\lambda}{2} = \frac{dx}{D}$

or

$$x_n = \frac{(2n-1)D\lambda}{2d}$$

$\therefore$ Fringe width of bright Fringe $\beta = x_{n+1} - x_n$

$$\beta = \frac{(2(n+1)-1)D\lambda}{2d} - \frac{(2n-1)D\lambda}{2d} = \frac{D\lambda}{d}$$

$$\text{Fringe width of bright fringe} = \frac{D\lambda}{d}$$

★ Here we can see that, fringe width of dark fringe is equal to the fringe width of bright fringe.

* At the sites of constructive interference -

$$I_{\max} \propto (a_1 + a_2)^2 = \text{Constant}$$

Hence all bright interference bands have the same intensity.

* At the sites of destructive interference -

$$I_{\min} \propto (a_1 - a_2)^2$$

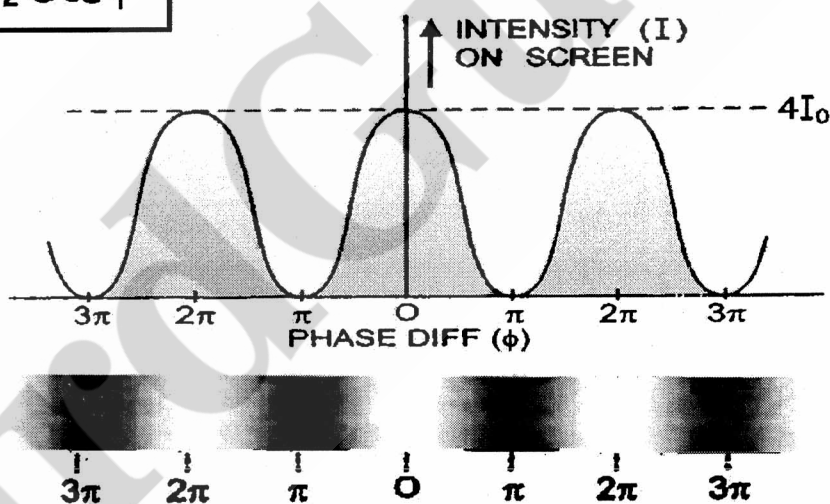
Hence all the dark fringes have same (zero) intensity.

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

$$a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2$$



Note-

When we use a source of white light, containing light of different colours (i.e. different wavelengths λ), then at a particular point, condition for constructive interference may be satisfied only for some particular value of λ . Therefore, colour corresponding to this value of λ alone, shall be visible at that point.

Hence the interference fringes will be coloured. The centre of coloured fringe pattern will be white.

As $\lambda_r > \lambda_v$ and $\beta = \frac{\lambda D}{a}$, therefore $\beta_r > \beta_v$, i.e. red fringe will be wider compared to the violet fringe.

Some Important Points-

- (1) The angular separation of interference fringes is $\frac{\lambda}{d}$. It is independent of position of the screen. The angular separation of the fringes increases with increase in λ and decreases when d increases.

Angular Position of n^{th} order -

(i) Bright Fringe $\frac{x_n}{D} = \frac{n\lambda}{d} \quad [n=1,2,\dots]$

(ii) Dark Fringe $\frac{x_n}{D} = (2n-1)\frac{\lambda}{2d} \quad [n=1,2,\dots]$

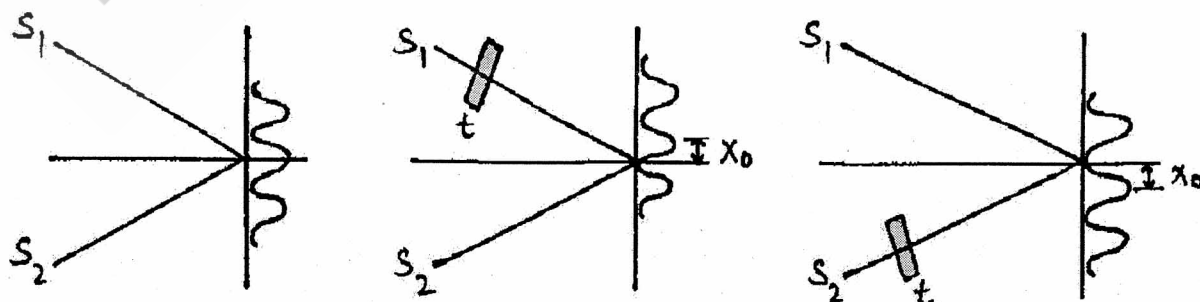
- (2) In Young's double slit experiment, when the apparatus is dipped in liquid then wavelength λ will change, it will become $\frac{1}{\mu}$ times of its value in air.

Fringe width in medium $\beta_m = \frac{D\lambda}{d\mu}$

- (3) If refracting slab of thickness t is placed in front of one of the two slits of Young's double slit experiment, then fringe pattern get shifted by

$$x_0 = \frac{\beta}{\lambda} (\mu-1)t$$

The shifting is towards the side in which the transparent plate is introduced.



Q.
CBSE
2003

How would the angular separation of interference fringes in Young's double slit experiment change when the distance between the slits and screen is halved?

Sol.

We know that -

$$\text{angular separation} = \frac{\lambda}{d}$$

Hence the angular separation of the interference fringes remains unchanged as it does not depend on D .

Q.
CBSE
2008

How does the fringe width of interference fringes change, when the whole apparatus of Young's experiment is kept in a liquid of refractive index 1.3?

Sol.

We know that -

$$\text{fringe width in medium } \beta_m = \frac{D\lambda}{d\mu} = \frac{\beta}{\mu}$$

Hence the new fringe width becomes $\frac{1}{1.3}$ times of original fringe width.

Q.
CBSE
2011

Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 7.2 mm. Calculate the wavelength of another source of laser light which produce interference fringes separated by 8.1 mm using same pair of slits.

Sol.

$$\text{As Fringe width } (\beta) = \frac{D\lambda}{d}$$

$$\Rightarrow \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} \quad [\because D \text{ and } d \text{ are same}]$$

$$\Rightarrow \lambda_2 = \lambda_1 \times \frac{\beta_1}{\beta_2} = 630 \times 10^{-9} \times \frac{8.1 \times 10^{-3}}{7.2 \times 10^{-3}} = 708.75 \text{ nm}$$

Q.
CBSE
2011

The intensity at the central maxima (0) in a Young's double slit experiment is I_0 . If the distance CP equals one third of fringe width of the pattern, show that the intensity at point P would be $I_0/4$.

Sol. Given that, the distance CP equals one-third of fringe width of the pattern. Hence

$$x = \frac{\beta}{3} = \frac{1}{3} \times \frac{D\lambda}{d} \Rightarrow \frac{x d}{D} = \frac{\lambda}{3}$$

$$\Rightarrow \text{Path difference} = BP - AP = \frac{dx}{D} = \frac{\lambda}{3}$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$\text{Hence Phase difference} = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

If intensity of the central fringe is I_0 , then intensity at point P, where phase difference is ϕ is given by -

$$I = I_0 \cos^2 \phi$$

$$\Rightarrow I = I_0 \cos^2\left(\frac{2\pi}{3}\right) = I_0 \left(-\frac{1}{2}\right)^2$$

or

$$\boxed{I = \frac{I_0}{4}}$$

Q.
CBSE
2010

In Young's double slit experiment the two slits 0.15 mm apart are illuminated by monochromatic light of wavelength 450 nm. The screen is 1 m away from the slits. Find the distance of the second bright fringe and dark fringe from the central maximum.

Sol. Given that - $d = 0.15 \text{ mm} = 1.5 \times 10^{-4} \text{ m}$.

$$\lambda = 450 \text{ nm} = 450 \times 10^{-9} \text{ m}. \quad D = 1 \text{ m}.$$

$$\text{Distance of } n^{\text{th}} \text{ bright fringe from central fringe} = \frac{nD\lambda}{d}$$

For second bright fringe $X_2 = \frac{2D\lambda}{d}$

$$X_2 = \frac{2 \times 1 \times 450 \times 10^{-9}}{1.5 \times 10^{-4}} = 6 \times 10^{-3} \text{ m.}$$

The distance of n^{th} order dark fringe from central fringe is given by-

$$x'_n = (2n-1) \frac{D\lambda}{2d}$$

For 2nd dark fringe $X'_2 = (2 \times 2 - 1) \frac{D\lambda}{2d}$

$$X'_2 = \frac{3 \times 1 \times 450 \times 10^{-9}}{2 \times 1.5 \times 10^{-4}} = 4.5 \times 10^{-3} \text{ m.}$$

Q. A beam of light, consisting of two wavelength 560 nm and 420 nm is used to obtain interference fringes in a Young's double slit experiment. Find the least distance from the central maximum, where the bright fringes, due to both the wavelengths coincide. The distance between the two slits is 4 mm and the screen is at a distance of 1 m from the slits.

Sol. Given that - $D = 1 \text{ m.}$, $d = 4 \times 10^{-3} \text{ m.}$
 $\lambda_1 = 560 \text{ nm}$, $\lambda_2 = 420 \text{ nm.}$

Let n^{th} order bright fringe of λ_1 coincide with $(n+1)^{\text{th}}$ order bright fringe of λ_2 .

$$\Rightarrow (x_n)_{\lambda_1} = (x_{n+1})_{\lambda_2}$$

$$\Rightarrow \frac{n D \lambda_1}{d} = \frac{(n+1) D \lambda_2}{d}$$

$$\text{or } n \lambda_1 = (n+1) \lambda_2$$

$$\Rightarrow \frac{n+1}{n} = \frac{\lambda_1}{\lambda_2} \Rightarrow 1 + \frac{1}{n} = \frac{560 \times 10^{-9}}{420 \times 10^{-9}}$$

$$\Rightarrow 1 + \frac{1}{n} = \frac{4}{3} \Rightarrow \frac{1}{n} = \frac{1}{3} \Rightarrow n = 3$$

∴ Least distance from the central fringe where bright fringe of two wavelength coincides is

$$(x_3)_{\lambda_1} = \frac{3D\lambda_1}{d} = \frac{3 \times 1 \times 560 \times 10^{-9}}{4 \times 10^{-3}}$$

$$(x_3)_{\lambda_1} = 0.42 \times 10^{-3} \text{ m} = 0.42 \text{ mm.}$$

Hence 3rd order bright fringe of λ_1 and 4th order bright fringe of λ_2 coincide at 0.42 mm distance from the central fringe.

Q.
Delhi
2006

In Young's double slit experiment, interference fringes are observed on a screen, kept at distance D from the slits. If the screen is moved towards the slits by $5 \times 10^{-2} \text{ m}$, the change in fringe width is found to be $3 \times 10^{-5} \text{ m}$. If the separation between the slits is 10^{-3} m . Calculate the wavelength of the light used.

Sol. Given that - $d = 10^{-3} \text{ m}$, $\Delta\beta = 3 \times 10^{-5} \text{ m}$
 $\Delta D = 5 \times 10^{-2} \text{ m}$.

∴ fringe width $\beta = \frac{D\lambda}{d}$

$$\Rightarrow \Delta\beta = \frac{\lambda}{d} \times \Delta D$$

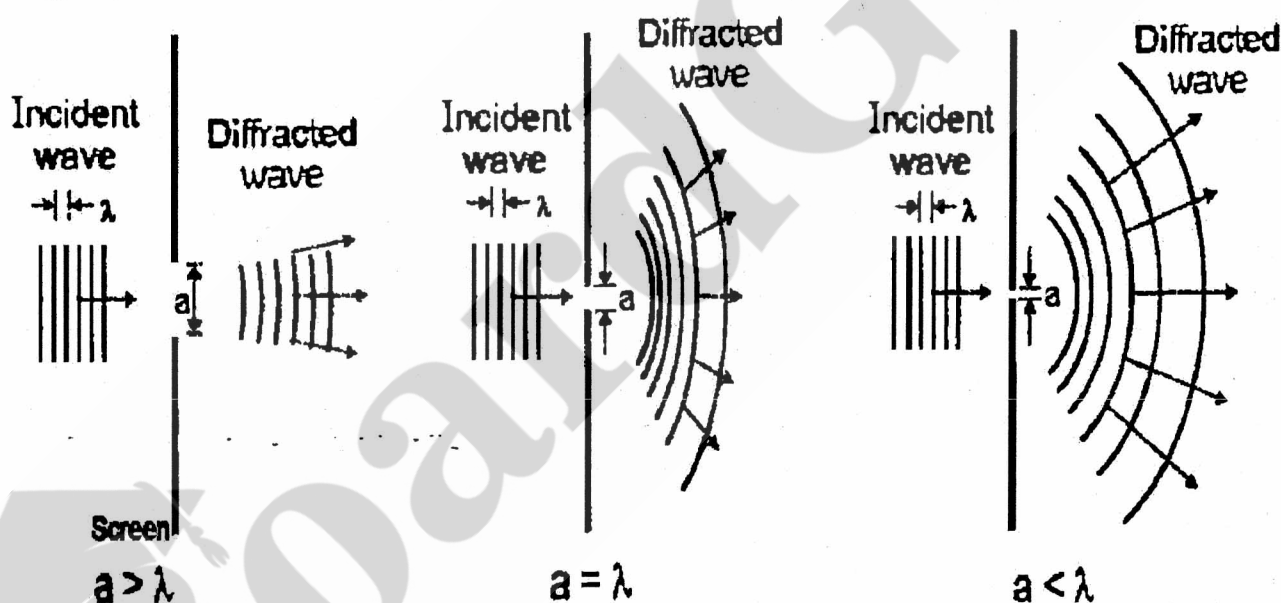
$$\Rightarrow \lambda = \frac{\Delta\beta \times d}{\Delta D} = \frac{10^{-3} \times 3 \times 10^{-5}}{5 \times 10^{-2}}$$

$$\lambda = 600 \times 10^{-9} \text{ m} = 600 \text{ nm.}$$

Diffraction of Light -

Diffraction of light is the phenomenon of bending of light around corners of an obstacle or aperture in the path of light. On account of this bending the light penetrates into the geometrical shadow of the obstacle.

The diffraction becomes much more effective when the dimensions of the aperture or the obstacle are comparable to the wavelength of light. Out of the given three cases, diffraction is most pronounced when slit width (a) is smaller than wavelength (λ) i.e. $a < \lambda$.

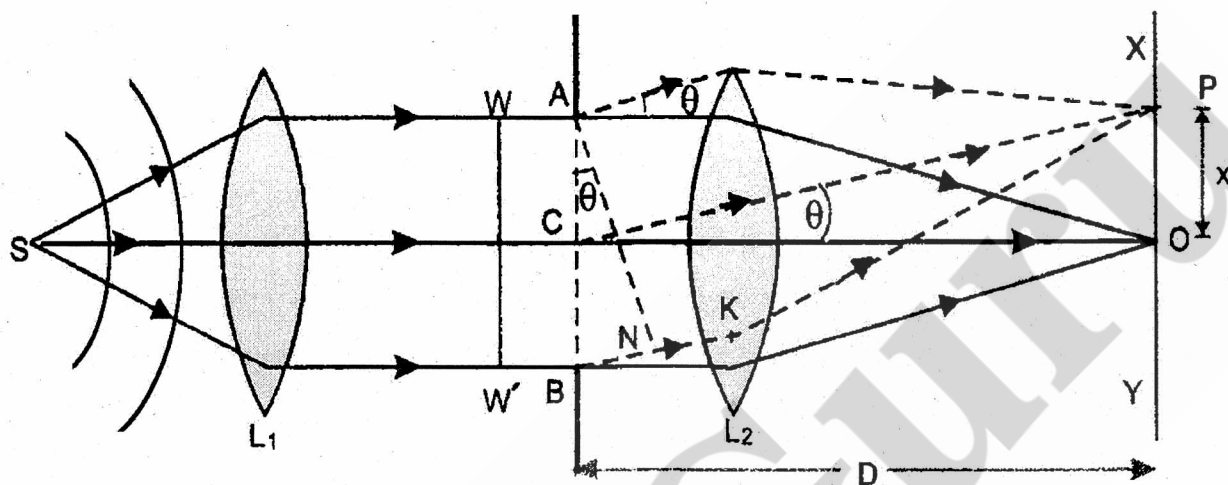


Diffraction of Light at a single slit -

In experimental setup S is a monochromatic light source held at the focus of a collimating lens L_1 . A parallel beam of light emerging from the lens with a plane wavefront WW' is made to fall on a single slit AB of width a .

The diffraction pattern is focussed on the screen XY with the help of a convex lens L_2 .

The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.



Explanation -

According to the Huygens principle each point on the plane wavefront AB sends out secondary wavelets in all the directions.

The secondary waves from points equidistant from the centre C of the slit lying in the portion CA and CB of wavefront travel the same distance in reaching O and hence the path difference between them is zero. These secondary waves superimpose each other, resulting in the maximum intensity at point O.

Position of Secondary Minima -

Consider a point P on the screen. Intensity at P will depend on the path difference between the secondary waves emitted from the corresponding point of the wavefront.

Now draw AN perpendicular to BK. Path difference between the secondary waves reaching P from A and B = BN = AB sin θ = $a \sin \theta$

If this path difference is λ , then P will be point of minimum intensity. This is because whole wavefront can be considered to be divided into two equal halves CA and CB and if the path difference between the secondary waves from A and B is λ , then the path difference between the secondary wave from A and C reaching P will be $\lambda/2$.

Also for every point in the upper half AC, there is a corresponding point in the lower half CB for which the path difference between the secondary waves reaching P is $\lambda/2$. Thus destructive interference takes place at P and therefore, P is a point of first secondary minima.

Similarly, if path difference $BN = 2\lambda$, then the point P will be the position of second secondary minimum and so on.

Hence For n^{th} secondary minimum -

$$\text{Path difference} = a \sin \theta = n\lambda$$

Position of Secondary Maxima -

If any other point P on the screen is such that the path difference $= a \sin \theta = 3\lambda/2$, then P will be the position of first secondary maximum.

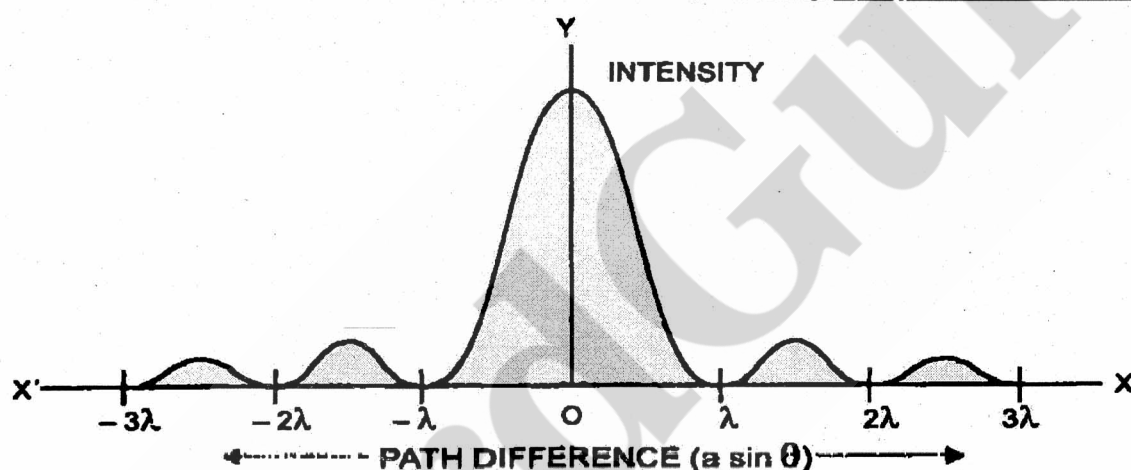
Here we can assume that the wavefront be divided into three equal parts so that the path difference between secondary waves from corresponding points in the first two parts will be $\lambda/2$. They will give rise to destructive interference.

The secondary waves from third part, remain unused and therefore they superimpose each other and produce first secondary maxima.

Similarly if $BN = 5\lambda/2$, then we get second secondary maximum of lower intensity and so on. Hence for n^{th} secondary maximum -

$$a \sin \theta = (2n+1) \frac{\lambda}{2}$$

Intensity distribution curve and diffraction Pattern



The point 0 corresponds to the position of the central bright maximum and the points with path difference $\dots a \sin \theta = \lambda, 2\lambda, \dots$ are secondary minima.

The secondary maxima are the points in between secondary minima and are of decreasing intensity. If intensity at 0 is I_0 , then the intensity at first secondary maximum is found to be $I_0/9$ and at second secondary maximum is $I_0/25$ and so on.

Width of central Maximum -

The width of central maximum is the distance between first secondary minimum on either side of 0.

If P is the position of 1st secondary minimum and $OP = x$ then - $a \sin \theta = 1 \times \lambda$

$$\text{or } \sin \theta = \frac{\lambda}{a}$$

If θ is small, then $\sin \theta \approx \tan \theta \approx \theta$

$$\text{Hence } \tan \theta = \frac{\lambda}{a} \Rightarrow \frac{x}{D} = \frac{\lambda}{a}$$

$$\text{or } x = \frac{D\lambda}{a}$$

$$\text{Width of central Maximum} = 2x = \frac{2D\lambda}{a}$$

$$\text{Angular width of central Maximum} = 2\theta = \frac{2x}{D} = \frac{2\lambda}{a}$$

Note- when source is emitting white light, then the diffraction pattern is coloured.

The central maximum is white, but other bands are coloured. As band width $\propto \lambda$, therefore, red band with higher wavelength is wider than the violet band with smaller wavelength.

Difference between Interference & Diffraction-

S. N.	Interference	Diffraction
1.	It is due to superposition of two distinct waves coming from two coherent sources.	It is produced as a result of superposition of the secondary wavelets coming from different parts of the same wavefront.
2.	In interference pattern all the bright fringes are of same intensity.	In diffraction pattern, all the bright bands are not of the same intensity.
3.	The width of the interference fringes may or may not be equal.	The fringe width of diffraction bands is unequal.

Q. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled?

Sol. $\therefore$ Angular width $(\theta) = \frac{\lambda}{a}$

Here θ is independent of separation between slit and screen (D). Hence the angular separation remains the same.

Fresnel Distance -

It is the minimum distance a beam of light can travel before its deviation from straight line path.

The angle of diffraction of central maximum can be given as- $\theta \approx \sin \theta = \lambda/a$

In travelling a distance z , the half angular width of beam would become $= \frac{z\lambda}{a}$

This beam width will become more than width of the slit. $(\frac{z\lambda}{a} \geq a)$ only when $z \geq \frac{a^2}{\lambda}$

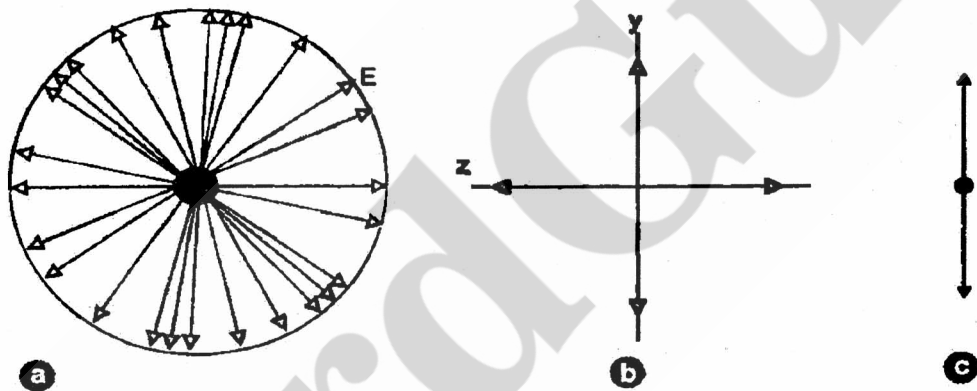
Hence

Fresnel distance $z_F = \frac{a^2}{\lambda}$

Polarization of Light -

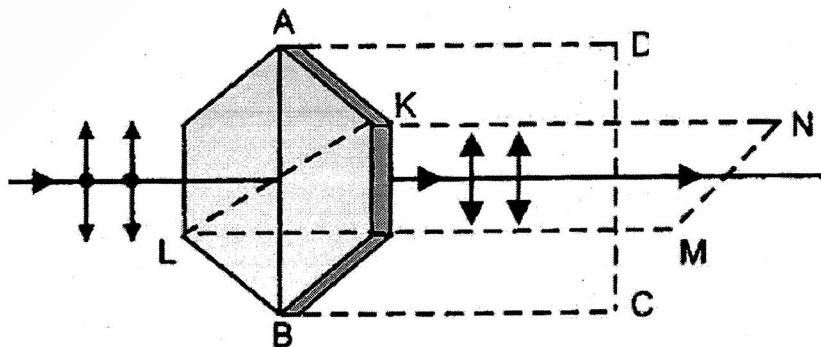
Light is an electromagnetic wave in which electric and magnetic field vectors vary sinusoidally, perpendicular to each other as well as perpendicular to the direction of propagation of Light wave.

The light propagated from common light source (sun or bulb) in a given direction consist of many independent waves whose plane of vibration are randomly oriented about the direction of propagation such light waves are said to be unpolarized.



When unpolarized light is passed through a tourmaline crystal cut with its face parallel to its crystallographic axis AB, then only those vibrations of light pass through the crystal which are parallel to AB. All other vibrations are absorbed.

The emergent light from the crystal which contain vibrations only in one direction is called plane polarized light.



Polarization -

The phenomena of restricting the vibrations of light (electric vector) in a particular direction perpendicular to the direction of wave motion is called polarization of light.

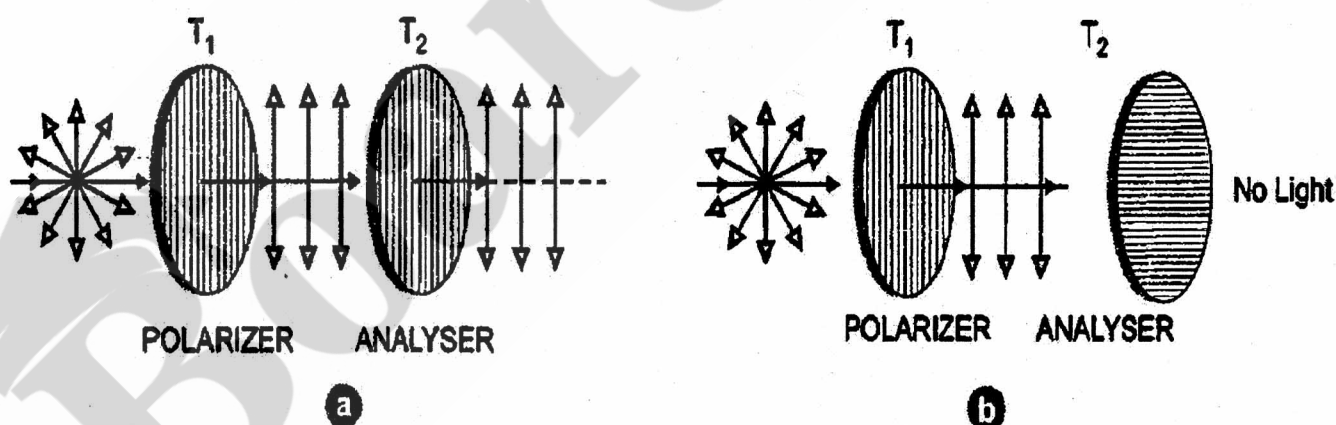
The tourmaline crystal acts as a polarizer.

The plane ABCD in which the vibrations of polarized light are confined is called the plane of vibration.

The plane KLMN which is perpendicular to the plane of vibration is defined as the plane of polarization.

Experimental Demonstration of Polarization-

In figure T_1 and T_2 are two thin plates of tourmaline.



A fine beam of ordinary light from the source S is passed through the plate T_1 and transmitted light is observed by the naked eye.

When the plate T_1 is rotated about the direction of propagation of light, the intensity of transmitted light remains the same.

Let the second plate T_2 be placed in the path

of light transmitted from T_1 , we observe that intensity of transmitted light by T_1 and T_2 remain unaffected only when T_1 and T_2 are set with their axes parallel.

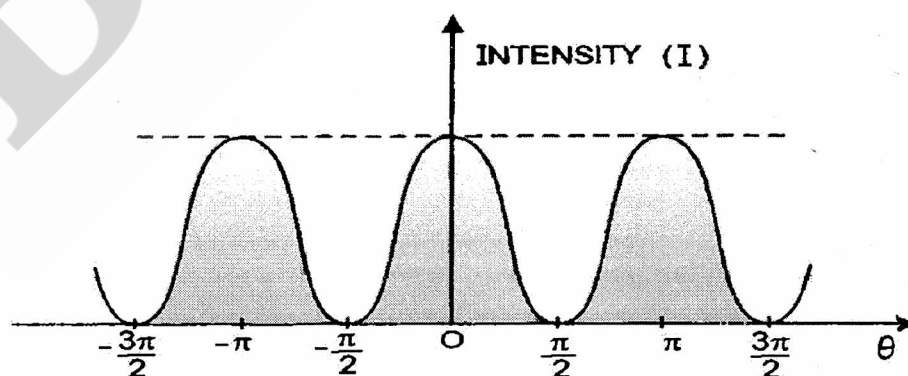
When T_2 is rotated gradually, intensity of light transmitted from T_2 goes on decreasing. As soon as the axes of the two crystals are at 90° to each other light is completely cut off. On rotating T_2 further, light reappears.

Hence light coming out of the crystal T_1 is said to be polarized and it has vibrations of electric vector which are restricted only in one direction (Parallel to the optic axis of crystal T_1).

Note -

The intensity of polarized light on passing through a tourmaline crystal changes, with the relative orientation of its crystallographic axes with that of polarizer, therefore light must consist of transverse waves.

Intensity of polarized light



θ = angle between plane of analyser and polarizer

Law of Malus -

According to Law of Malus, when a beam of completely plane polarized light is incident on an analyzer, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of the angle (θ) between plane of transmission of analyser and polarizer.

$$I \propto \cos^2 \theta$$

Proof -

Suppose $\vec{a}$ is amplitude of electric vector transmitted by the polarizer along OP. The plane of analyser OA makes an angle θ with OP. The two rectangular components of $\vec{a}$ are $a \cos \theta$ and $a \sin \theta$, which are parallel and perpendicular to plane of transmission of analyser respectively.

As the component transmitted through analyser is only $a \cos \theta$, therefore, intensity of light transmitted from analyser is -

$$I = K (a \cos \theta)^2 = K a^2 \cos^2 \theta$$

Hence

$$I = I_0 \cos^2 \theta$$

or

$$I \propto \cos^2 \theta$$

I_0 = Intensity of incident light

(i) when polariser and analyser are parallel ($\theta = 0^\circ$ or 180°)

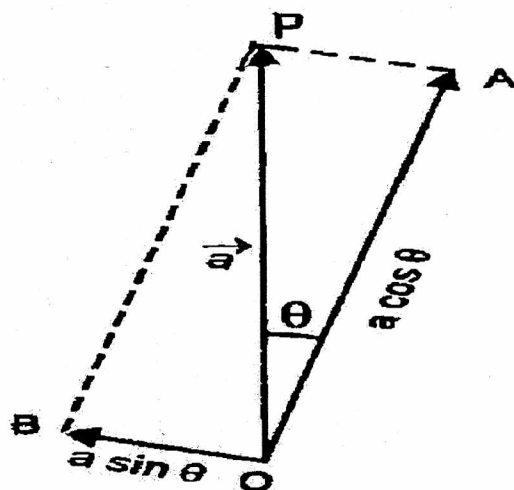
so that -

$$I = I_0$$

(ii) when polariser and analyser are perpendicular ($\theta = 90^\circ$)

so that

$$I = 0$$



Note:-

When unpolarized light of intensity I_0 gets polarized on passing through a polaroid, its intensity become half. (i.e. $I = \frac{I_0}{2}$)

Proof -

In unpolarized light, vibrations are probable in all the directions in a plane perpendicular to the direction of propagation. Therefore θ can have any value from 0 to 2π . Therefore-

$$I_{av} = \frac{1}{2\pi} \int_0^{2\pi} I_0 \cos^2 \theta d\theta = \frac{I_0}{2\pi} \int_0^{2\pi} \frac{(1 + \cos 2\theta)}{2} d\theta$$

$$I_{av} = \frac{I_0}{2\pi} \times \frac{2\pi}{2}$$

 $\Rightarrow$

$$I_{av} = \frac{I_0}{2}$$

Q.
CBSE
2011

Between two polaroids placed in crossed position, a third polaroid is introduced. The axis of the third polaroid makes an angle of 30° with the axis of the first polaroid. Find intensity of transmitted light from the system assuming I_0 to be the intensity of the polarized light incident at the first polaroid.

Sol. The intensity of transmitted light from P_1 is -

$$I_1 = \frac{I_0}{2}$$

The intensity of transmitted light from P_2 is -

$$I_2 = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 30^\circ = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow I_2 = \frac{3I_0}{8}$$

The intensity of transmitted light from P_3 is -

$$I_3 = I_2 [\cos(90^\circ - \theta)]^2 = \frac{3I_0}{8} [\cos(90^\circ - 30^\circ)]^2$$

$$I_3 = \frac{3I_0}{8} \times \left(\frac{1}{2}\right)^2$$

 $\Rightarrow$

$$I_3 = \frac{3I_0}{32}$$

Q.
CBSE
2010

Light from a sodium Lamp is passed through two polaroid sheets P_1 and P_2 kept one after the other. Keeping P_1 fixed P_2 is rotated so that its pass axis can be at different angles θ , with respect to the pass axis P_1 . The given data for the intensity of light coming out of P_2 as a function of the angle θ .

S.N.		1	2	3	4	5
1.	θ	0°	30°	45°	60°	90°
2.	I	$\frac{I_0}{2}$	$\frac{3I_0}{8}$	$\frac{I_0}{2\sqrt{2}}$	$\frac{I_0}{8}$	0

One of these observations is not in agreement with the expected theoretical variation of I . Identify this observation and write the correct expression.

Sol. The observation 3 is not in agreement with the expected theoretical variation of I .

For angle $\theta = 45^\circ$, intensity of transmitted light from P_2 is-

$$I = \frac{I_0}{2} \cos^2 45^\circ$$

$$I = \frac{I_0}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \boxed{I = \frac{I_0}{4}}$$

Q.
CBSE
2012

Two polaroids A and B are kept in crossed position. How should a third polaroid C be kept between the so that the intensity of polarized light transmitted by polaroid B reduces to $\frac{1}{8}$ th of the intensity of unpolarized light incident on A?

Sol. Let θ be the angle between the pass axis of A and C.

$$\text{Intensity of light passing through A} = \frac{I_0}{2}$$

Intensity of light passing through C is -

$$I = \frac{I_0}{2} \cos^2 \theta$$

Intensity of light passing through B is -

$$I' = \left(\frac{I_0}{2} \cos^2 \theta \right) \cos^2 (90^\circ - \theta)$$

$$I' = \frac{I_0}{2} (\cos \theta \sin \theta)^2$$

But given that $\frac{I_0}{2} (\cos \theta \sin \theta)^2 = \frac{I_0}{8}$

$$(\sin \theta \cos \theta)^2 = \frac{1}{4}$$

$$\left(\frac{\sin 2\theta}{2} \right)^2 = \frac{1}{4}$$

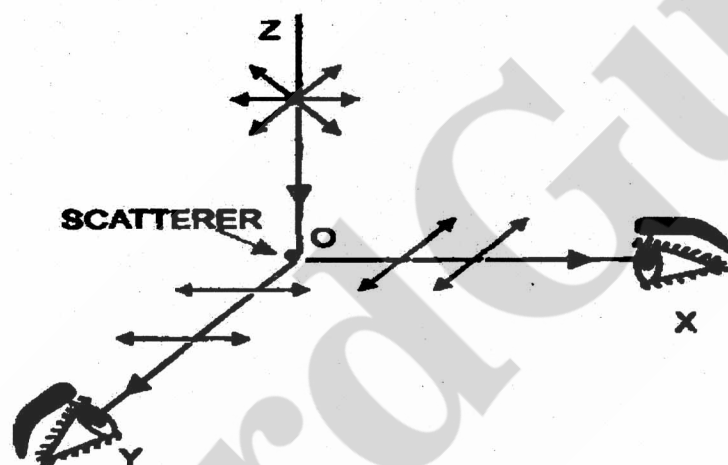
$$\sin 2\theta = 1 \quad \Rightarrow \quad 2\theta = \sin^{-1}(1)$$

$$\theta = 45^\circ$$

Polarization by Scattering-

When a beam of white light is passed through a medium containing particles whose size is of the order of wavelength of light, then the beam gets scattered.

When the scattered light is seen in a direction perpendicular to the direction of incidence it is found plane polarized. This phenomenon is called polarization by scattering.



When a beam of unpolarized light is incident along z axis on a scatterer at O . As light waves are transverse in nature, all the possible directions of vibrations of electric vector in unpolarised light are present.

When we look along x -axis, then because of transverse nature of light we can see only the vibrations of electric vector which are parallel to y -axis.

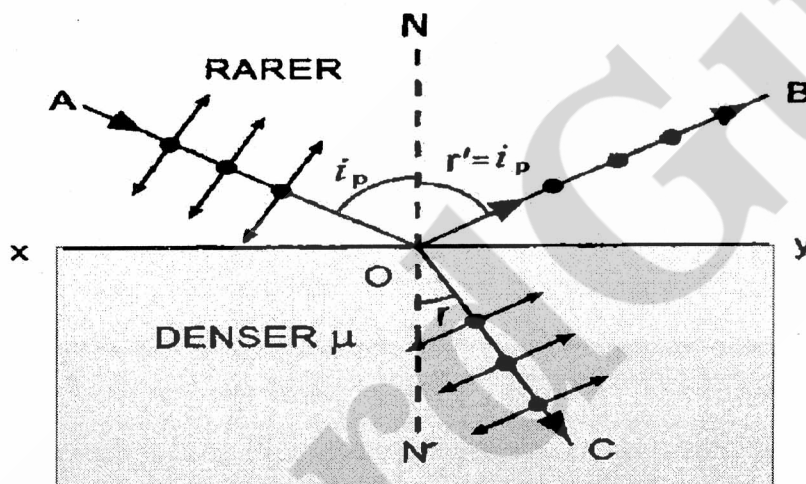
Similarly when we look along y -axis, the vibrations of electric vector seen are perpendicular to y axis or parallel to x -axis.

Hence light scattered in a direction perpendicular to the incident light is always plane polarized.

Polarization by Reflection-

When unpolarized light is reflected from a surface, the reflected light may be completely polarized, partially polarized or unpolarized. This would depend on the angle of incidence.

If angle of incidence is 0° or 90° , then the reflected beam remain unpolarized. For angles of incidence between 0° and 90° , the reflected beam is polarized to varying degree.



The angle of incidence at which the reflected light is completely plane polarized is called polarizing angle or Brewster's angle. It is represented by i_p .

The value of i_p depends on the wavelength of light used.

As shown in figure when unpolarized light is incident along AO at $\angle i_p$ on the interface XY separating air from a medium of refractive index μ . The reflected light along OB is completely plane polarized.

The vibrations of electric vector perpendicular to the plane of incidence remain always parallel to

the reflecting surface, whatever be the angle of incidence. Therefore condition of their reflection is not changed with the change in the angle of incidence.

However the other set of oscillations of electric vector in the plane of incidence make different angles with the reflecting surface, as angle of incidence of the unpolarized light is changed. At polarizing angle (i_p), most of these vibrations of electric vector get transmitted and are not reflected.

Hence refracted light along oc is a mixture of polarized light and unpolarized light i.e. it is partially polarized.

Brewster's Law -

According to this Law, when unpolarized light is incident at polarizing angle (i_p) on an interface separating air from a medium of refractive index μ , then the reflected light is fully polarized (Perpendicular to the plane of incidence), provided

$$\mu = \tan i_p$$

This relation represents Brewster's Law.

★ When light is incident at polarizing angle, the reflected component along OB and refracted component along OC are mutually perpendicular to each other.

Proof of Brewster's Law -

In figure

$$\angle BOY = \angle YOC = 90^\circ$$

$$(90^\circ - i_p) + (90^\circ - r) = 90^\circ$$

or $90^\circ - i_p = r$ ($r = \text{angle of refraction}$)

According to Snell's Law -

$$\mu = \frac{\sin i}{\sin r}$$

When $i = i_p$ and $r = (90^\circ - i_p)$

$$\therefore \mu = \frac{\sin i_p}{\sin(90^\circ - i_p)}$$

or $\mu = \frac{\sin i_p}{\cos i_p} = \tan i_p$

$\Rightarrow \boxed{\mu = \tan i_p}$ This proves Brewster's Law.

For air-water interface $\mu_w = 4/3$

$$\therefore i_p = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow i_p = 53.1^\circ$$

Thus light reflected in air from surface of water will be completely polarised, when angle of incidence of unpolarized light on the surface of water is 53.1°

Polaroids -

A polaroid is a material which polarizes light. Tourmaline is a natural polarising material.

Use of plane polarized Light and Polaroids -

- (1.) The major use of polaroids is to avoid glare of light. When we use polarized sun glasses with their vibration plane - vertical, the most of the polarized light reflected from glazed surfaces is cut off.
- (2.) To avoid the dazzling light of a car approaching from the opposite side during night driving, polaroids are fitted on the wind shield and on the cover glasses of head lights of each car.
- (3.) Clear photographs of white clouds are obtained by fitting polaroids in front of the camera lens.
- (4.) Polaroids are useful in three dimensional motion pictures.
- (5.) In CD players, polarized laser beam acts as needle for producing sound from compact disc (CD).

Very Important Questions

1 Mark Questions-

Q.1. What type of wavefront will emerge from a
(a) Point source (b) distant light source

(Delhi 2009)

Q.2. Draw a diagram to show refraction of a plane wavefront incident on a convex lens and hence draw the refracted wavefront.

(Delhi 2009)

Q.3. Draw the wavefront coming out from a convex lens when a point source of light is placed at its focus.

(CBSE 2009, 03)

Q.4. Define a wavefront.

(CBSE 2009, 02)

Q.5. Sketch the shape of wavefront emerging from a point source of light and also mark the rays.

(RBSE 2004)

Q.6. Why are coherent sources are required to create interference of light?

(CBSE 2001)

Q.7. State Huygen's principle of diffraction of light.

(CBSE 2011)

Q.8. If the angle between the axis of polarizer and the analyser is 45° ; write the ratio of the intensities of original light and the transmitted light after passing through the analyzer.

Answer - $I/I_0 = 1/4$

(Delhi 2009)

Q.9. Unpolarized light of intensity I is passed through a polaroid. What is the intensity of the light transmitted through the polaroid?

(CBSE 2009, 01)

Q.10 Unpolarized light is incident on a plane surface of glass of refractive index μ at angle i . If the reflected light gets totally polarized, write the relation between the angle i and refractive index
(Delhi 2009)

Q.11. A parallel beam of monochromatic light falls normally on a single narrow slit. How does the angular width of the principal maximum in the resulting diffraction pattern depend on width of slit?
(CBSE 2008)

2 Marks Questions -

Q.1. Write down two conditions to obtain the sustained interference fringe pattern of light.
What is the effect on interference fringes in Young's double slit experiment, when monochromatic source is replaced by a source of white light?
(CBSE 2008)

Q.2. Yellow light ($\lambda = 6000 \text{ \AA}$) illuminates a single slit of width $1 \times 10^{-4} \text{ m}$. Calculate the distance between two dark lines on either side of the central maximum when the diffraction pattern is viewed on a screen kept 1.5 m away from the slit.
(CBSE 2011)

Q.3. Define the term 'linearly polarized light'. When does the intensity of transmitted light become maximum when a polaroid sheet rotated between two crossed polaroids?
(CBSE 2005, 07)

Q.4. Explain with the help of diagram, how plane polarized light is obtained by scattering?
(RBSE 2004, 10) (CBSE 2006)

3 Marks Questions-

Q.1. Define a wavefront. Use Huygen's geometrical construction to show the propagation of plane wave front from a rarer medium (1) to a denser medium (2) undergoing refraction.

Hence derive Snell's Law of refraction. (CBSE 2012)

Q.2. Using Huygens geometrical construction of wavefronts show how a plane wave gets reflected from a surface. Hence verify Laws of reflection.

(RBSE 2008)(CBSE 2011)

Q.3. Describe Young's double slit experiment to produce interference pattern due to a monochromatic source of light. Deduce the expression for the fringe width.

(CBSE 2003, 2001)

Q.4. In a Young's double slit experiment, the two slits are kept 2mm apart and the screen is placed 140 cm. away from the plane of the slits. The slits are illuminated with light of wavelength 600 nm. Find the distance of the third bright fringe from the central maximum in the interference pattern obtained on the screen.

If the wavelength of the incident light were changed to 480 nm, find out the shift in the position of third bright fringe from the central maximum.

Answer - (i) $y_3 = 1.26 \text{ mm}$ (ii) $\Delta y = 0.252 \text{ mm}$ (CBSE 2008)

Q.5. (a) With the help of diagram describe briefly how the transverse nature of light can be demonstrated by the phenomenon of polarization?

(b) When unpolarized light passes from air to a transparent medium, under what condition does the reflected light get polarized? (Delhi 2011)

Q.6. (a) Light passes through two polaroids P_1 and P_2 with pass axis of P_2 making an angle θ with pass axis of P_1 . For what value of θ is the intensity of emergent light zero?

(b) A third Polaroid is placed between P_1 and P_2 with its pass axis making an angle β with the pass axis of P_1 . Find a value of β for which the intensity of light from P_2 is $I_0/8$, where I_0 is the intensity of light on the polaroid P_1 .

(RBSE 2005) (CBSE 2011)

Answer - (a) $\theta = 90^\circ$ (b) $\theta = 45^\circ$

Q.7. Define Brewster's angle. Obtain the relation between this angle and the refractive index for the given pair of medium. (Delhi 2010)

Q.8. In a single slit diffraction experiment, a slit of width d is illuminated by red light of wavelength 650 nm. For what value of d will-

(a) the first minimum fall at an angle of diffraction of 30° and

(b) the first maximum fall at an angle of diffraction of 30° ?

(CBSE 2009)

Answer - (a) $d = 1.3 \times 10^{-6} \text{ m}$

(RBSE 2006)

(b) $d = 1.95 \times 10^{-6} \text{ m}$

Q.9. In a single slit diffraction experiment, when a tiny circular object is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?

What is the difference between interference pattern and diffraction pattern?

(CBSE 2009, 03)

- Q.10. In the observed diffraction pattern due to a single slit, how will the width of central maximum be affected if -
- (i) the width of the slit is doubled
 - (ii) the wavelength of the light used is increased?
- (RBSE 2009, 01)

Q.11 (a) show that the angular width of the first diffraction fringe is half of that of the central fringe.

- (b) How does the width of the slit affect the size of the central diffraction band?
- (CBSE 2010, 07)

5 Marks Questions-

Q.1. (a) When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply reduction in its energy? Explain.

(b) State Huygen's principle. Using the geometric construction of secondary wavelets, explain the refraction of a plane wavefront incident at a plane surface. Hence verify Snell's law of refraction.

(CBSE 2006)

Q.2. (a) In a Young's double slit experiment, derive the condition for (i) constructive interference and (ii) destructive interference at a point on the screen

(b) A beam of light consisting of two wavelengths 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, Calculate the least distance from the central bright maximum where

the bright fringe of the two wavelengths coincide

Answer (b) 12 mm.

(CBSE 2012, 06) (RBSE 2004)

Q.3. (a) The intensity of central maxima in Young's double slit experiment is I_0 . Show that the intensity at a point where the path difference is $\lambda/3$ is $I_0/4$.

(b) What are coherent sources? Why are they necessary for observing an interference pattern? How are the two coherent sources obtained in the Young's double slit experiment?

(CBSE 2010, 2003)

Q.4 (a) Plot a graph of the intensity distribution v/s path difference in Young's double slit experiment. Compare this with the intensity distribution of fringes due to diffraction at a single slit. What important difference do you observe.

(b) In Young's double slit experiment, the two slits are 0.03 cm. apart and the screen is placed at a distance of 1.5 m away from the slit. The distance between the central bright fringe and 4th bright fringe is 1 cm. Calculate the wavelength of light used.

(CBSE 2007, 2000)

Answer - (b) $\lambda = 500 \text{ nm}$.

Q.5. In Young's double slit experiment monochromatic light of wavelength 630 nm illuminates the pair of slits and produces an interference pattern in which two consecutive fringes are separated by 8.1 mm. Another source of monochromatic light produces the interference pattern in which the two consecutive fringes are separated by 7.2 mm. Find the wavelength of second light source.

Answer - $\lambda_2 = 560 \text{ nm}$.

(CBSE 2009)

- Q.1. What are Coherent sources of light ?
- Q.2. In a single-slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band ?
- Q.3. Draw a graph showing the variation of intensity of polarised light transmitted by an analyser.
- Q.4. Two coherent waves of equal amplitude produce interference pattern in Young's double slit experiment. What is the ratio of intensity at a point where phase difference is $\pi/2$ to intensity at centre ?
Ans. $\frac{I_{\pi/2}}{I_{\text{center}}} = \frac{1}{2}$
- Q.5. What is the effect on interference fringes if red light is replaced by blue light ?
- Q.6. For which of the red or blue colours of light, the polarising angle is greater ?
 $(i_p)_B > (i_p)_R$
- Q.7. In Young's double slit experiment using monochromatic light of wavelength λ , the intensity of light at a point on the screen where path difference is λ , is K units. Find out the intensity of light at a point where path difference is $\lambda/3$.
Ans. $I' = \frac{K}{4}$
- Q.8. Give reason for the value of the Brewster angle for a transparent medium is different for lights of different colours.

Q. 9. The distance between two slits in Young's interference experiment is 0.03 cm . The fourth bright fringe is obtained at a distance of 1 cm from central fringe on a screen placed at a distance of 1.5 m from slits. Calculate the wavelength of light used.

Ans. $\lambda = 5 \times 10^{-7} \text{ m}$

Q.10. Two wavelengths of sodium light 590 nm and 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture $2 \times 10^{-4} \text{ m}$. The distance between the slit and the screen is 1.5 m . Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in two cases.

Ans. $\Delta X = 0.065 \text{ mm}$.

Q.11. A parallel beam of light of 600 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1.2 m away. It is observed that the first minimum is at a distance of 3 mm from the centre of the screen. Calculate the width of the slit.

Q.12. Which of the following waves can be polarized (i) Heat waves (ii) Sound waves? Give Reason.

Q.13. Why is the interference pattern not detected, when two coherent sources are far apart?

Q.14. The polarising angle of a medium is 60° . What is the refractive index of the medium?

Ans. $\mu = 1.732$

Q.15. How can you distinguish between an unpolarised light and a linearly polarised light beam using a polaroid?

PHYSICS

Unit - 11. Dual Nature of Matter & Radiation

Class - 12th

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11. Dual Nature of Radiation & Matter

Introduction -

The discovery of phenomena like interference, diffraction and polarisation of light established that the light is of wave nature. The Maxwell's equations of electromagnetism and Hertz experiment on the production and detection of electromagnetic waves in 1886 strongly, supported the concept of wave nature of light.

In 20th century the discoveries of photoelectric effect by Hertz, Compton effect by Compton were explained by quantum theory of light. According to which, the light consist of the packets of energy which travel in straight line, with the speed of light. Each packet of energy is called photon ($h\nu$). Thus the particle nature of light was established.

Free Electrons -

In metals, the electrons in the outer shells (Called valance electrons) of the atoms are loosely bound. They are free to move easily within the metal surface but can not leave the metal surface. Such loosely bound electrons are called free electrons.

The free electrons are held inside the metal, by attractive force of the surface called restraining forces.

Work Function (ϕ_0).

Work function of a metal is the minimum energy required by an electron to just escape from the metal surface so as to overcome the restraining forces at the surface.

Work function is represented by ϕ_0 and measured in eV.

Electron Emission -

The phenomena of emission of electrons from the surface of a metal is called electron emission.

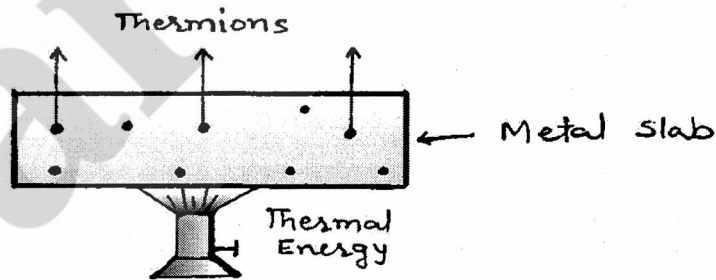
The electron emission from a metal surface is possible if the energy possessed by electrons is more than the work function of the metal.

The required energy for the electron emission from the metal surface can be supplied to the free electrons by one of the following physical processes:

(i) Thermionic emission -

It is the phenomena of emission of electrons from the metal surface when heated suitably. The energy required for the emission of electrons from metal surface is being supplied by thermal energy.

The emitted electrons are called thermal electrons or thermions. The number of thermions emitted depends on the temperature of the metal surface.



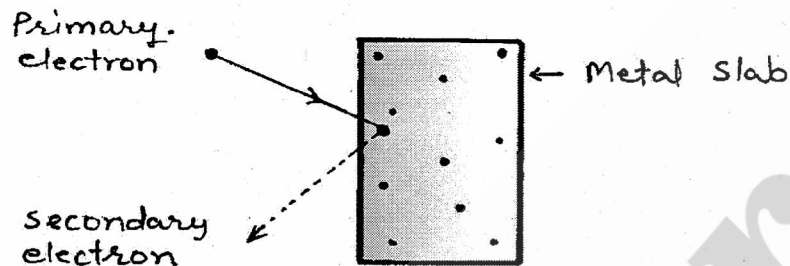
(ii) Secondary Emission -

It is the phenomena of emission of electrons from the surface of metal in large numbers when fast moving electrons (primary electrons) strike the metal surface.

The fast moving electrons are high energy electrons. As they fall on metal surface, they transfer their energy to the free electrons of the metal by

which their energy becomes more than the work function of the metal so they are emitted from the metal surface.

The electrons so emitted are called secondary electrons.

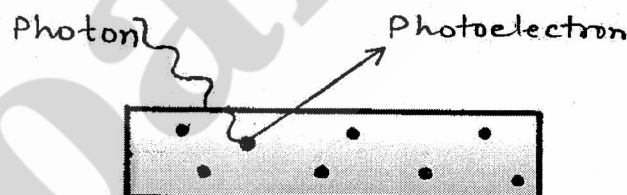


(iii) Photoelectric Emission -

It is the phenomena of emission of electrons from the surface of metal when light radiation of suitable frequency fall on it.

Here the energy to the free electrons for their emission is being supplied by light photon. The emitted electrons are called photoelectrons.

The number of photoelectrons emitted depends on the intensity of the incident light.



(iv) Field Emission / Cold Cathode Emission -

It is the phenomena of emission of electrons from the metal surface under the application of a strong electric field.

When a very strong electric field (10^8 Vm^{-1}) is applied to metal, it emits electrons.

Photoelectric Effect -

The emission of electrons from the surface of metals, when radiations of suitable frequency fall on them.

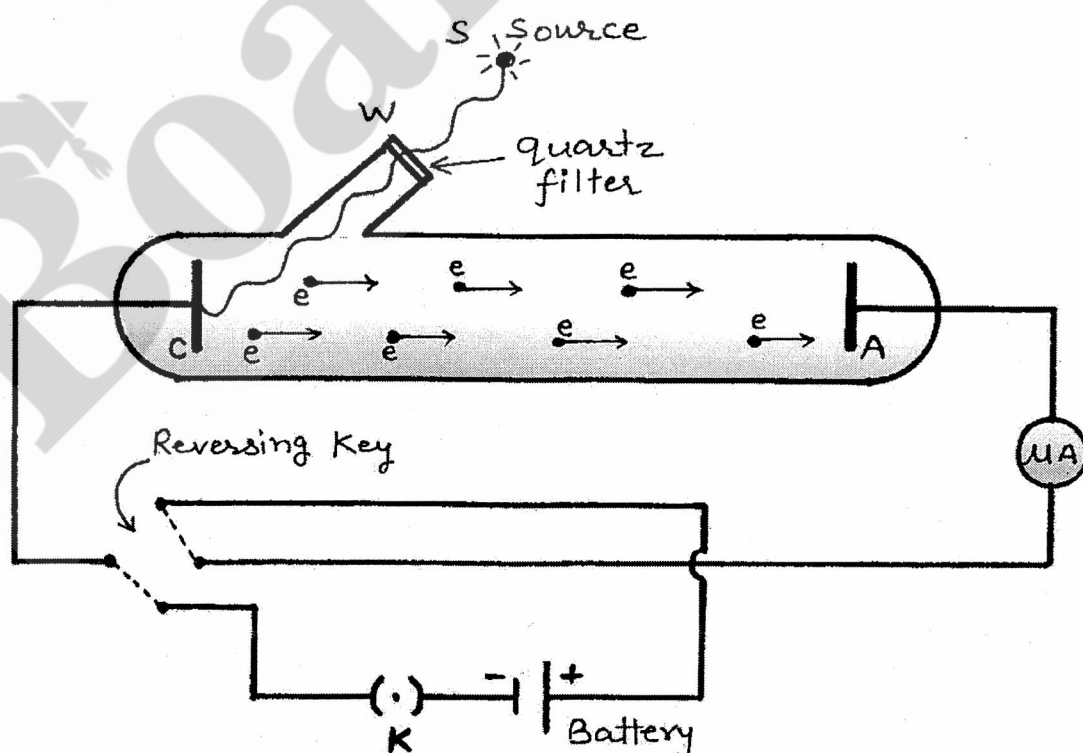
The emitted electrons are called photo-electrons and the current so produced is called photoelectric current.

Experimental Study of Photoelectric Effect -

The apparatus consists of an evacuated glass or quartz tube which encloses a photosensitive plate C (called emitter) and a metal plate A (called collector).

A transparent window W is sealed on the glass tube which can be covered with a filter for a light of particular radiation. This will allow the light of particular wavelength to pass through it.

The plate A can be given a desired positive or negative potential with respect to plate C, using the arrangement as shown in figure.



Working -

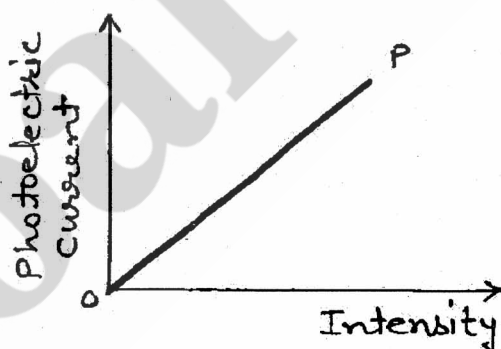
When a monochromatic radiations of suitable frequency obtained from source S, fall on the photosensitive plate C, the photoelectrons are emitted from C, which get accelerated towards the plate A (collector) if it is kept at positive potential.

These electrons flow in the outer circuit resulting in the photoelectric current. Due to it, the microammeter shows a deflection. The reading of microammeter measures the photoelectric current.

This experimental arrangement can be used to study the variation of photoelectric current with the following quantities.

(i) Effect of Intensity of the incident radiation -

By varying the intensity of the incident radiations, keeping the frequency constant, it is found that the photoelectric current varies linearly with the intensity of the incident radiation.



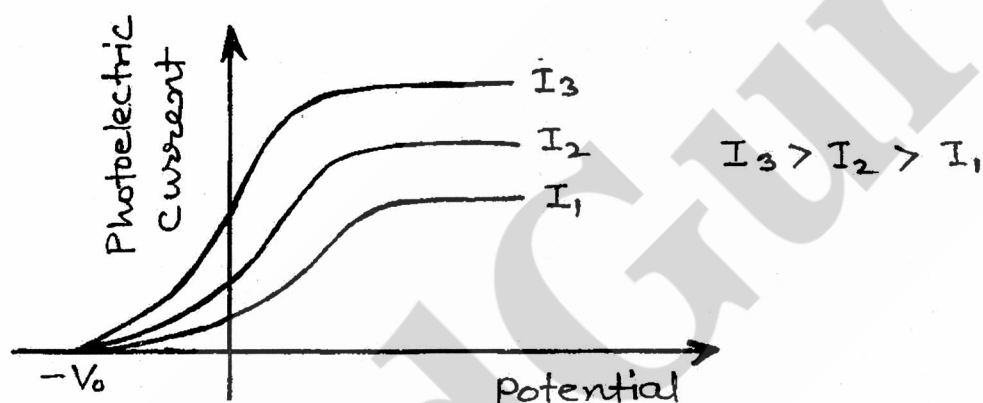
Also the number of photoelectrons emitted per second is directly proportional to the intensity of the incident radiations.

(ii) Effect of potential of plate A w.r.t. plate C -

It is found that the photoelectric current increases gradually with the increase in positive potential of plate A.

At one stage for a certain positive potential of Plate A, the photoelectric current becomes maximum or saturates. After this if we increase the positive potential of plate A, there will be no increase in the photoelectric current.

This maximum value of current is called saturation current. The saturation current corresponds to the state when all the photoelectrons emitted from C reach the plate A.



Now apply a negative potential on plate A w.r.t. plate C. We will note that the photoelectric current decreases, because the photoelectrons emitted from C are repelled and only energetic photoelectrons are reaching the plate A.

By increasing the negative potential of plate A, the photoelectric current decreases rapidly and becomes zero at a certain value of negative potential V_0 on plate A.

This maximum negative potential V_0 , given to the plate A w.r.t. plate C at which the photoelectric current becomes zero is called stopping potential or cut off potential.

$$K_{\max} = eV_0 = \frac{1}{2} m V_{\max}^2$$

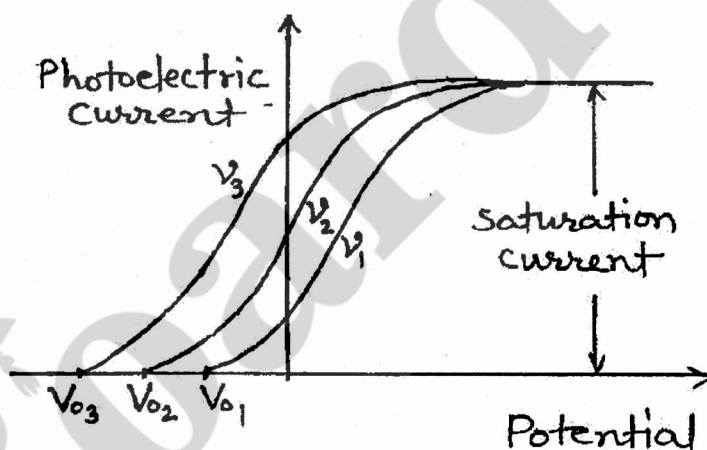
where e = charge on electron, m = mass of electron
 $V_{\max}$ = maximum velocity of emitted photoelectron.

The value of stopping potential is independent of the intensity of the incident radiation. It means, the maximum kinetic energy of emitted photoelectrons depends on the radiation source and nature of material of plate C but is independent of the intensity of incident radiation.

(iii) Effect of frequency of the incident radiation -

When we take the radiations of different frequencies but of same intensity, then the value of stopping potential is different for radiation of different frequency.

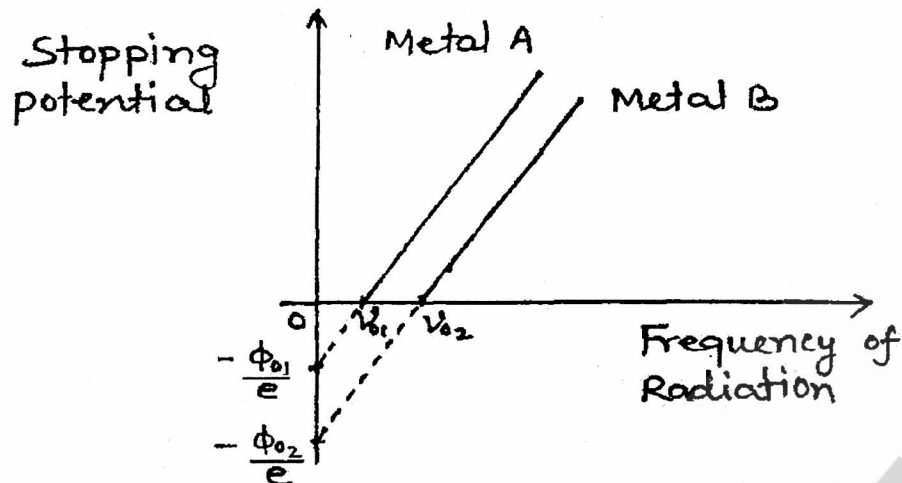
The value of stopping potential is more negative for radiation of higher incident frequency. The value of saturation current depends on the intensity of incident radiation but is independent of the frequency of the incident radiation.



(iv) Effect of frequency on stopping potential -

For a given photosensitive material, the stopping potential varies linearly with the frequency of the incident radiation.

For every photosensitive material, there is a certain minimum cut off frequency ν_0 (threshold frequency) for which the stopping potential is zero.



the intercept on the potential axis $= -\frac{\phi_0}{e} = -\frac{h\nu_0}{e}$

Hence work function $\phi_0 = e \times \text{magnitude of intercept on the potential axis}$

Note - It is found that, if the incident radiation is of higher frequency than that of threshold frequency for which the emission of photoelectron is just possible, the photoelectric emission starts instantaneously, even if the incident light is very dim.

The time lag between the incidence of radiation and emission of photo-electrons is less than 10^{-9} sec.

Laws of Photo-electric emission -

- (1) For a given metal and frequency of incident radiation, the number of photoelectrons ejected per second is directly proportional to the intensity of the incident light.
- (2) For a given metal, there exists a certain minimum frequency of the incident radiation below which no emission of photoelectrons takes place. This frequency is called threshold frequency.
- (3) Above the threshold frequency, the maximum kinetic energy of the emitted photoelectron is independent of the intensity of the incident light but depends only upon the frequency of incident light.
- (4) The photoelectron emission is an instantaneous process. The time lag between the incidence of radiations and emission of photoelectrons is very small, less than even 10^{-9} second.



The stopping potential in an experiment on photo-electric effect is 2V. What is the maximum kinetic energy of the photoelectrons emitted.?

Sol. Given that $V_0 = 2V$

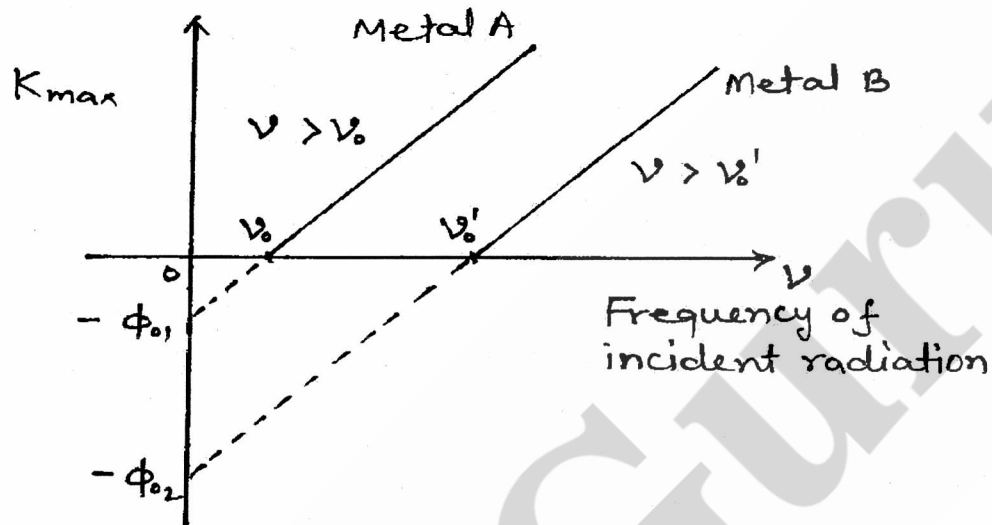
so Maximum Kinetic Energy $K_{\max} = eV_0$

$$K_{\max} = e \times 2 = 2 \text{ eV.}$$

Q.
RBSE
2007

Show graphically how the maximum kinetic energy of electrons emitted from a photosensitive surface varies with the frequency of incident radiations?

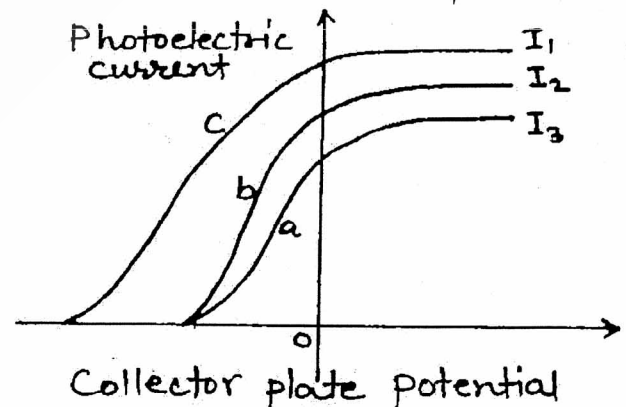
Sol.



Q.
Delhi
2009

The figure shows a plot of three curves a, b, c showing the variation of photocurrent I vs Collector plate potential for three different intensities I_1 , I_2 and I_3 having frequencies ν_1 , ν_2 and ν_3 respectively incident on a photo-sensitive surface.

Point out two curves for which the incident radiations have same frequency but different intensities.



Sol. The photoelectric current is directly proportional to the intensity of incident radiation. Energy of photoelectrons or cut-off potential depends on incident radiation.

Curve a and b have got same cut-off potential, so for these two curves, frequencies will be same.

Einstein's Photoelectric Equation -

According to the plank's quantum theory -

Light radiations consist of tiny packets of energy called quanta. One quantum of light radiation is called a photon which travels with the speed of light.

$$\text{Energy of a Photon} = E = h\nu$$

where h is plank's constant and ν is the frequency of light radiation.

Einstein assumed that one photoelectron is ejected from a metal surface if one photon of suitable light radiation fall on it.

When a photon of light of frequency ν , incident on a photosensitive metal surface. The energy of the photon ($h\nu$) is spent in two ways :

- (i) A part of photon energy is used to free the electron from the metal surface which is equal to the work function (ϕ_0) of the metal.
- (ii) The rest of the energy of the photon is used in giving the maximum kinetic energy $K_{\max}$ to the emitted photoelectron.

Max. Kinetic Energy of photoelectron

$$K_{\max} = \frac{1}{2} m v_{\max}^2$$

$$\therefore h\nu = \phi_0 + \frac{1}{2} m v_{\max}^2$$

or

$$K_{\max} = \frac{1}{2} m v_{\max}^2 = h\nu - \phi_0$$

This equation is called Einstein's photoelectric equation

Also $\phi_0 = h\nu_0$ where ν_0 is threshold frequency so that we can also write

$$K_{\max} = h(\nu - \nu_0)$$

Relation between Cut off Potential, Frequency of incident photon and Threshold Frequency -

By Einstein's photoelectric equation -

$$\text{Max. Kinetic Energy } K_{\max} = h\nu - \phi_0 \quad \text{--- (1)}$$

If V_0 is the cut off potential or stopping potential and e is the charge on the electron, then

$$K_{\max} = eV_0 \quad \text{--- (2)}$$

If ν_0 is the threshold frequency then work function $\phi_0 = h\nu_0$ --- (3)

so by combining equation (1), (2) and (3), we get -

$$eV_0 = K_{\max} = h\nu - h\nu_0$$

$$\boxed{eV_0 = h(\nu - \nu_0)}$$

$$\text{Also } eV_0 = K_{\max} = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)$$

or

$$\boxed{eV_0 = K_{\max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}$$

Q.
CBSE
2007

Ultraviolet radiation of different frequencies ν_1 and ν_2 are incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$) respectively. The Kinetic energy of the emitted electrons is same in both the cases. which one of the two radiations will have higher frequency?

Sol. K.E. of Photoelectron $K_{\max} = h\nu - W$

Here K.E. of electrons in both the cases is same.

$$\therefore (K.E.)_1 = (K.E.)_2 \Rightarrow h\nu_1 - W_1 = h\nu_2 - W_2$$

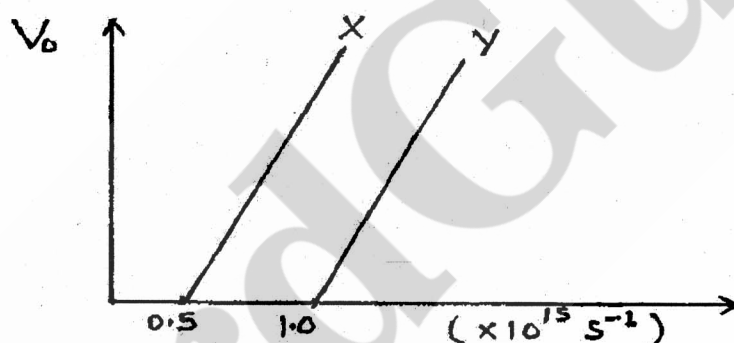
Here $W_1 > W_2$ so that

$$\boxed{\nu_1 > \nu_2}$$

Q.
CBSE
2008

The following graph shows the variation of stopping potential V_0 with the frequency ν of the incident radiation for two photosensitive metals X and Y.

- (a) which of the metals has larger threshold wavelength? Give reason.
- (b) Explain giving reasons, which metal gives out electrons, having larger Kinetic energy for the same wavelength of the incident radiation.
- (c) If the distance between the light source and metal X is halved, how will the Kinetic energy of electrons emitted from it change?



Sol. (a) From graph, threshold frequency for material X is

$$\nu_x = 0.5 \times 10^{15} \text{ s}^{-1}$$

Threshold frequency for material Y is

$$\nu_y = 1 \times 10^{15} \text{ s}^{-1}$$

It is clear that $\nu_y > \nu_x$

Threshold wavelength $\lambda \propto \frac{1}{\nu}$ so $\lambda_x > \lambda_y$

(b) $\therefore K E_{\text{max}} = h\nu - h\nu_0$

Here ν_0 is higher for metal Y for given wavelength and frequency ν .

$\therefore K E_{\text{max}}$ for metal X is greater than the metal Y

(c) No effect, because the Kinetic energy depends only on frequency of incident light.

Q.
RBSE
2005

Radiations of frequency 10^{15} Hz are incident on two photosensitive surfaces A and B. Following observations are recorded.

Surface A - No photo-electric emission take place.

Surface B - Photoemission takes place but photo-electrons have zero energy.

Explain the above observations on the basis of Einstein's photo-electric equation. How will the observation with surface B change when the wavelength of incident radiations is decreased?

Sol. Einstein's photoelectric equation states that -

$$K_{\max} = h(\nu - \nu_0)$$

For surface A -

No photoemission takes place from surface A, it means $\nu < \nu_0$ (incident frequency is less than threshold frequency because $K_{\max}$ can never be negative)

For surface B -

$$K_{\max} = 0$$

$\Rightarrow$

$$h(\nu - \nu_0) = 0$$

or

$$\boxed{\nu = \nu_0}$$

Hence frequency of incident light is equal to the threshold frequency of the photosensitive material. With the decrease of wavelength of light of incident beam, frequency increases and hence energy of incident photon increases that in turn increases the maximum K.E. of photoelectron by given photosensitive material

as

$$\boxed{K.E._{\max} = h\nu - \phi}$$

Important Graphs of Photoelectric Effect-

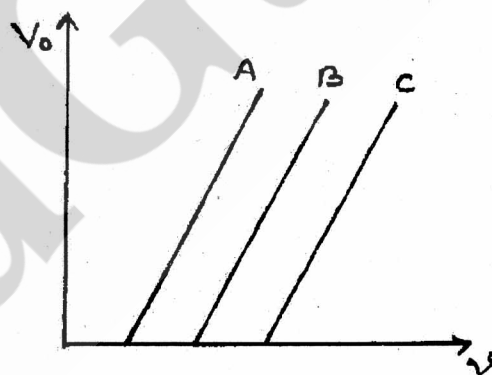
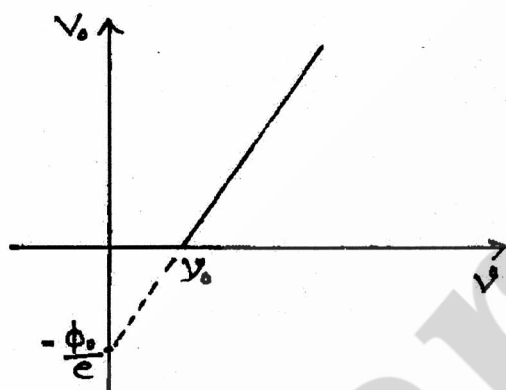
(i) Frequency (ν) and Stopping Potential (V_0) graph -

As we know $eV_0 = h\nu - \phi_0$
 or $V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$

So $V_0 \propto \nu$

Therefore ν - V_0 graph is a straight line.

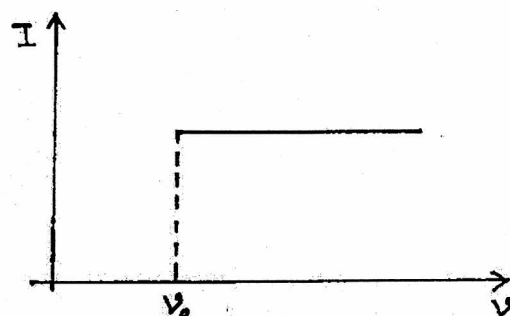
- * Comparing the above relation with $y = mx + c$, we note that the slope of ν - V_0 graph is h/e .
 The x-intercept is ν_0 and the y-intercept is $-\phi_0/e$.



- * Frequency (ν) and stopping potential (V_0) graphs for different materials will be represented by parallel straight lines, because the slope of each straight line is h/e (constant) but the threshold frequency for different materials is different.

(ii) Frequency (ν) and photoelectric current graph-

The graph shows that the photoelectric current is independent of frequency of the incident light, till intensity remains constant.

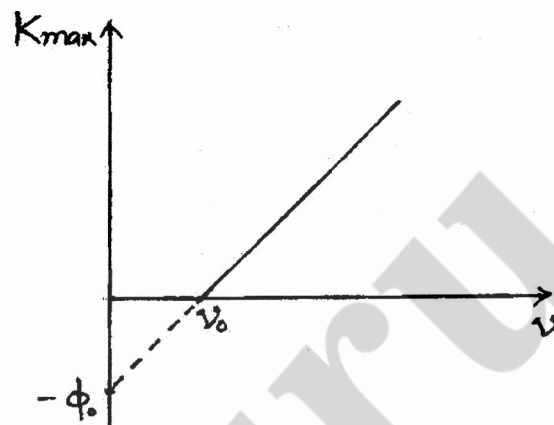


(iii) Frequency (ν) and maximum Kinetic energy graph.

As $K_{\max} = h\nu - \phi_0$

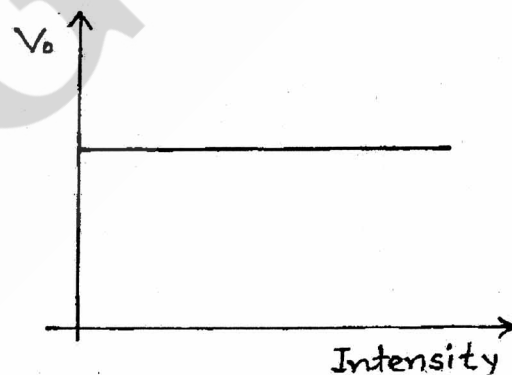
So $K_{\max} \propto \nu$

The ν - $K_{\max}$ graph is a straight line. Comparing the above relation with $y = mx + c$, we note that slope of ν - $K_{\max}$ graph is h , x -intercept is ν_0 and y -intercept is ϕ_0 .



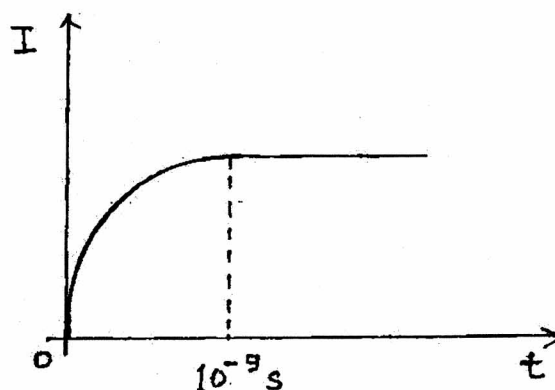
(iv) Intensity and stopping potential graph.

This graph shows that the magnitude of the stopping potential is independent of the intensity of the incident radiation, till the frequency of the radiation remains constant.



(v) Time and photoelectric Current graph.

This graph shows that the photoelectric current attains the saturation value in 10^{-9} sec. from the start of radiation falling on photosensitive material, irrespective of the intensity of incident light.



Particle Nature of Light (The photon) -

- (1) In interaction of radiation with matter, radiation behaves as it is made of particles like photons.
- (2) Each photon has energy $E (= h\nu = hc/\lambda)$ and momentum $P (= \frac{h\nu}{c} = \frac{h}{\lambda})$.
- (3) Irrespective of the intensity of radiation, all the photons of a particular frequency ν or wavelength λ have the same energy $E (= h\nu)$ and the same momentum.
- (4) By increasing the intensity of radiation of a given frequency or wavelength on a metal surface, there is only increase in the number of photons per second falling on that surface, while each photon is having the same energy.
- (5) All the photons emitted from a source of light travel through space with the same speed which is equal to the speed of light.
- (6) The frequency of photon which shows a definite energy (or color), does not change when photon travels through different media.
- (7) The velocity of photon in different media is different which is due to change in its wavelength.
- (8) Photons are not deflected by electric and magnetic fields. Hence photons are electrically neutral.
- (9) The rest mass of a photon is zero.

According to the theory of relativity, the mass m of a particle moving with velocity v , comparable with the velocity of light c is given by -

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad \text{or} \quad m_0 = m\sqrt{1 - v^2/c^2}$$

where m_0 is the rest mass of particle.
for photon $v = c$ so that $m_0 = 0$.

Failure of Wave Theory of Light to explain photoelectric Effect -

Huygen's wave theory of light could not explain the photoelectric emission due to the given reasons.

- (1) According to wave theory of light, if the intensity of light is increased, there will be increase in amplitude of waves and hence energy carried by waves will increase.

When beam of light falls on a metal surface, the energy carried by light is distributed uniformly over the entire surface of metal. All the free electrons present in the surface of metal will receive the light energy and they will be ejected out of metal surface.

When the waves of light of higher intensity falls on a metal surface it will impart more energy to the electrons in metal. Due to which the Kinetic energy of the ejected electrons will increase.

This is against the experimental facts that the maximum Kinetic energy of the ejected photoelectrons is independent of the intensity of incident light.

- (2) According to the wave theory, by increasing the intensity of the incident wave of light, the energy can be increased and photoelectric emission will take place even if the frequency of the incident light is less than threshold frequency.

But it is against the experimental fact that no photoelectric emission takes place from a metal surface if the frequency of the incident light is less than the threshold frequency, no matter whatever be the intensity.

(3) When the wave of light falls on a metal surface, the energy of the wave of light will not go to a particular electron in metal but will be distributed to all the electrons present in the illuminated portion of metal surface. As a result of which the electrons will take some time in accumulating the energy required for their emission from metal surface.

This is also against the experimental fact that the emission of photoelectrons takes place immediately after the light is incident on the metal.

Dual Nature of Radiation -

The radiation sometimes behaves as a wave and sometimes as a particle, hence radiation has dual nature. In a particular experiment, the radiation has a particular nature (Wave nature or Particle nature)

The various phenomena related to radiation can be divided into three categories:

- (1) The phenomena like interference, diffraction and polarisation etc. can be explained on the basis of electromagnetic (Wave) nature of radiation only.
- (2) The phenomena like photoelectric effect, Compton effect etc. can be explained on the basis of quantum (particle) nature of radiation.
- (3) The phenomena like rectilinear propagation, reflection, refraction etc. can be explained on the basis of either of the wave nature or of the particle nature of radiation.

De-Broglie Dualistic Hypothesis -

According to de-Broglie a moving material particle sometimes act as a wave and sometimes as a particle or a wave is associated with moving particle. The wave associated with moving particle is called matter wave or de-Broglie wave whose wavelength called de-Broglie wavelength is given by.

$$\lambda = \frac{h}{mv}$$

where m and v are the mass and velocity of the particle and h is Planck's constant.

Derivation of de-Broglie wavelength -

According to Plank's quantum theory, the energy of a photon of a radiation of frequency ν and wavelength λ is -

$$E = h\nu \quad \text{--- (1)}$$

If photon is considered to be a particle of mass m , then according to Einstein mass-energy relation, the energy associated with it is given by -

$$E = mc^2 \quad \text{--- (2)}$$

From eq. (1) and (2) we get -

$$h\nu = mc^2 \quad \text{or} \quad m = \frac{h\nu}{c^2}$$

Momentum of Photon $P = \text{mass} \times \text{velocity}$

$$P = \frac{h\nu}{c^2} \times c = \frac{h\nu}{c} = \frac{h}{\lambda}$$

or

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$

De-Broglie assumed that the above equation is equally applicable to both the photon of radiation and other material particle.

Q.
CBSE
2008

An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photoelectrons emitted from this surface have the de-Broglie wavelength λ_1 , prove that $\lambda = \left(\frac{2mc}{h}\right) \lambda_1^2$.

Sol.

For negligible work function, by Einstein's photoelectric equation

$$\frac{hc}{\lambda} = K_{\max} + 0 \quad (\phi_0 \text{ is very small})$$

$$\frac{hc}{\lambda} = \frac{p^2}{2m} \quad \left(\because K.E. = \frac{p^2}{2m} \right)$$

where p is momentum of electron

$$p = \sqrt{\frac{2mhc}{\lambda}}$$

$$\therefore \text{de-Broglie wavelength } \lambda_1 = \frac{h}{p}$$

$$\lambda_1 = \frac{h}{\sqrt{\frac{2mhc}{\lambda}}} = \sqrt{\frac{h\lambda}{2mc}}$$

squaring both sides, we get -

$$\lambda_1^2 = \frac{h\lambda}{2mc}$$

or

$$\lambda = \left(\frac{2mc}{h}\right) \lambda_1^2$$

Hence Proved.

De-Broglie Wavelength of an Electron -

Consider an electron of mass m and charge e . Let v be the velocity acquired by electron when accelerated from rest through a potential difference of V volt. Then

$$\text{Kinetic Energy of Electron} = \frac{1}{2}mv^2$$

$$\text{Work done on the electron} = eV$$

$$\therefore \frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

If λ is the de-Broglie wavelength associated with the electron, then -

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2eV/m}} = \frac{h}{\sqrt{2meV}}$$

By substituting the values of m , e and h we get -

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}} = \frac{12.27 \times 10^{-10}}{\sqrt{V}} \text{ m}$$

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

Q.
CBSE
2010

An electron is accelerated through a potential difference of 100V. What is the de-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond.

Sol. From de-Broglie equation $\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$

for $V = 100 \text{ V}$ $\lambda = \frac{12.27}{\sqrt{100}} = 1.227 \text{ \AA}$

This wavelength belong to the x-ray part of electromagnetic radiation.

Q.
CBSE
2010

The ratio between the de-Broglie wavelengths, associated with protons, accelerated through a potential of 512 V and α -particles accelerated through a potential of x volt, is found to be one. Find the value of x .

Sol. De-Broglie Wavelength of accelerated charge particle -

$$\lambda = \frac{h}{\sqrt{2mqV}} \Rightarrow \lambda \propto \frac{1}{\sqrt{mqV}}$$

Ratio of wavelengths of proton and α -particle.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)}$$

Here $\frac{m_\alpha}{m_p} = 4$, $\frac{q_\alpha}{q_p} = 2$, $\frac{V_\alpha}{V_p} = \frac{x}{512}$, $\frac{\lambda_p}{\lambda_\alpha} = 1$

So that $1 = \sqrt{4 \times 2 \times \left(\frac{x}{512}\right)} = \sqrt{\frac{x}{64}}$

$\Rightarrow x = 64 \text{ V}$

Q.
Delhi
2011

show graphically the variation of de-Broglie wavelength (λ) with the potential (V) through which an electron is accelerated from rest.

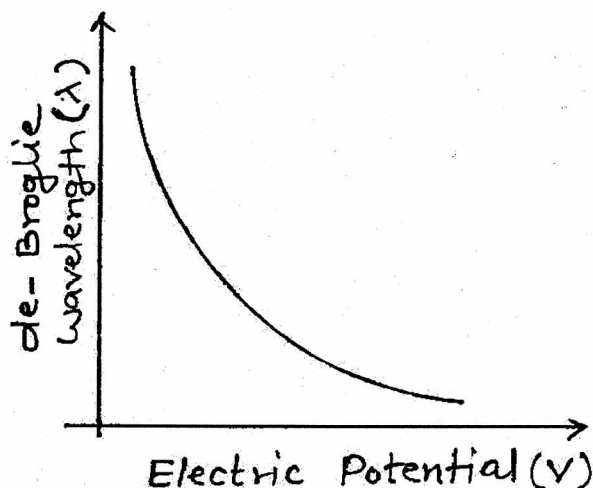
Sol. Kinetic energy $K = eV$
where V = Potential diff.

$$P = \sqrt{2mK} = \sqrt{2meV}$$

$\therefore$ de-Broglie wavelengths

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2meV}}$$

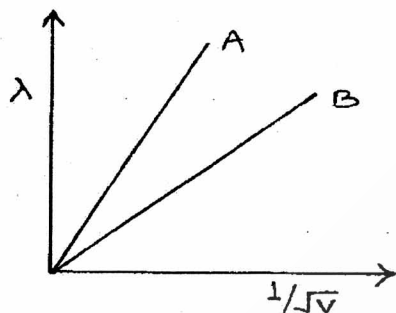
$$\lambda \propto \frac{1}{\sqrt{V}}$$



Q.
CBSE
2010

The two lines marked A and B in the given figure, shows a plot of de-Broglie wavelength λ vs $1/\sqrt{V}$, where V is the accelerating potential, for two nuclei ${}^1\text{H}^2$ and ${}^1\text{H}^3$.

- (a) What does the slope of the lines represent?
(b) Identify which lines corresponded to these nuclei.



Sol.

de-Broglie wavelength of a charged particle is given by -

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \lambda \sqrt{V} = \frac{h}{\sqrt{2mq}} = \text{Constant}$$

(a) The slope of the line represent $\frac{h}{\sqrt{2mq}}$

(b) $\because {}^1\text{H}^2$ and ${}^1\text{H}^3$ carry same charge

$$\therefore \lambda \sqrt{V} \propto \frac{1}{\sqrt{m}}$$

The lighter mass i.e. ${}^1\text{H}^2$ is represented by line of greater slope i.e. A and similarly ${}^1\text{H}^3$ by line B.

Q. (a) Determine the de-Broglie wavelength of a proton whose Kinetic energy is equal to the rest mass energy of an electron. Mass of a proton is 1836 times that of electron.

(b) In which region of electromagnetic spectrum does this wavelength lie?

Sol. (a) de-Broglie matter wave equation is given by -

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}}$$

According to question, Kinetic energy of proton

$$K = m_e c^2 \quad [\text{Einstein's relation}]$$

$$\lambda = \frac{h}{\sqrt{2m(m_e c^2)}} = \frac{h}{\sqrt{2} c \sqrt{m \cdot m_e}}$$

$$\lambda = \frac{h}{\sqrt{2} c m_e \sqrt{1836}} \quad (\because m = 1836 m_e)$$

$$\lambda = \frac{6.63 \times 10^{-34}}{1.414 \times 3 \times 10^8 \times 9.1 \times 10^{-31} \times 42.8}$$

$$\lambda = 4 \times 10^{-14} \text{ m.}$$

(b) This region of electromagnetic spectrum is X-ray.

Experimental Demonstration of Wave nature of Electron -

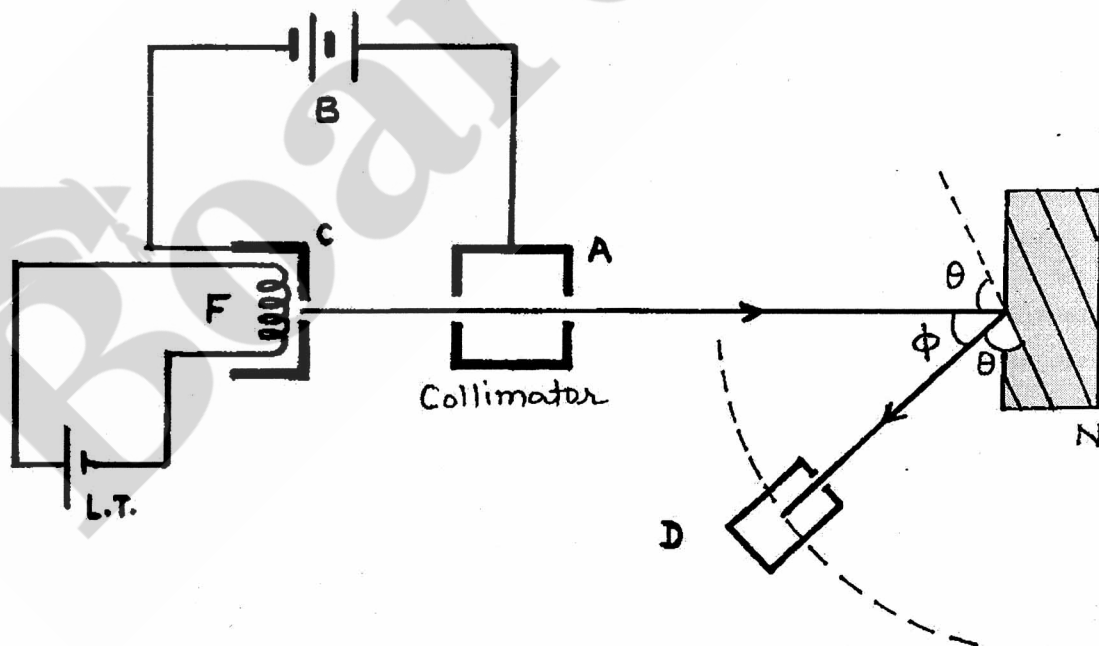
The wave nature of moving electrons has been established experimentally by Davisson and Germer in 1927.

Apparatus -

The apparatus consist of a filament F of tungsten coated with barium oxide, which on heating with current from low tension battery emits large number of electrons.

C is a hollow metallic cylinder with a hole along the axis, it surrounds the filament and is kept at negative potential, so that the electrons emitted from filament may form a convergent beam of electrons. It act as a cathode.

A is a cylinder with fine hole along its axis. It is kept at positive potential w.r.t. cathode, and is called anode.



The cathode and anode form an electron gun, by which a fine beam of electrons can be obtained, under accelerating potential applied between cathode and anode.

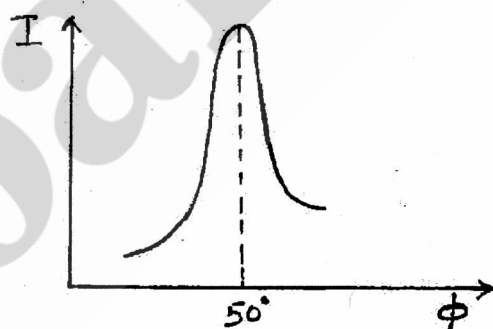
N is a nickel crystal cut along cubical diagonal. D is an electron detector. It can be rotated on a circular scale and is connected to a sensitive galvanometer which records the current.

Working -

A fine beam of accelerated electrons obtained from electron gun is made to fall normally on the surface of nickel crystal. The incident electrons are scattered in different directions by the atoms of the crystal.

The intensity of the electron beam, scattered in a given direction is found by the use of detector. By rotating the electron detector on circular scale at different positions, the intensity of the scattered beam is measured for different values of scattering angle ϕ , the angle between the incident and the scattered electron beam.

The experiment was performed by varying the accelerating voltage from 44 V to 68 V. It was noticed that at accelerating voltage 54 V, the variation of intensity (I) and scattering angle (ϕ) is as shown in figure.



From the graph it is noted that at accelerating voltage 54 V, there is a sharp peak in the intensity of the scattered electrons for scattering angle $\phi = 50^\circ$.

The appearance of peak in a particular direction is due to constructive interference of electrons scattered from different layers of regularly spaced atoms of the crystal i.e. the diffraction of electrons takes place. This establishes the wave nature of electrons.

Here for the scattering angle $\phi = 50^\circ$, the angle of glancing θ (angle between the scattered beam of electron with the plane of atoms of the crystal), for the electron beam will be given by -

$$\theta + \phi + \theta = 180^\circ$$

$$\text{or } \theta = \frac{1}{2}(180^\circ - 50^\circ) = 65^\circ$$

Now for the nickel crystal, the interatomic separation is - $d = 0.91 \text{ \AA}$

According to Bragg's Law for first order diffraction maxima ($n=1$), we have -

$$2d \sin \theta = 1 \times \lambda$$

$$\therefore \lambda = 2 \times 0.91 \times \sin 65^\circ = 1.65 \text{ \AA}$$

According to de-Broglie hypothesis, the wavelength of the wave associated with electron is given by

$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}}$$

$$\lambda = 1.66 \text{ \AA}$$

This shows that the estimated value of de-Broglie wavelength and the experimental value determined by Davisson and Germer are very close. This prove the existence of de-Broglie waves for the moving electrons.

Q. crystal diffraction experiment can be performed either by using electrons accelerated through appropriate voltage or by using x-rays. If the wavelength of these probes (electrons or x-rays) is $1 \text{ \AA}$, estimate which of the two has greater energy.

Sol. For an electron beam, the de-Broglie wave equation states that -

$$\lambda = \frac{h}{\sqrt{2emv}} = \frac{h}{\sqrt{2mk}}$$

$$\Rightarrow K = \frac{h^2}{2m\lambda^2}$$

For x-ray photon of same wavelength $\lambda = 1 \text{ \AA}$.

$$E' = h\nu = \frac{hc}{\lambda}$$

$$\text{Hence } \frac{K}{E'} = \frac{h^2/2m\lambda^2}{hc/\lambda} = \frac{h}{2mc\lambda}$$

$$\frac{K}{E'} = \frac{h}{2mc\lambda} = \frac{6.6 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 3 \times 10^8 \times 1 \times 10^{-10}}$$

$$\frac{K}{E'} = \frac{11}{911}$$

Here $\frac{K}{E'} < 1$

So that

$$K < E'$$

Energy possess by x-ray is more than electron.

Doppler's Effect in Light -

According to Doppler's effect, whenever there is a relative motion between a source of light and observer, the apparent frequency of light received by observer is different from the true frequency of light emitted actually from the source of light.

The apparent frequency of light increases when the distance between source of light and observer is decreasing and the apparent frequency of light decreases if the distance between source of light and observer is increasing.

Suppose a source of light emit waves of freq. ν and wavelength λ . If c is velocity of light in vacuum and the relative velocity between source and observer is v then -

$$\boxed{\frac{\Delta \nu}{\nu} = -\frac{\Delta \lambda}{\lambda} = -\frac{v}{c}}$$

where $\Delta \nu = \nu' - \nu$
 $\Delta \lambda = \lambda' - \lambda$

Here ν' and λ' are apparent frequency and wavelength respectively.

- * When source and observer approach each other, $\Delta \nu$ is +ve, i.e. apparent freq. increases or apparent wavelength decreases. This is called blue shift.
In this case velocity is +ve.
- * When source and observer move away from each other, $\Delta \nu$ is negative i.e. apparent freq. decreases or apparent wavelength increases. This is called Red Shift.
In this case velocity is -ve.

[Q.] The spectral line for a given element in the light received from a distant star shifted towards longer wavelength side by 0.025%. Calculate the velocity of star in the line of sight.

Sol. Here $\frac{\Delta\lambda}{\lambda} = 0.025\% = \frac{0.025}{100} = 2.5 \times 10^{-4}$

As we know that $\frac{\Delta v}{v} = -\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$

$\therefore -\frac{v}{c} = -\frac{\Delta\lambda}{\lambda} \Rightarrow v = \frac{\Delta\lambda}{\lambda} \times c$

So that $v = 2.5 \times 10^{-4} \times 3 \times 10^8 = 7.5 \times 10^4 \text{ m/s}$

As shift is towards longer wavelength side, the star is moving away.

[Q.] With what velocity should a galaxy move with respect to us so that the sodium line at 589.0 nm is observed at 589.6 nm?

Sol. Here $\lambda = 589.0 \text{ nm}$ and $\lambda' = 589.6 \text{ nm}$

From $v = \frac{\Delta\lambda}{\lambda} \times c = \frac{(589.6 - 589.0) \times 10^{-9} \times 3 \times 10^8}{589.0 \times 10^{-9}}$

$v = 3.06 \times 10^5 \text{ m/s}$

The galaxy is moving away from us, as apparent wavelength increases.

Very Important Questions

1 Mark Questions -

Q.1. Define the term 'stopping potential' in relation to photoelectric effect. (CBSE 2011,08)

Q.2. Show the variation of photocurrent with collector plate potential for different frequencies but same intensity of incident radiation. (RBSE 2011)

Q.3. Show the variation of photocurrent with collector plate potential for different intensities but same frequency of incident radiation. (CBSE 2005)

Q.4. The stopping potential in an experiment on photoelectric effect is 1.5 V. What is the maximum kinetic energy of the photoelectrons emitted? Answer = 1.5 eV (CBSE 2009)

Q.5. The maximum kinetic energy of a photoelectron is 3 eV. What is the stopping potential? Answer = 3 V (CBSE 2006)

Q.6. Show graphically how the stopping potential for a given photosensitive surface varies with frequency of incident radiations? (RBSE 2007)

Q.7. State de - Broglie hypothesis. (CBSE 2012)

Q.8. Name an experiment which shows wave nature of electron. Which phenomenon was observed in this experiment using an electron beam? (CBSE 2010)

Q.9. An electron and alpha particle have the same kinetic energy. How are the de-Broglie wavelength associated with them related?

(Delhi 2008)

Q.10. De-Broglie wavelength associated with an electron accelerated through a potential difference V is λ . What will be its wavelength when the accelerating potential is increased to $4V$?

Answer = $6.135 \text{ \AA}$

(CBSE 2006)

Q.11. With what purpose was famous Davisson-Germer experiment with electrons performed?

(Delhi 2006)

2 Marks Questions -

Q.1. Write Einstein's photoelectric equation. State clearly the three features observed in photoelectric effect which can be explained on the basis of above equation.

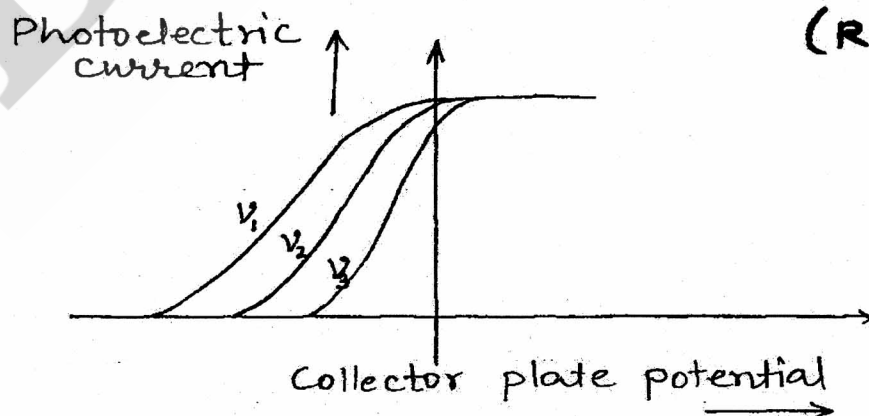
(CBSE 2010, 05)

Q.2. The given graph shows variation of photoelectric current with collector plate potential for different frequencies of incident radiations.

(a) Which physical parameter is kept constant for the three curves?

(b) Which frequencies (ν_1 , ν_2 or ν_3) is the highest?

(RBSE 2009)



Q.3. Plot a graph showing variation of stopping potential (V_0) with the frequency (ν) of the incident radiation for a given photosensitive material. Hence state the significance of the threshold frequency in photoelectric equation.

Using the principle of energy conservation, write the equation relating the energy of incident photon, threshold frequency and the maximum kinetic energy of the emitted photoelectrons.

(Delhi 2009)

Q.4. Write Einstein's photoelectric equation in terms of the stopping potential and the threshold frequency for a given photosensitive material. Draw a plot showing the variation of stopping potential v/s the frequency of incident radiation.

(RBSE 2008)

Q.5. Write Einstein's photoelectric equation. Explain the terms (a) Threshold frequency (b) Stopping potential.

(CBSE 2008)

Q.6. An α -particle and a proton are accelerated from rest by the same potential. Find the ratio of their de-Broglie wavelengths.

(CBSE 2010)

Q.7. An electron is accelerated through a potential difference of 144 V. What is the de-Broglie wavelength associated with it? To which part of electromagnetic spectrum does this wavelength correspond?

(Delhi 2010)

Answer = $1 \text{ \AA}$, X-ray

Q.8. For what kinetic energy a neutron, will the associated de-Broglie wavelength be $1.32 \times 10^{-10} \text{ m}$.

Answer = K.E. = $7.5 \times 10^{-21} \text{ J}$

(CBSE 2008)

Q.9. Derive an expression for the de-Broglie wavelength associated with an electron accelerated through a potential V . Draw a schematic diagram of a localized-wave describing the wave nature of the moving electron.

(CBSE 2009)

Q.10. Calculate the ratio of the accelerating potential required to accelerate a proton and an α -particle to have the same de-Broglie wavelength associated with them.

Answer : $V_p : V_\alpha = 8 : 1$

(Delhi 2009)

3 Marks Questions .

Q.1 Define the terms 'cut-off voltage' and 'threshold frequency' in relation to the phenomenon of photoelectric effect.

Using Einstein's photoelectric equation show how the cut off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph.

(CBSE 2012, 07)

Q.2. In a plot of photoelectric current versus anode potential, how does

(a) The saturation current vary with anode potential for incident radiations of different frequencies but same intensity?

(b) The stopping potential vary for incident radiations of different intensities but same frequency?

(c) Photoelectric current vary for different intensities but same frequency of incident radiations?

(Delhi 2007)

Q.3. The mass of a particle moving with velocity $5 \times 10^6 \text{ m/s}$ has de-Broglie wavelength associated with it to be 0.135 nm . Calculate its mass.

In which region of the electromagnetic spectrum does this wavelength lie?

Answer = $9.82 \times 10^{-31} \text{ kg}$. , X-ray

(RBSE 2011)

Q.4. A proton and an α -particle are accelerated through the same potential. Which of the two has
(a) Greater value of de-Broglie wavelength associated with it and

(b) less Kinetic Energy

(CBSE 2009)

Answer = (a) $\lambda_p : \lambda_\alpha = 2\sqrt{2} : 1$ (b) $(K.E)_p < (K.E)_\alpha$

Q.5. Draw the schematic diagram of the experimental arrangement used by Davisson and Germer to establish the wave nature of electrons. Explain briefly how the de-Broglie relation was experimentally verified in case of electrons?

(RBSE 2007, 10)

Q.1. Write the basic features of photon of electromagnetic radiation on which Einstein's photoelectric equation is based.

Q.2. An electron and a proton, each have de-Broglie wavelength of 1 nm. (a) Find the ratio of their momenta. (b) Compare the Kinetic energy of the proton with that of the electron.

Ans. (a) 1:1 (b) $\frac{K.E_p}{K.E_e} = 5.4 \times 10^{-4}$

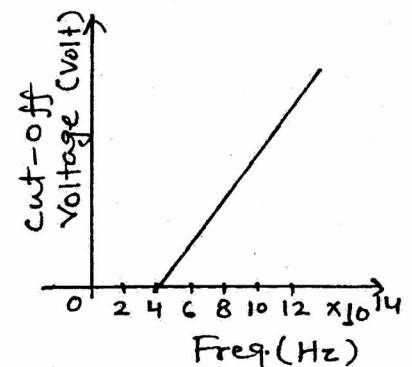
Q.3. In a photoelectric effect the yellow light is just able to emit electrons, will green light emit photo-electrons? What about red light?

Q.4. Light from source is incident on a metallic surface. The maximum Kinetic energy is E and current is I. If the distance of source from metal is doubled, what will be the values of Kinetic energy and current?

Hint $\rightarrow$ Intensity $\propto \frac{1}{r^2}$ and Photo current $\propto$ Intensity.

Ans. $I' = \frac{I}{4}$

Q.5. For photoelectric effect in sodium, the figure shows the plot of cut-off voltage versus frequency of incident radiation. Calculate (i) threshold freq. (ii) work function for sodium.



Ans. (i) $\nu_0 = 4 \times 10^{14}$ Hz (ii) $\phi_0 = 1.65$ eV.

Q.6. Calculate the de-Broglie wavelength of a neutron of Kinetic energy 150 eV. mass of neutron $= 1.67 \times 10^{-27}$ kg.

PHYSICS

Unit – 12. Atom

Class – 12th

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12. ATOMS

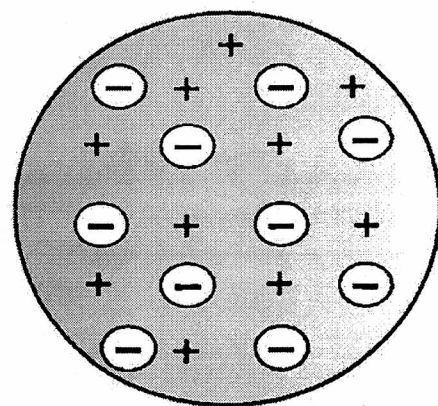
In this unit, we shall discuss the models of atoms in some detail. The first contribution in this regard come from Dalton, who proposed that matter is made of atoms, which are indivisible. J.J. Thomson proposed a structure for the atom, which is modified by Rutherford and later by Niels Bohr.

Thomson Model of Atom -

According to the Thomson model, every atom consist of a positively charged sphere of radius of the order of 10^{-10}m in which entire mass and positive charge of the atom are uniformly distributed. Inside this sphere, the electrons are embedded like seeds in a watermelon. or like plums in a pudding. The number of electron is such that their negative charge is equal to the positive charge of the atom. Thus... the atom is electrically neutral.

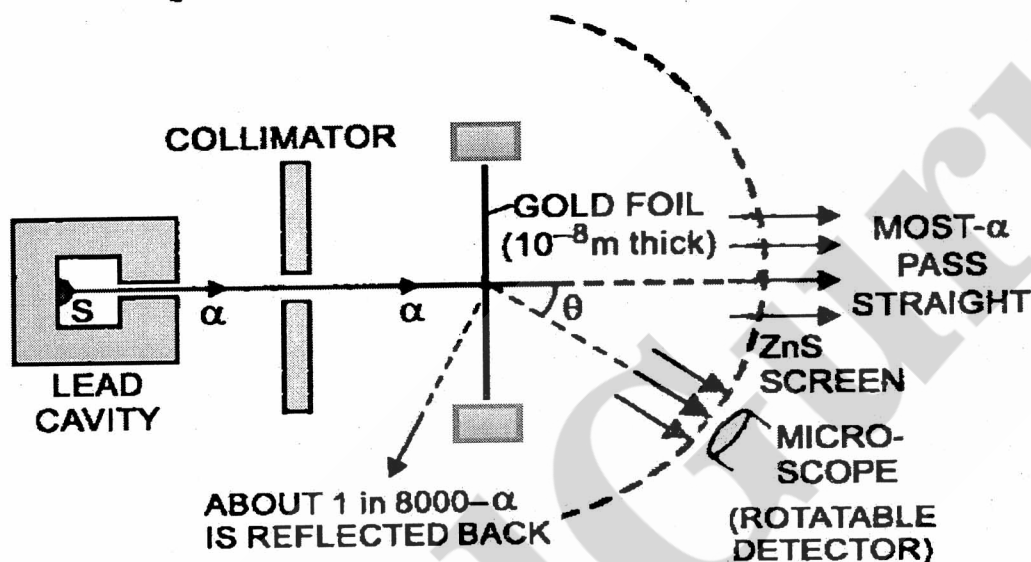
Limitations of Thomson Atom model -

- (i) It could not explain the origin of spectral series of hydrogen and other atoms, observed experimentally.
- (ii) It could not explain the large angle scattering of α -particles from thin metal foils, as observed by Rutherford.



Rutherford's α -ray Scattering Experiment:

The experimental setup used by Rutherford and his collaborators, Geiger and Marsden is shown in fig -



S is a piece of radioactive source (${}_{83}\text{Bi}^{214}$) contained in a lead cavity. The α -particles emitted by the source are collimated into a narrow beam with the help of a lead slit (collimator).

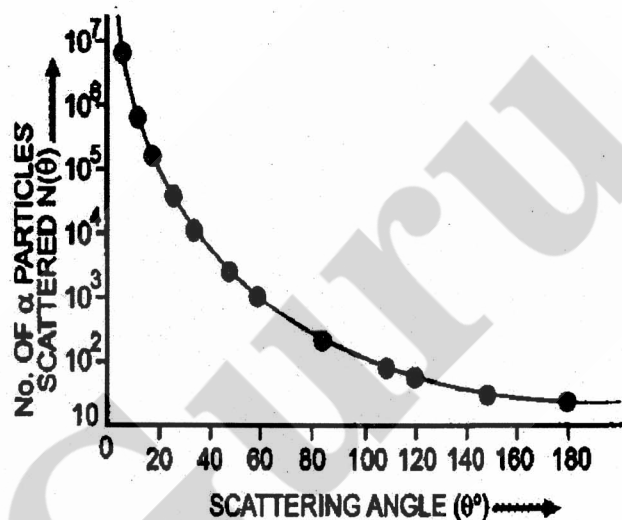
The collimated beam is allowed to fall on a thin gold foil of thickness of the order of $2.1 \times 10^{-8} \text{ m}$. The α -particles scattered in different directions are observed through a rotatable detector consisting of a zinc sulphide screen and a microscope.

The α -particles produce bright flashes on the ZnS screen. These are observed in the microscope and counted at different angles from the direction of incidence of the beam. The angle θ of deviation of an α -particle from its original direction is called its scattering angle θ .

Observations -

A graph is plotted between the scattering angle θ and the number of α -particles N , scattered at $\angle\theta$ for a large number of α -particles. We find that -

- (i) Most of the α -particles pass straight through the gold foil. It means they do not suffer any collision with gold atoms.
- (ii) Only about 0.14% of incident α -particles scatter by more than 1° .
- (iii) About one α -particle in every 8000 α particles deflects by more than 90° .



Explanation -

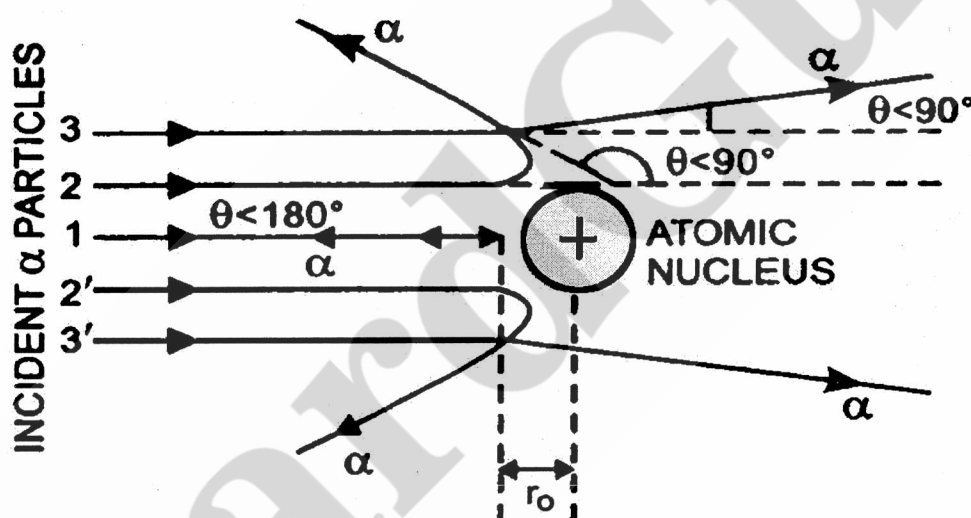
An α -particle is over 7000 more massive than an electron, and in this experiment, α -particle is travelling at a high speed, therefore, very strong force alone could have deflected them through large angles. This led Rutherford to postulate that the entire positive charge of the atom must be concentrated in a tiny central core of the atom. This tiny central core of each atom was called atomic nucleus.

An α -particle carries two units of positive charge and has mass of a helium atom. Charge on gold nucleus = Ze , where Z of gold is 79. As gold nucleus is about 50 times heavier than an α -particle, we assume that it would remain stable in the scattering process.

Therefore, the trajectory of α -particle can be computed using Newton's law of motion and Coulomb force of repulsion between α -particle and gold nucleus i.e.

$$F = \frac{1}{4\pi\epsilon_0} \frac{(2e)(2e)}{r^2}$$

where r is the distance of α -particle from the centre of the nucleus. The magnitude and direction of the force on an α -particle changes continuously as it approaches the nucleus first and then moves away from it.



As shown in fig an α -particle (1) tending to collide with the nucleus, slow down due to repulsive force of the nucleus, finally stops and is then repelled back. This α -particle, therefore retrace its path, scattering through 180° .

The α -particles 2 and 2' tending to hit the nucleus at its periphery, experience strong repulsive force and get scattered through large angles ($\theta > 90^\circ$).

The α -particles 3 and 3', which pass at a distance from the nucleus experience small repulsive forces and get scattered through small angles.

The α -particles which pass at large distances from the nucleus go almost undeviated.

We can show that the number of α -particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\sin^4(\frac{\theta}{2})$.

$$N(\theta) \propto \frac{1}{\sin^4(\frac{\theta}{2})}$$

Distance of Closest Approach -

When α -particle is directed towards the nucleus, the Kinetic energy of α -particle goes on decreasing and in turn electrical potential energy goes on increasing due to Coulomb's repulsive force between nucleus and α -particle.

At a certain distance r_0 from the nucleus, K.E. of α -particle reduces to zero. The particle stops and it cannot go closer to the nucleus. It is repelled by the nucleus and therefore, it retraces its path, turning through 180° . This distance r_0 is known as the distance of closest approach.

Electrical potential at distance r_0 due to nucleus

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{Ze}{r_0}$$

Potential Energy of α -particle at distance r_0 from the nucleus = Potential $\times$ Charge

$$P.E. = \frac{Ze}{4\pi\epsilon_0 r_0} \times 2e$$

Kinetic Energy of α -particle of mass m moving with velocity v is $\Rightarrow$

$$K.E. = \frac{1}{2}mv^2$$

As at the distance of closest approach -

$$K.E. = P.E.$$

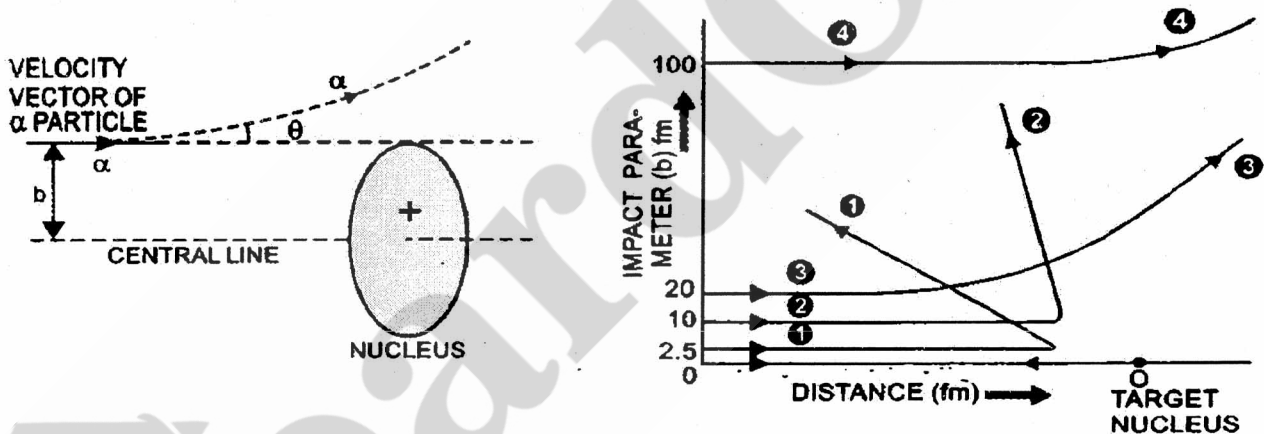
$$\frac{1}{2}mv^2 = \frac{ze \times 2e}{4\pi\epsilon_0 r_0}$$

or

$$r_0 = \frac{ze \times 2e}{4\pi\epsilon_0 \left(\frac{1}{2}mv^2\right)}$$

Impact Parameter (b) -

It is defined as the perpendicular distance of the initial velocity of α -particle from the central line of the nucleus, when the particle is far away from the nucleus.



When the impact parameter is large, an α -particle will deviate through a much smaller angle. However, when impact parameter is small, force experienced is large and the α -particle will scatter through a large angle.

Rutherford calculated analytically the relation between the impact parameter b and scattering angle θ , which is given by -

$$b = \frac{1}{4\pi\epsilon_0} \frac{ze^2 \cot \theta/2}{K.E.}$$

$$\left[K.E. = \frac{1}{2}mv^2 \right]$$

Rutherford's Atom Model -

- (1.) Every atom consist of a tiny central core, called the atomic nucleus in which the entire positive charge and almost entire mass of the atom are concentrated.
- (2.) The size of the nucleus is of the order of 10^{-15} m. which is very small as compared to the size of the atom which is of the order of 10^{-10} m.
- (3.) The atomic nucleus is a number of electrons. As atom is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.
- (4.) These electrons revolve around the nucleus in various circular orbits. The centripetal force required by electron for revolution is provided by the electrostatic force of attraction between the electrons and the nucleus.

Energy of the electron in orbit -

Let F_c = Centripetal force required to keep a revolving electron in orbit

F_e = electrostatic force of attraction between the revolving electron and the nucleus

then for a dynamically stable orbit in a hydrogen atom ($z=1$) -

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{e \cdot e}{4\pi\epsilon_0 r^2} \Rightarrow mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$$

$$\text{K.E. of the electron in the orbit} = \frac{1}{2} mv^2$$

Hence

$$K.E. = \frac{e^2}{8\pi\epsilon_0 r}$$

potential energy of electron in orbit -

$$U = \frac{e(-e)}{4\pi\epsilon_0 r} = \frac{-e^2}{4\pi\epsilon_0 r}$$

∴ Total energy of electron in hydrogen atom -

$$E = K.E. + U$$

$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$$

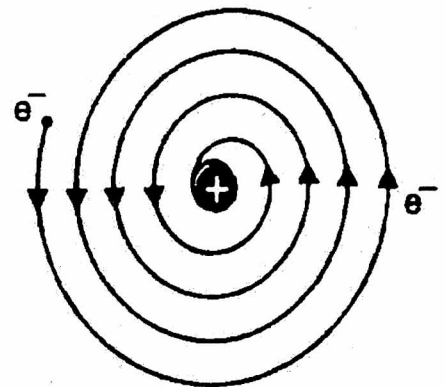
or

$$E = -\frac{e^2}{8\pi\epsilon_0 r}$$

Hence the total energy of electron in orbit of hydrogen atom is negative. Hence, the electron is bound to the nucleus i.e. the electron is not free to leave the orbit around the nucleus.

Limitation of Rutherford Atom Model -

- (i) According to the classical EM theory, the revolving electrons must radiate energy in the form of EM waves. As revolving electron loses energy continuously, it must spiral inwards and finally fall into the nucleus, but as matter is stable, we cannot expect the atoms to collapse.



- (ii) As the revolving electrons spiral inwards, their angular velocities and hence their frequencies of revolution would change continuously. Therefore, frequency of EM waves emitted must change continuously.

Therefore, atoms should emit continuous spectrum but we observe only a line spectrum.

Bohr Model of Hydrogen Atom-

There are three basic postulates of this model-

- (1) Every atom consist of a central core called nucleus, in which entire positive charge and almost entire mass of the atom are concentrated. A suitable number of electrons revolve around the nucleus in circular orbits. The centripetal force required for revolution is provided by the electrostatic force of attraction between the electron and the nucleus.

Centripetal force = Electrostatic force of attraction

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(e)}{r^2}$$

$$\therefore \boxed{\frac{mv^2}{r} = \frac{KZe^2}{r^2}} \quad \left[\begin{array}{l} K = 1/4\pi\epsilon_0 \\ Z = 1 \end{array} \right]$$

- (2) According to Bohr, electron can revolve only in certain discrete non radiating orbits, called stationary orbits, for which total angular momentum of the revolving electron is an integral multiple of $h/2\pi$, where h is plank's constant.

Thus the angular momentum of a orbiting electron is quantised.

$$\boxed{mvr = \frac{nh}{2\pi}} \quad n = 1, 2, 3, \dots$$

Here n is called principle quantum number.

* The electron, while revolving in such orbits, shall not lose energy i.e. its energy would stay constant.

(3.) The emission / absorption of energy occurs only when an electron jumps from one of its specified non-radiating orbit to another. The difference in the total energy of electron in the two orbits is absorbed when the electron jumps from an inner to an outer orbit and emitted when electron jumps from outer to the inner orbit.

$$h\nu = E_2 - E_1$$

where ν is the frequency of radiation emitted on jumping from outer to inner orbit of energy E_2 and E_1 respectively.

Radius of Bohr's stationary Orbits -

We know that for stationary orbits -

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

By putting the value of v in $\frac{mv^2}{r} = \frac{Kze^2}{r^2}$

$$\Rightarrow \frac{m}{r} \times \frac{n^2 h^2}{4\pi^2 m^2 r^2} = \frac{Kze^2}{r^2}$$

$$r = \frac{n^2 h^2}{4\pi^2 m K e^2}$$

for hydrogen atom
 $Z = 1$

It shows that - $r \propto n^2$

Hence the radius of stationary orbits are in the ratio $1^2 : 2^2 : 3^2 : \dots$ and so on i.e. $1 : 4 : 9 : \dots$. Clearly the stationary orbits are not equally spaced.

Velocity of electron in Bohr's Stationary Orbit-

As we know that - $\frac{mv^2}{r} = \frac{Kze^2}{r^2}$

$$\text{or } r = \frac{Kze^2}{mv^2} \quad \text{--- (1)}$$

$$\text{and also } r = \frac{nh}{2\pi mv} \quad \text{--- (2)}$$

$\therefore$ by equation (1) and (2) we get -

$$\frac{Kze^2}{mv^2} = \frac{nh}{2\pi mv} \quad \text{or } v = \frac{2\pi Kze^2}{nh}$$

for hydrogen atom, $z = 1$

$$v = \frac{2\pi ke^2}{nh}$$

As $v \propto \frac{1}{n}$, hence the orbital velocity of electron in outer orbits is smaller as compared to its value in the inner orbits.

Frequency of electron in Bohr's Stationary Orbit-

It is the number of revolutions completed per second by the electron in a stationary orbit, around the nucleus. It is represented by ν .

$$\text{As } v = r\omega = r(2\pi\nu)$$

$$\nu = \frac{v}{2\pi r} = \frac{2\pi Kze^2}{nh \cdot 2\pi r} = \frac{Kze^2}{nhr}$$

$$\nu = \frac{Kze^2}{nhr}$$

$$\Rightarrow \nu = \frac{Ke^2}{nhr}$$

$z = 1$
for hydrogen

The frequency of electron in subsequent stationary orbits is smaller as $\nu \propto \frac{1}{n}$.

Total Energy of electron in Bohr's Stationary Orbit

Kinetic energy of electron revolving in a stationary orbit is -

$$\text{K.E. of electron} = \frac{1}{2}mv^2 = \frac{Kze^2}{2r} \quad \left[\text{by } \frac{mv^2}{r} = \frac{Kze^2}{r^2} \right]$$

Potential energy of electron = Potential $\times$ Charge

$$\text{P.E. of electron} = \frac{Kze}{r} \times (-e) = -\frac{Kze^2}{r}$$

Total energy of electron in the orbit -

$$E = \text{K.E.} + \text{P.E.}$$

$$E = \frac{1}{2} \frac{Kze^2}{r} - \frac{Kze^2}{r} = -\frac{Kze^2}{2r}$$

by putting $r = \frac{n^2 h^2}{4\pi^2 m K z e^2}$ we get -

$$E = -\frac{2\pi^2 m K^2 z^2 e^4}{n^2 h^2}$$

by substituting the standard values, we get -

$$E = -\frac{13.6}{n^2} \text{ eV} \quad E \propto \frac{1}{n^2}$$

Hence the total energy of electron in a stationary orbit is negative, which means that the electron is bound to the nucleus and is not free to leave it.

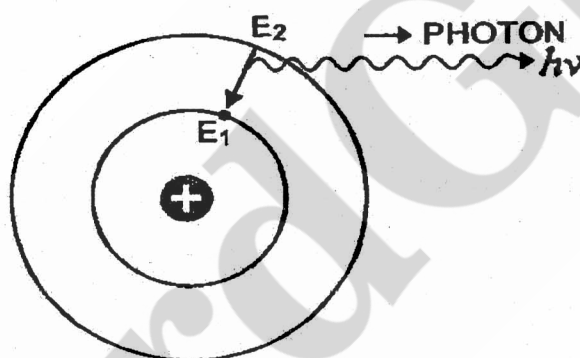
* When $n=1$ then this state of lowest energy of the atom is called ground state. The energy of this state is $E_1 = -13.6 \text{ eV}$.

* Therefore, the minimum energy required to free the electron from the ground state of hydrogen atom is 13.6 eV. This is called ionisation energy of hydrogen atom.

* As n increases, the value of negative energy decreases i.e. energy is progressively larger in the outer orbits.

Origin of Spectral Lines -

At room temperature, most of the hydrogen atoms are in ground state. When a hydrogen atom receives energy by processes such as electron collisions or heat, the atom may require sufficient energy to raise the electron to higher energy state i.e. from $n=1$ to $n=2, 3, 4, \dots$. The atom is then said to be in an excited state.



From these excited states, the electron can fall back to a state of lower energy and emitting a photon of particular energy (= difference in energies of the two states).

Let E_1 and E_2 are the total energy of electron in the inner and outer orbit respectively.

When an electron jumps from an outer to an inner orbit, the energy of radiation emitted is given by -

$$h\nu = E_2 - E_1$$

$$h\nu = \frac{2\pi^2 m K^2 z^2 e^4}{h^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\frac{hc}{\lambda} = \frac{2\pi^2 m K^2 z^2 e^4}{h^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\text{or } \frac{1}{\lambda} = \frac{2\pi^2 m K^2 e^4 Z^2}{ch^3} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Now $\frac{1}{\lambda} = \bar{\nu}$, where $\bar{\nu}$ is the wave number of radiation emitted i.e. number of complete wave in unit length.

$$\text{and } \frac{2\pi^2 m K^2 e^4}{ch^3} = R \quad (\text{Rydberg Constant})$$

$$\text{or } R = 1.097 \times 10^7 \text{ m}^{-1}$$

Hence

$$\boxed{\bar{\nu} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]}$$

For hydrogen
 $Z = 1$

Above equation is called Rydberg formula for the spectrum of hydrogen atom.

* By above formula it is clear that wavelength/frequencies/wave numbers of radiations emitted by the excited hydrogen atom are not continuous. They have specific values depending upon the values of n_1 and n_2 .

* Frequency of radiation emitted when the atom makes transition from the higher energy state (n_2) to the lower energy state (n_1) -

$$\nu = \frac{c}{\lambda}$$

$\Rightarrow$

$$\boxed{\nu = RC \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]}$$

Q.CBSE
2010

Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its (a) second permitted energy level to the first level (b) the highest permitted energy level to the first permitted level.

Sol. (a) Energy of Photon released = $E_2 - E_1$
 $= -3.4 - (-13.6) = 10.2 \text{ eV}$

(b) The highest permitted energy level to the first permitted level = $E_\infty - E_1$
 $= 0 - (-13.6) = 13.6 \text{ eV}$

Ratio of energies of Photon = $\frac{10.2}{13.6} = \frac{3}{4}$

Q.CBSE
2010

What is the ratio of radius of the orbits corresponding to first excited state and ground state in a hydrogen atom?

Sol. For first excited state $n = 2$
 Ground state occurs for $n = 1$

$\therefore r \propto n^2$

Hence $\frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{1}\right)^2$

$r_1 : r_2 = 4 : 1$

Q.CBSE
2002

The radius of innermost electron orbit of a hydrogen atom is $5.3 \times 10^{-11} \text{ m}$. What is the radius of orbit in the second excited state?

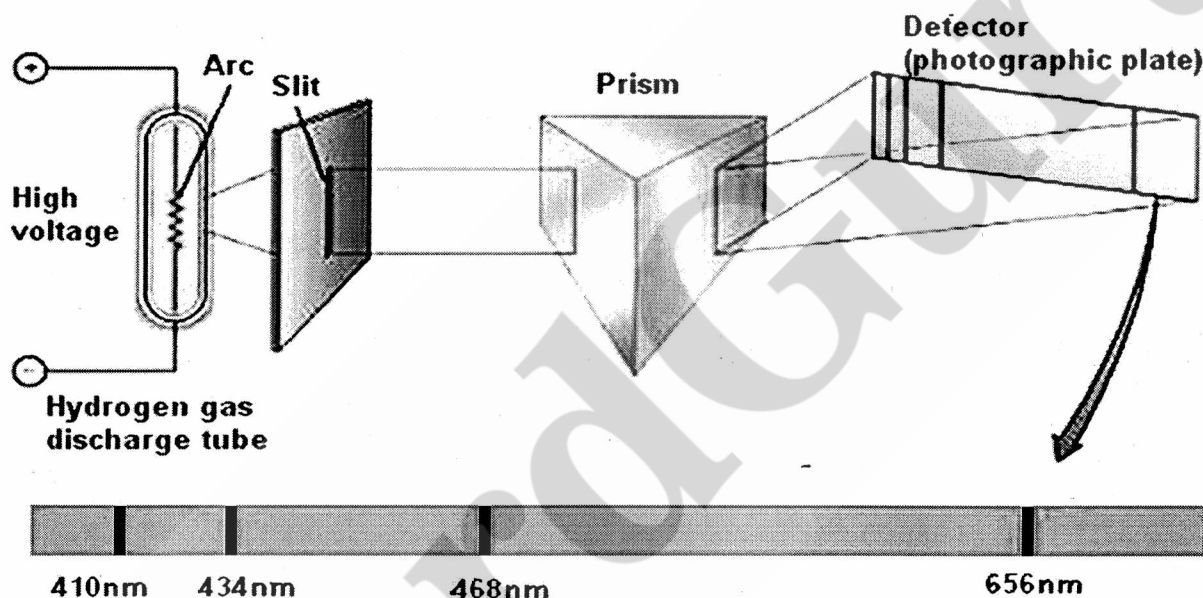
Sol. The radius of atom whose principle quantum number is n , is given by $r = n^2 r_0$
 Here r_0 is radius of innermost ($n=1$) orbit.

For second excited state $n = 3$

$\therefore r = (3)^2 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} \text{ m}$

Bohr's Explanation of spectral series of Hydrogen Atom -

When an atomic gas or vapour at low pressure is excited usually by passing an electric current through it, the gas/vapour emits radiations of certain specific wavelengths only. This kind of spectrum is called "line emission spectrum" and it consists of a few bright lines on a dark background.



When the white light is passed through the same gas/vapour, we observe a bright background crossed by a few dark lines signifying the missing wavelengths or the wavelengths that are absorbed by the gas. They form a "Line absorption spectrum".

It was found that missing wavelengths are the same as the wavelength present in the emission spectrum of the gas/vapour.

Bohr gave his theory of hydrogen atom about spectral series which had observed experimentally by various scientists. Bohr offered a theoretical explanation of these spectral series as follows :

1. Lyman Series -

Bohr postulates that Lyman series is obtained when an electron jumps to the first orbit ($n_1 = 1$) from any outer orbit ($n_2 = 2, 3, 4, \dots$)

Wave numbers of spectral lines of Lyman series were calculated as -

$$\bar{\nu} = R \left[\frac{1}{1^2} - \frac{1}{n_2^2} \right] \quad (n_2 = 2, 3, 4, \dots)$$

These values of $\bar{\nu}$ lie in the "ultra violet region" of the spectrum.

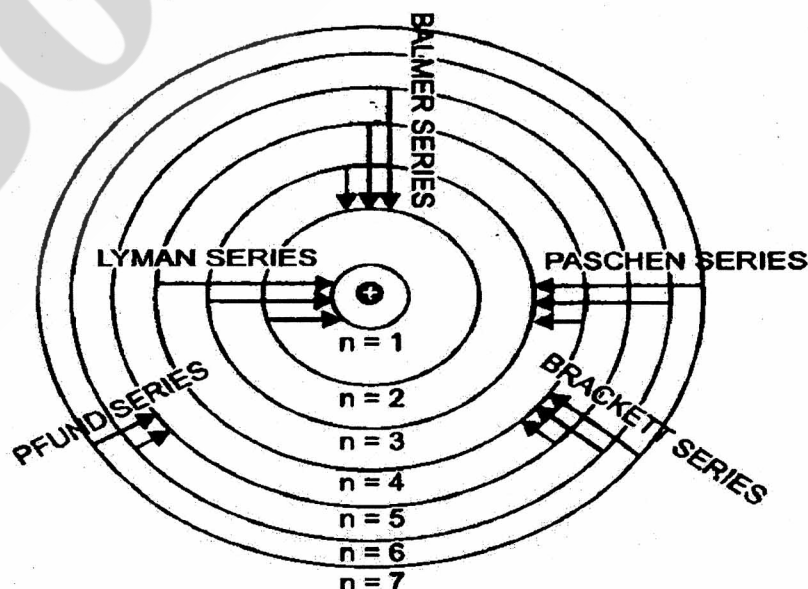
2. Balmer Series -

According to Bohr, Balmer series is obtained when an electron jumps to the second orbit ($n_1 = 2$) from any outer orbit ($n_2 = 3, 4, 5, \dots$)

Wave number of these spectral lines were calculated as -

$$\bar{\nu} = R \left[\frac{1}{2^2} - \frac{1}{n_2^2} \right] \quad (n_2 = 3, 4, 5, \dots)$$

This set of spectral lines lie in the visible part of the spectrum.



3. Paschen Series -

According to Bohr, paschen series is obtained when an electron jumps to the 3rd orbit ($n_1 = 3$) from any outer orbit ($n_2 = 4, 5, 6, \dots$).

Bohr calculated the wave number of spectral lines of Paschen series from the relation:

$$\bar{\nu} = R \left[\frac{1}{3^2} - \frac{1}{n_2^2} \right] \quad (n_2 = 4, 5, 6, \dots)$$

The value of $\bar{\nu}$ lie in the infrared region of the spectrum.

4. Brackett Series -

According to Bohr, Brackett series is obtained when an electron jumps to the 4th orbit ($n_1 = 4$) from any outer orbit ($n_2 = 5, 6, 7, \dots$).

Wave number $\bar{\nu} = R \left[\frac{1}{4^2} - \frac{1}{n_2^2} \right] \quad (n_2 = 5, 6, 7, \dots)$

Brackett series was discovered in the infrared region of the spectrum.

5. Pfund Series -

According to Bohr Pfund series is obtained when an electron jumps to the 5th orbit ($n_1 = 5$) from any outer orbit ($n_2 = 6, 7, 8, \dots$).

Wave number $\bar{\nu} = R \left[\frac{1}{5^2} - \frac{1}{n_2^2} \right] \quad n_2 = 6, 7, 8, \dots$

Pfund series was discovered in the infrared region of the spectrum.

Energy Level Diagram -

A diagram which represents the total energies of electron in different stationary orbits of an atom is called the energy level diagram of that atom.

In this diagram, total energies of electron in various stationary orbits are represented by the parallel horizontal lines drawn according to some suitable energy scale.

Total energy of electron in n^{th} orbit of hydrogen atom is -

$$E = -\frac{13.6}{n^2} \text{ eV}$$

putting $n = 1, 2, 3, \dots$ we get the energies of electrons in various stationary orbits as:

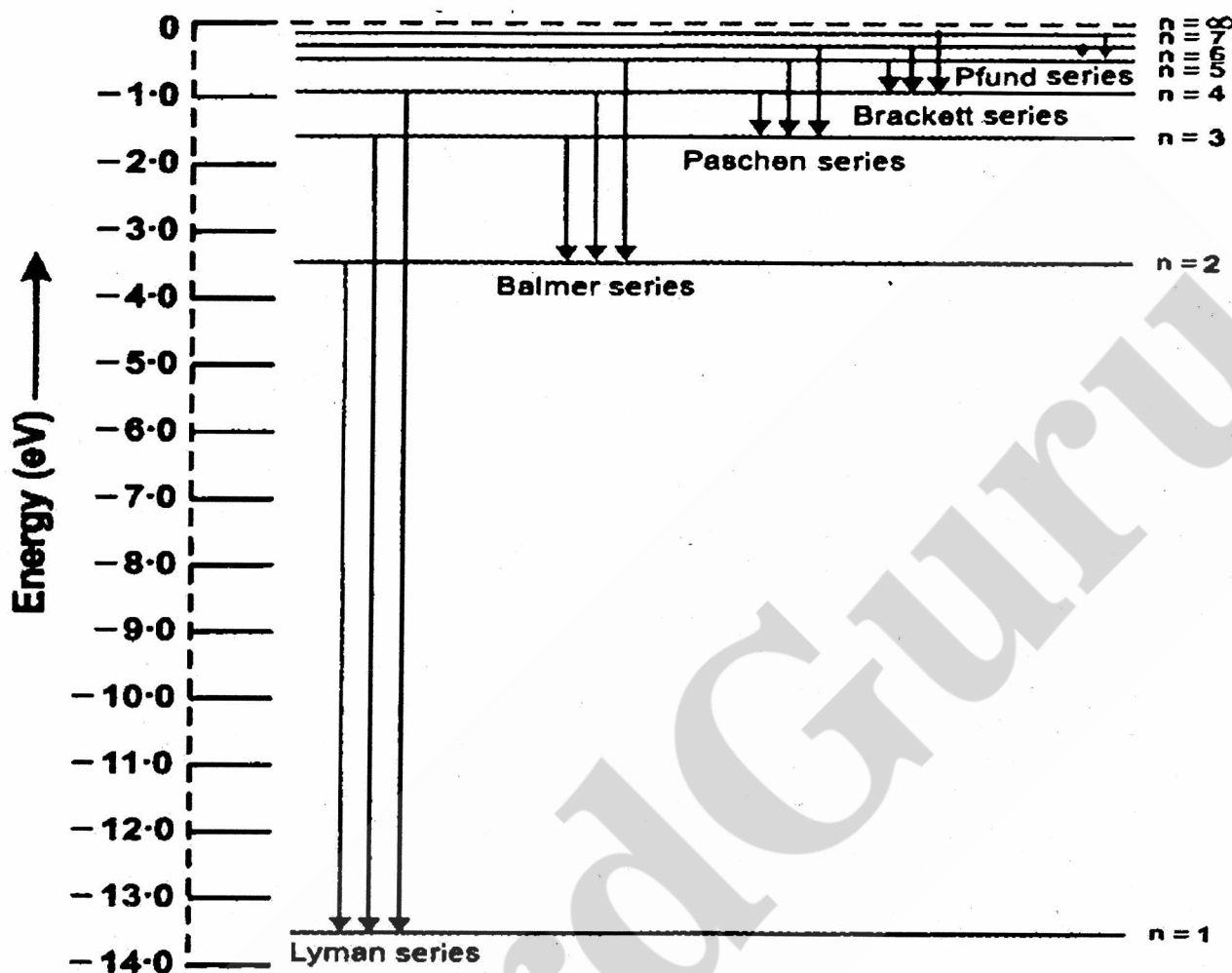
$$E_1 = -\frac{13.6}{1^2} = -13.6 \text{ eV}, \quad E_2 = -\frac{13.6}{2^2} = -3.4 \text{ eV}$$

$$E_3 = -\frac{13.6}{3^2} = -1.51 \text{ eV}, \quad E_4 = -\frac{13.6}{4^2} = -0.85 \text{ eV}$$

$$E_5 = -\frac{13.6}{5^2} = -0.54 \text{ eV}, \quad E_6 = -\frac{13.6}{6^2} = -0.37 \text{ eV}$$

Clearly as n increases E_n becomes less negative until at $n = \infty$, $E_n = 0$.

The energy level diagram is shown in figure for hydrogen atom. The highest energy state corresponds to $n = \infty$ and has energy $E = 0 \text{ eV}$. This is the energy of the atom, when the electron is removed ($r = \infty$) from the nucleus and the electron is at rest.



* As n increases energies of the excited state come closer together.

Q. Calculate the wavelength of H_{α} line ($n_2 = 3$) in Balmer series of hydrogen atom, given Rydberg Constant $R = 1.097 \times 10^7 \text{ m}^{-1}$

Sol. For Balmer series, wavelength is given by-

$$\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n_2^2} \right]$$

for H_{α} line $n_2 = 3$, so that

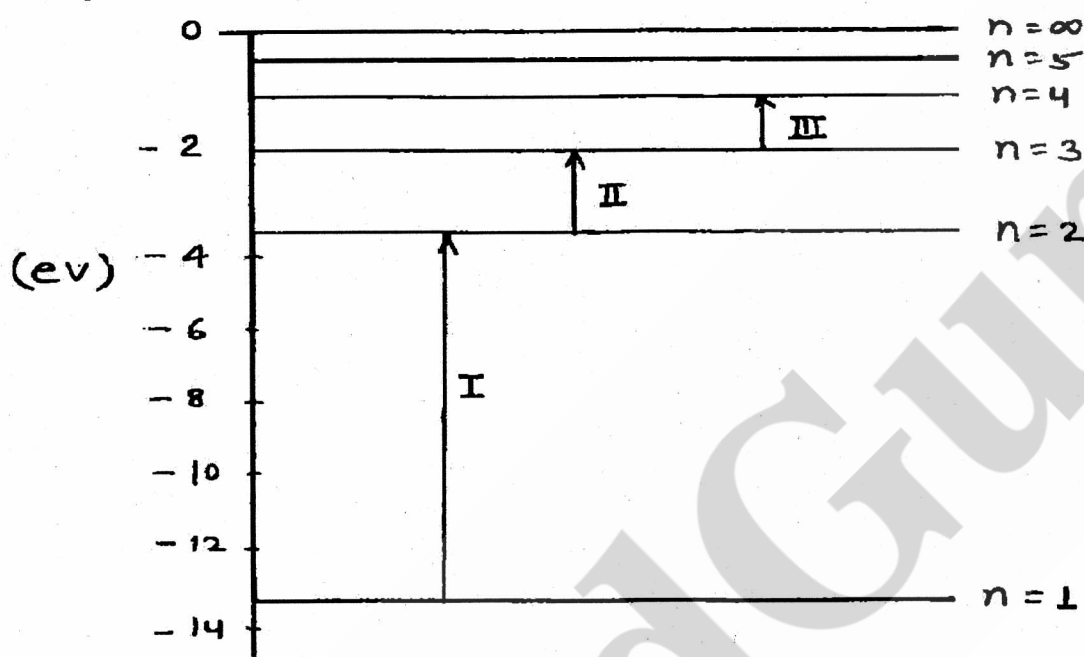
$$\frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{3^2} \right]$$

$$\frac{1}{\lambda} = 0.1524 \times 10^7$$

$$\lambda = \frac{1}{0.1524 \times 10^7} = 6563 \text{ \AA}$$

Q.
Delhi
2009

Photons, with a continuous range of frequencies are made to pass through a sample of hydrogen. The transitions shown here, indicate three of the spectral absorption lines in the continuous spectrum.



- (a) Identify the spectral series of the hydrogen emission spectrum, to which each of these three lines corresponds.
- (b) Which of these lines corresponds to the absorption of radiation of maximum wavelength?

Sol.

(a) Ist spectrum represent Lyman series because in this spectrum, electron jumps from $n=1$ to $n=2$.

IInd spectrum represent Balmer series because in this spectrum, electron jumps from $n=2$ to $n=3$.

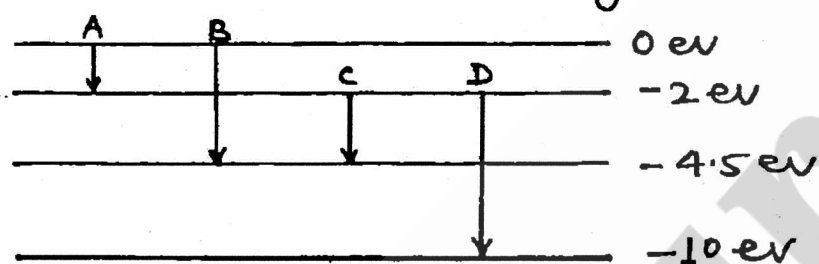
IIIrd spectrum represents Paschen series as the electron jumps from $n=3$ to $n=4$ in this spectrum.

(b) Spectral line III (Paschen series) corresponds to the absorption of radiation of maximum wavelength.

Q.
CBSE
2011

(a) The energy levels of an atom are shown in fig. which of them will result in the emission of a photon of wavelength 275 nm?

(b) Which transition corresponds to emission of radiation of maximum wavelength?



Sol. (a) When $\lambda = 275 \text{ nm} = 275 \times 10^{-9} \text{ m}$ then

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{275 \times 10^{-9}}$$

$$E = 4.5 \text{ eV}$$

$\therefore$ Transition B will result in the emission of photon of $\lambda = 275 \text{ nm}$.

(b) Maximum wavelength has minimum energy. Transition A provides energy of 2 eV, which is minimum.

De Broglie's Explanation of Bohr's second postulate of Quantization

The second postulate of Bohr model says that angular momentum of electron orbiting around the nucleus is quantized

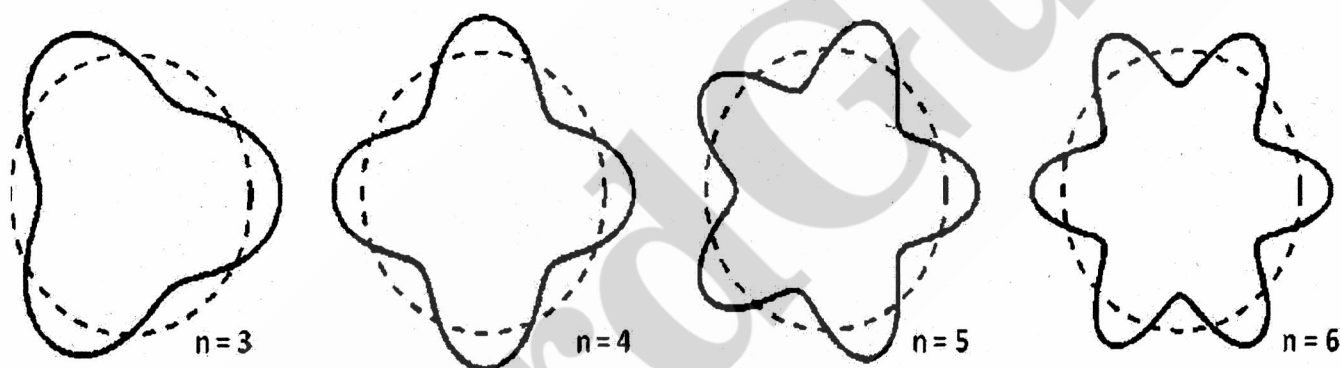
$$\text{i.e. } mvr = \frac{nh}{2\pi} \text{ where } n = 1, 2, 3, \dots$$

Louis de Broglie explained this puzzle. According to de Broglie, the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave.

We know that when a string fixed at two ends is plucked, a large number of wavelengths are excited. But only those waves which have nodes at the two ends form the standing waves and survive.

It means that in a string, standing waves form when total distance travelled by a wave down the string and back is any integer multiple of the wavelengths.

Waves with other wavelengths interfere with themselves upon reflection and their amplitudes vanish quickly.



Hence, according to de Broglie, a stationary orbit is that which contains an integral number of de Broglie waves associated with the revolving electron.

For an electron revolving in n^{th} circular orbit of radius r ,

For the permissible orbit, $2\pi r = n\lambda$

According to de Broglie $\lambda = \frac{h}{mv}$

where v is speed of electron revolving in n^{th} orbit

$$\therefore 2\pi r = \frac{nh}{mv} \Rightarrow mvr = \frac{nh}{2\pi}$$

Angular momentum $mvr = n\left(\frac{h}{2\pi}\right)$

Hence angular momentum of electron revolving in n^{th} orbit must be an integral multiple of $h/2\pi$, which is the quantum Condition proposed by Bohr in second postulate.

Limitation of Bohr's Theory -

- (1.) This theory is applicable to only simplest atom like hydrogen, with $z=1$. The theory fails in case of atoms of other elements for which $z > 1$.
- (2.) The theory does not explain why orbits of electrons are taken as circular, while elliptical orbits are also possible.
- (3.) Bohr's theory does not say anything about the relative intensities of spectral lines.
- (4.) Bohr's theory does not take into account the wave properties of electrons.

Very Important Questions

1 Mark Questions-

- Q.1. Write the expression for Bohr's radius in hydrogen atom. (Delhi 2010, 01)
- Q.2. State Bohr's quantisation condition for defining stationary orbits. (RBSE 2009, CBSE 2010, 03)
- Q.3. In the Rutherford scattering experiment, the distance of closest approach for an α -particle is d_0 . If α -particle is replaced by a proton, how much Kinetic energy in comparison to α -particle will it require to have the same distance of closest approach d_0 ?
Hint :: Proton (H^+), α -particle (He^{2+}) (CBSE 2009, 2000)
- Q.4. In hydrogen atom, if the electron is replaced by a particle which is 200 times heavier but has the same charge, how would its radius change ? (CBSE 2002, 08)
- Q.5. The energy of the electron in the ground state of hydrogen atom is -13.6 eV.
(a) What does the negative sign signify ?
(b) How much energy is required to take an electron in this atom from the ground state to the first excited state ? (CBSE 2009)

2 Mark Questions -

Q.1. The ground state energy of hydrogen atom is -13.6 eV . The Photon emitted during the transition of electron from $n=2$ to $n=1$ state is incident on the photosensitive material of unknown work function. The photoelectrons are emitted from materials with a maximum kinetic energy of 8 eV . Calculate the threshold wavelength of the material used.

Answer - $\lambda = 564.5 \text{ nm}$.

(CBSE 2008)

Q.2. Using Bohr's postulates for hydrogen atom, show that the total energy (E) of the electron in the stationary state can be expressed as the sum of kinetic energy (K) and potential energy (U), where $K = -2U$. Hence deduce the expression for the total energy in the n^{th} energy level of hydrogen atom.

(CBSE 2002, 12)

Q.3. (a) Using Bohr's second postulate of quantization of orbital angular momentum show that the circumference of the electron in the n^{th} orbital state in hydrogen atom is n times the de-broglie wavelength associated with it.

(b) The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state?

(Delhi 2012, RBSE 2006)

Answer: (b) $n = 3$

Q.4. The ground state energy of hydrogen atom is -13.6 eV . If an electron makes a transition from an energy level -0.85 eV to -1.51 eV , calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?

(CBSE 2001, 12)

Q.5. In a Geiger-Marsden experiment calculate the distance of closest approach to the nucleus of $Z = 80$, when an α -particle of 8 MeV energy incident on it before it comes rest and reverse its direction.

How will the distance of closest approach be affected when the kinetic energy of the α -particle is doubled?

(CBSE 2003, 09)

Answer - $r_0 = 2.88 \times 10^{-19} \text{ m}$

Q.6. State the basic assumptions of the Rutherford model of the atom. Explain in brief why this model cannot account for the stability of an atom?

(Delhi 2002, 10)

Q.7. Using the Bohr's postulates, derive the expression for the -

- (a) speed of the electron in the n^{th} orbit,
- (b) radius of the n^{th} orbit of the electron in hydrogen atom.

(CBSE 2007, 10)

Q.8. The energy of the electron in the ground state of hydrogen is -13.6 eV . Calculate the energy of the photon that would be emitted if the electron were to make a transition corresponding to the emission of the first line of the (a) Lyman series

(b) Balmer series of the hydrogen spectrum

Answer - (a) 10.2 eV (b) 1.9 eV (CBSE 2009)

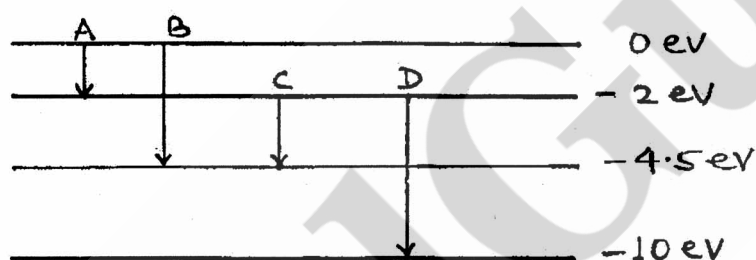
Q.9. The ground state energy of hydrogen atom is -13.6 eV .

(a) What is the K.E. of an electron in the 2nd excited state?

(b) If the electron jumps to the ground state from the 2nd excited state, calculate the wavelength of the spectral line emitted.

Answer (a) 1.5 eV (b) $\lambda = 1030 \text{ \AA}$ (CBSE 2003, 08)

Q.10. The energy levels of an atom are as shown in figure below.



(a) Which of them will result in the transition of a photon of wavelength 275 nm ?

(b) Which transition corresponds to emission of radiation of maximum wavelength?

Answer: (a) transition B (CBSE 2001, 09)

(b) transition A.

Q.11 (a) Using postulates of Bohr's theory of hydrogen atom, show that -

(i) The radius of orbits increase as n^2

(ii) The total energy of the electron increases as $1/n^2$ where n is the principal quantum number of the atom.

(b) Calculate the wavelength of H_α line in Balmer series of hydrogen atom, given that Rydberg Constant $= 1.097 \times 10^7 \text{ m}^{-1}$.

Answer - (b) $\lambda = 6563 \text{ \AA}$ (CBSE 2002, 06)

PHYSICS

Unit - 13. Nuclei

Class - 12th

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13. NUCLEI

Atomic Masses -

The unit in which atomic and nuclear masses are measured is called atomic mass unit (a.m.u.)

One 'a.m.u.' is defined as $\frac{1}{12}$ th of the mass of an atom of ${}^{12}_6\text{C}$. It is denoted by 'u'.

As Avogadro's number = 6.023×10^{23}

$$\therefore \text{Mass of one atom of } {}^{12}_6\text{C} = \frac{12}{6.023 \times 10^{23}} \text{ g}$$

By definition 1 a.m.u. = $\frac{1}{12} \times$ mass of one atom of ${}^{12}_6\text{C}$

$$\therefore 1 \text{ amu} = \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}} = 1.66 \times 10^{-24} \text{ g.}$$

$$1 \text{ a.m.u.} = 1.66 \times 10^{-27} \text{ Kg.}$$

In terms of a.m.u.,

mass of an electron $m_e = 0.00055 \text{ u.}$

mass of a Proton $m_p = 1.0073 \text{ u.}$

mass of a neutron $m_n = 1.0086 \text{ u.}$

Electron Volt (eV) -

It is the small unit of energy. One electron volt is the energy gained by an electron, when accelerated through a potential difference of one volt.

$$\text{Energy} = \text{Work done} = \text{Charge} \times \text{Potential}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \times 1$$

$$\text{or } 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

According to the Einstein Mass-energy equivalence -
 $E = mc^2$

taking $m = 1 \text{ a.m.u} = 1.66 \times 10^{-27} \text{ Kg}$ and $c = 3 \times 10^8 \text{ m/s}$

$$E = (1.66 \times 10^{-27})(3 \times 10^8)^2 = 1.49 \times 10^{-10} \text{ J}$$

$$\text{Hence } E = \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} = 931.15 \times 10^6 \text{ eV}$$

$$\therefore \boxed{1 \text{ a.m.u.} = 931.15 \text{ MeV}}$$

Discovery of Neutron -

A neutron is a neutral particle carrying no charge and having mass equal roughly to the mass of a proton.

The neutron was discovered experimentally by Chadwick. Chadwick observed that when beryllium was bombarded with alpha particles, some neutral particles were emitted, whose mass is roughly equal to the mass of a proton. A free neutron is unstable and has a mean life time of 1000 s. Neutron is stable inside the nucleus.

Atomic Number (Z) -

It is the number of protons present inside the nucleus of an atom of the element. It is represented by Z.

$$Z = \text{Number of Protons} = \text{Number of electrons}$$

Mass Number (A) -

It is the total number of protons and neutrons present inside the atomic nucleus of the element. It is represented by A.

$$A = \text{Number of nucleons (neutron + Proton)}$$

$$A - Z = \text{number of neutrons}$$

A nuclide is represented as ${}_Z X^A$, where X is the chemical symbol of the element.

Examples - ${}_6\text{C}^{12}$, ${}_{79}\text{Au}^{197}$, ${}_{92}\text{U}^{235}$ etc.

Nuclear Size -

Experimental measurement shows that the volume of a nucleus is proportional to its mass number A .

If R is the radius of the nucleus, then its volume $\propto A$

$$\frac{4}{3} \pi R^3 \propto A$$

or

$$R \propto A^{1/3}$$

or

$$R = R_0 A^{1/3}$$

Where R_0 is a constant whose value is $1.2 \times 10^{-15} \text{ m}$.

* As A is different for different elements, therefore atomic nuclei of different elements have different sizes.

Nuclear Density -

Density of a nucleus is the ratio of mass of nucleus to its volume.

If m is average mass of a nucleon and R is the nuclear radius, then mass of nucleus = $m A$

where A is the mass number of the element.

$$\text{Volume of Nucleus} = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi (R_0^3 A)$$

$$\text{Density of Nucleus} = \frac{m A}{\frac{4}{3} \pi R_0^3 A} = \frac{3 m}{4 \pi R_0^3}$$

$$\text{or } \rho = \frac{3 \times 1.66 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3} = 2.29 \times 10^{17} \text{ Kg/m}^3$$

* Hence density of nucleus is same for all the elements, which is very large as compared to density of ordinary matter.

Isotopes -

Isotopes of an element are that atoms which have the same atomic number but different atomic weights.

Example - ${}_1\text{H}^1, {}_1\text{H}^2, {}_1\text{H}^3$ are isotopes of hydrogen,
 ${}_6\text{C}^{10}, {}_6\text{C}^{11}, {}_6\text{C}^{12}, {}_6\text{C}^{13}, {}_6\text{C}^{14}$ are the isotopes of carbon.

As isotopes of an element have the same atomic number, hence these have identical chemical properties, but due to different atomic weight their physical properties are different.

The atomic weight of an element is the weighted average of the masses of all its isotopes. For example, neon has two isotopes of masses 20 and 22 which occur in the ratio 9:1.

$$\therefore \text{Average atomic weight of neon} = \frac{20 \times 9 + 22 \times 1}{(9+1)} \\ = 20.2$$

But mass number is always represented by an integer as it represents the number of nucleons in the nucleus.

Isobars -

Isobars are the atoms of different elements which have the same atomic weight but different atomic number.

Example: ${}_{11}\text{Na}^{22}$ and ${}_{10}\text{Ne}^{22}$ are isobars, ${}_{20}\text{Ca}^{40}$ and ${}_{18}\text{Ar}^{40}$ are isobars etc.

The chemical properties of isobars are different due to different atomic number but their physical properties may be identical due to same atomic weight.

Isotones -

Isotones are the atoms of different elements which contain the same number of neutrons, hence in their case, $(A-Z) = N$ is the same.

Examples - ${}_{17}\text{Cl}^{37}$ and ${}_{19}\text{K}^{39}$ are isotones, similarly ${}^3_1\text{H}$ and ${}^4_2\text{He}$ are isotones.

Nuclear Binding Energy -

Binding energy of a nucleus is the energy with which nucleons are bound in the nucleus. It is measured by the work required to separate the nucleons at infinite distance apart from the nucleus, so that they may not interact each other.

Mass Defect -

The difference between the sum of the masses of neutrons and protons forming a nucleus and mass of the nucleus is called mass defect.

This mass defect appears in the form of binding energy, which is responsible for binding the nucleon together in the nucleus.

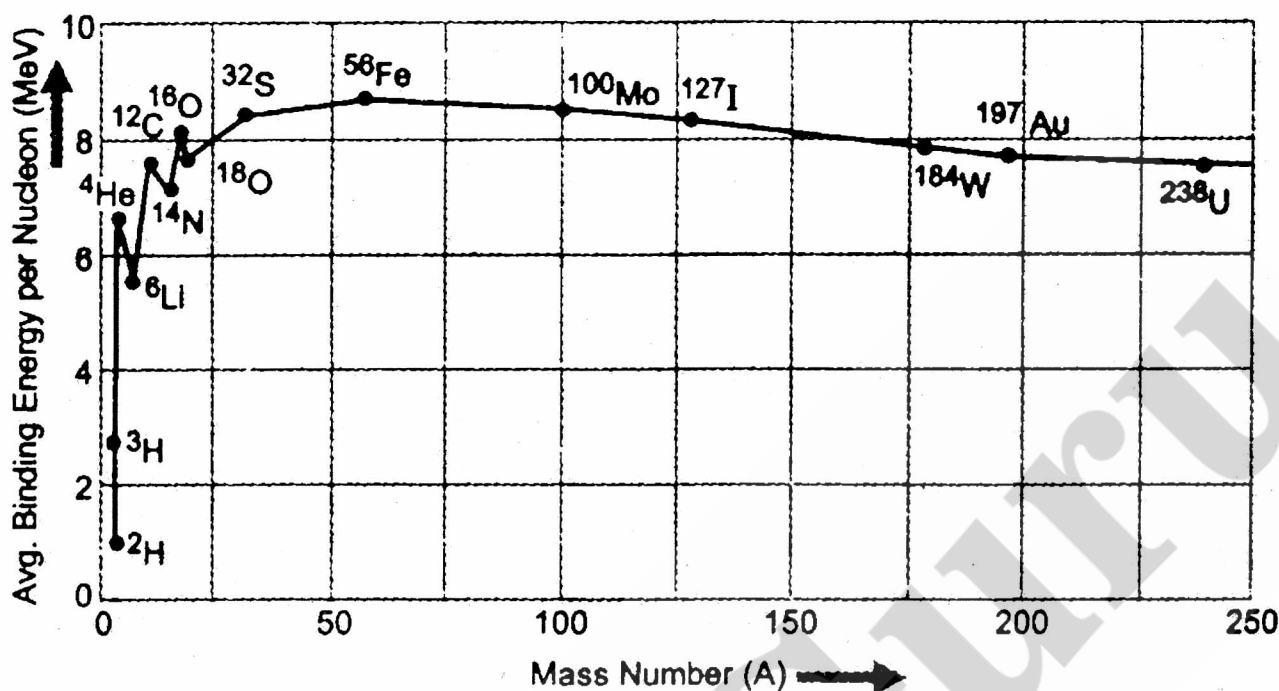
Nuclear Binding energy = mass defect $\times$ (Velocity of light)²

$$\text{B.E.} = [Zm_p + (A-Z)m_n - m_N] \times c^2$$

or B.E. (in Mev) = [Mass defect in a.m.u.] $\times$ 931

Average Binding Energy per nucleon of a Nucleus -

It is the average energy we have to spend to remove a nucleon from the nucleus to infinite distance. It is given by the total binding energy divided by the mass number of the nucleus.



The graph between binding energy per nucleon and mass number (A) shows the following facts:

- (i) Average binding energy of light nuclei like ^1H , ^2H , ^3H is small.
- (ii) There are sharply defined peaks corresponding to ^4He , ^{12}C , ^{16}O which represents that these nuclei are relatively more stable than the other nuclei in their neighbourhood.
- (iii) The B.E. Curve has a maximum value in the range $A = 30$ to $A = 140$ corresponding to average binding energy per nucleon = 8.5 MeV. The peak value of the maximum is 8.8 MeV/N for ^{56}Fe . Hence iron is the most stable element on the earth.
- (iv) As the mass number increases the B.E./nucleon decreases and gradually falling to about 7.6 MeV per nucleon for ^{238}U . Hence heavy nuclei are relatively less stable.

Importance of Binding energy Curve -

- (i) When we move from the heavy nuclei region to the middle region of the graph, there will be an increment in the overall binding energy and hence release of energy. This indicates that energy can be released when a heavy nucleus ($A \approx 240$) breaks into two roughly equal parts. This process is called nuclear fission.
- (ii) When we move from lighter nuclei to heavier nuclei we find that there is an increment in the overall binding energy and hence release of energy. This indicates that energy can be released when two or more lighter nuclei fuse (mix) together to form a heavy nucleus. This process is called nuclear fusion.

Packing Fraction -

Packing fraction of a nucleus is defined as the mass excess per nucleon.

$$\text{Packing Fraction} = \frac{\text{Mass Excess}}{\text{Mass Number}} = \frac{M - A}{A}$$

Where M = Actual weight of nuclei
 A = Atomic mass number

- * For values of $A < 20$, packing fraction is positive.
- * As A increases, packing fraction becomes negative, and becomes positive again at values of A of about 180.

Q. calculate the amount of energy released during the α -decay of - ${}_{92}\text{U}^{238} \longrightarrow {}_{90}\text{Th}^{234} + {}_2\text{He}^4$

Given that - $m_{\text{U}} = 238.05079 \text{ u}$, $m_{\text{Th}} = 234.04363 \text{ u}$,
and $m_{\text{He}} = 4.00260 \text{ u}$.

Sol. Mass defect = $[238.05079 - (234.04363 + 4.00260)]$
= 0.00456 u .

Hence Energy Released = $0.00456 \times 931 \text{ MeV}$
= 4.25 MeV .

Nuclear Forces-

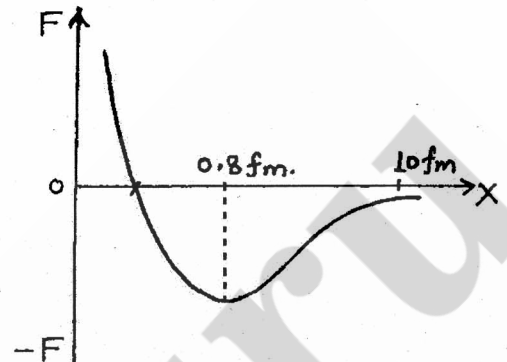
Nuclear forces are the strong forces of attraction which hold together the nucleons (neutrons and protons) in the tiny nucleus of an atom, inspite of strong electrostatic force of repulsion between protons.

Important characteristics of Nuclear forces-

- (i) Nuclear forces act between a pair of neutrons, a pair of protons and also between a neutron, proton pair with the same strength. This shows that nuclear forces are independent of charge.
- (ii) Nuclear forces are the strongest forces in nature. The nuclear forces is 100 times and 10^{38} time stronger than the electrostatic and gravitational forces respectively.
- (iii) The nuclear forces are very short range forces. They are operative upto distance of a few fermi.
- (iv) Nuclear forces are non central forces. This shows that the distribution of nucleons in a nucleus is not spherically symmetric.

(v) The variation of nuclear forces with the distance between nucleons is shown in figure:

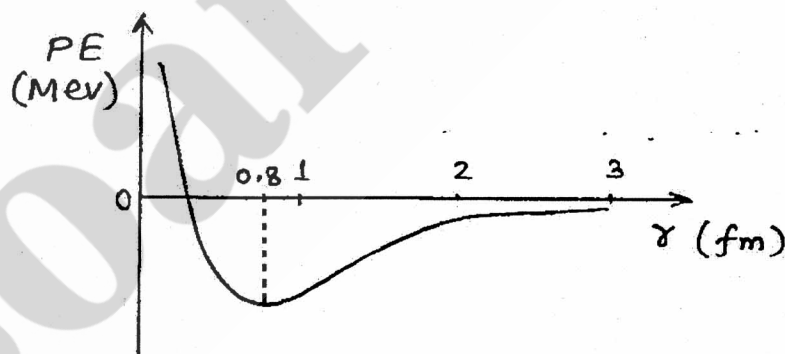
(a) Nuclear forces are negligible, when distance between nucleons is more than 10 fermi.



(b) When nucleons are brought closer, nuclear force of attraction develops, which increase rapidly with decreasing distance. At distance 0.8 fermi forces becomes strongly attractive.

(c) When distance between nucleons becomes less than 0.8 fm, the nuclear forces becomes strongly repulsive.

(vi) The variation of potential energy of a pair of nucleons as a function of their separation. It shows that nuclear force is of attractive nature for separation between the nuclei greater than 1 fm and repulsive when separation is less than 1 fm.



* P.E. is positive for repulsive forces and is negative for attractive forces.

Nature of Nuclear forces-

A Japanese Physicist Yukawa postulated that nuclear forces between nucleons arise on account of continuous exchange of particles called mesons between the nucleons.

Later on these particles were detected experimentally. They were found to have a mass about 270 times the mass of an electron.

Three types of mesons were detected. Mesons carrying no charge were called neutral Pi meson (π^0). Mesons carrying unit positive charge were called positive pi meson (π^+), and mesons carrying unit negative charge were called negative pi meson (π^-).

Meson theory of nuclear forces:

- (i) All nucleons consist of identical cores surrounded by a pulsating cloud of π meson.
- (ii) The difference between a proton and a neutron is essentially in the composition of their respective meson clouds.
- (iii) The force between neutron and proton is due to exchange of charged meson between them.

$$p^+ + \pi^- = n^0$$

$$n^0 + \pi^+ = p^+$$

Thus proton and neutron continuously exchange their by absorbing and emitting π mesons.

$$n^0 \longrightarrow p^+ + \pi^- \quad \text{and} \quad p^+ \longrightarrow n^0 + \pi^+$$

- (iv) The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral pi meson (π^0) between them-

$$p \longrightarrow p' + \pi^0 \quad \text{or} \quad p + \pi^0 \longrightarrow p'$$

$$n \longrightarrow n' + \pi^0 \quad \text{or} \quad n + \pi^0 \longrightarrow n'$$

Nuclear Stability -

Nuclear stability depends on following factors:

- (i) The stability of a nucleus is determined by the value of its binding energy per nucleon. Higher the B.E./nucleon more stable is the nucleus.
- (ii) The stability of a nucleus is also determined by its neutron to proton ratio. More is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are needed to provide the strong attractive forces necessary to keep the nucleus stable.
- (iii) The stability of a nucleus is also determined by the consideration, that it contains an even or odd number of protons and neutrons. A detailed study shows that stable nuclei often contains even number of protons or neutrons or both.

Radioactivity -

It is the property by which a heavy element disintegrates itself without being forced by any external agent to do so. The element exhibiting this property is called radioactive element.

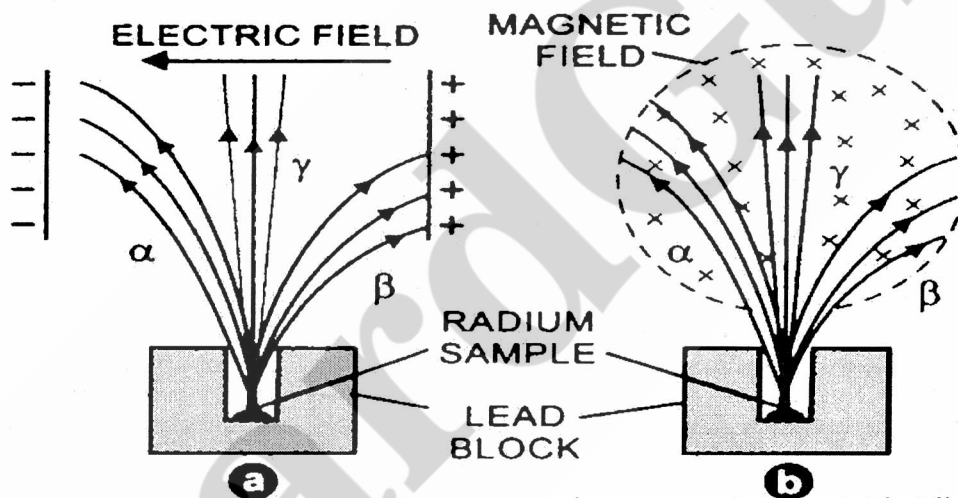
Examples - Radium, Thorium, Actinium, Polonium etc.

- * Natural elements with atomic number greater than 82 are all radioactive, i.e. their nuclei are unstable.
- * Experimental data shows that the radiations emitted by radioactive element consist of three kinds, which are called α , β and γ rays. All the three types of radiations need not necessarily be emitted by one radioactive element.

Nature of α , β and γ rays -

The sample of a radioactive element is placed in a small cavity drilled in a lead block. The Lead block absorbs all the α , β and γ rays, except those through the opening.

The radiations coming out of the cavity are subjected to an electric field, provided by two charged plates. The α -rays are deflected through smaller angles towards the negative plate. The β -rays are deflected through larger angles towards the positive plate. The γ -rays remains undeflected.



As opposite charges attract each other, it was concluded that α -rays consist of a stream of positively charged particle, whereas β -rays consist of negatively charged particles. Since γ -rays were undeflected hence it consist of uncharged particles.

Same results were obtained when these radiations were subjected to the action of a magnetic field.

Properties of α -rays-

- (1) A α -particle carries double the positive charge of proton and mass of α -particle is roughly four times that of hydrogen atom. Hence α -particle is a doubly ionised helium atom.
- (2) The velocity of α -particle ranges between 1.4×10^7 m/s to 2.1×10^7 m/s.
- (3) Because of large mass and charge the penetrating power of α -particles is very small, it being $1/100$ times that due to β -rays and $1/10000$ times that due to γ -rays. α -particles can easily be stopped by an aluminium foil only 0.02 mm thick.
- (4) Because of large mass and large velocity, α -particles have large ionising power. Each α -particle produces thousands of ions before being absorbed.
- (5) α -particles produce fluorescence in certain substances, like barium-platinocyanide and zinc-sulphide.
- (6) α -particles affect photographic plate slightly.
- (7) α -particles cause burns on human body.
- (8) α -particles on being stopped, produce heating effect.
- (9) α -particles are deflected by electric & magnetic fields.

Properties of β -rays-

- (1) A β -particle carries 1.6×10^{-19} C of negative charge and mass of β -particle is 9.1×10^{-31} kg, which is the same as that of electron.
- (2) The velocity of β -particles ranges from 33 % to 99 % of the velocity of light.
- (3) The β -particles affect a photographic plate.

- (4) Because of small mass the penetrating power of β -particles is very large. They can easily pass through a few millimeter of aluminium sheet.
- (5) The β -particles ionise the gas through which they pass, but their ionising power is only $1/100$ times that of the α -particles.
- (6) The β -particles can also produce fluorescence in a substance like barium platinocynide and zinc sulphide
- (7) β -particles are deflected by electric and magnetic fields

Properties of γ -rays -

- (1) γ -rays are not deflected by electric and magnetic fields, hence they are uncharged particles (Photon).
- (2) The rest mass of γ -ray photon is zero.
- (3) γ -rays travel with the speed of light.
- (4) γ -rays have very large penetrating power. They can pass through several centimetre of iron and lead.
- (5) γ -rays have very small ionising power.
- (6) γ -rays can produce fluorescence in a substrate like willimite.
- (7) γ -rays affect a photographic plate more than α and β -particles.
- (8) γ -rays can knock out electrons from the surface of a metal, on which they may fall.
- (9) γ -rays can cause nuclear reactions.

Laws of Radioactive disintegration -

Rutherford and Soddy from their experimental study regarding the radioactive disintegration formulated the following laws, known as law of radioactive disintegration:

- (1) Radioactivity is a spontaneous process which does not depend upon external factors like temperature, pressure etc.
- (2) During disintegration of an atom, either a α -particle or a β -particle is emitted. Both of these particles are never emitted simultaneously. Also at a time, an atom will not emit more than one α -particle or more than one β -particle.
- (3) The emission of α -particle from an atom will change it, into a new atom whose charge number is reduced by two and mass number is reduced by four.
- (4) The emission of β -particle from an atom will change it, into a new atom whose charge number is raised by one, without any change in its mass number.
- (5) The number of atoms disintegrated per second (rate of disintegration) at any instant is directly proportional to the number of radioactive atoms actually present in the sample at that instant.

Let-

N_0 = total number of atoms present in a sample at time $t=0$.

N = total number of atoms left undecayed in the sample at time t

dN = a small number of atoms that disintegrate in a small interval of time dt .

$$\therefore \text{Rate of disintegration of element } (R) = -\frac{dN}{dt}$$

According to radioactive law of disintegration -

$$-\frac{dN}{dt} \propto N$$

$$\therefore R = -\frac{dN}{dt} = \lambda N$$

where λ is a constant, called the disintegration constant or the decay constant.

$$\text{Now } \frac{dN}{dt} = -\lambda N \quad \Rightarrow \quad \frac{dN}{N} = -\lambda dt$$

$$\text{or } \int \frac{dN}{N} = \int -\lambda dt$$

$$\log_e N = -\lambda t + C$$

where C is a constant of integration.

At $t = 0$, $N = N_0$ so that

$$\log_e N_0 = 0 + C \quad \Rightarrow \quad C = \log_e N_0$$

Hence we get - $\log_e N = -\lambda t + \log_e N_0$

$$\log_e N - \log_e N_0 = -\lambda t$$

$$\text{or } \log_e \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore \boxed{N = N_0 e^{-\lambda t}}$$

Hence it is clear that $N = 0$ when $t = \infty$.

Disintegration Constant -

Substituting $t = 1/\lambda$ in above equation we get -

$$N = N_0 e^{-\lambda \times \frac{1}{\lambda}} \quad \Rightarrow \quad N = N_0 e^{-1} = \frac{N_0}{e}$$

Thus the disintegration constant of a radioactive element is the reciprocal of the time at the end of which, the number of atoms left undecayed in a radioactive sample reduces to $(1/e)$ times the original number of atoms (N_0).

Half Life of Radioactive Element -

Half life of a radioactive element is defined as the time during which half the number of atoms present initially in the sample decay. It is represented by T .

Thus when - $t = T$, $N = N_0/2$

$\therefore$ by equation $N = N_0 e^{-\lambda t}$

we get - $\frac{N_0}{2} = N_0 e^{-\lambda T}$

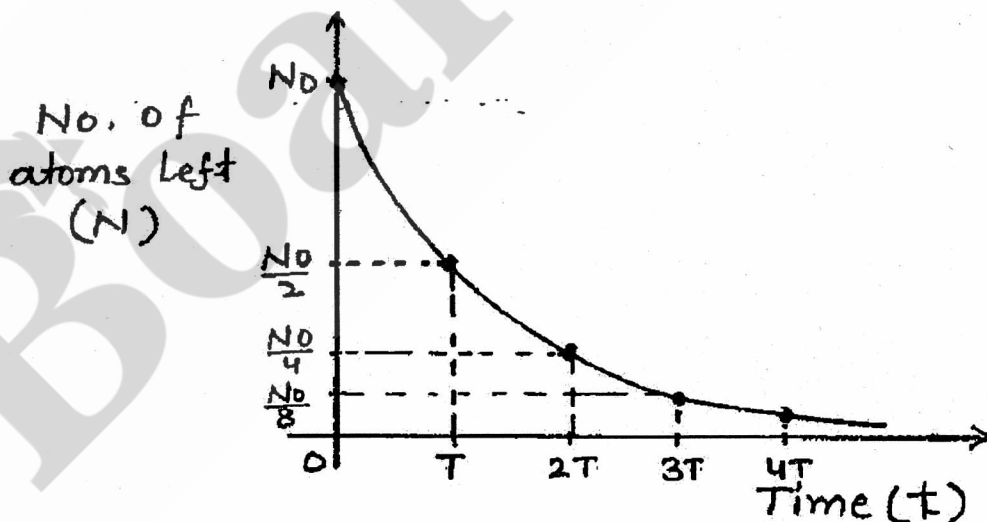
$$\text{or } \frac{1}{2} = e^{-\lambda T} \Rightarrow 2 = e^{\lambda T}$$

by taking log of both sides, we get -

$$\log_e 2 = \lambda T$$

$$\text{or } T = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$

$$T = \frac{0.693}{\lambda}$$



*

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$n \rightarrow$ number of half life times

Average / Mean Life of Radioactive Element -

Average life time of a radioactive element can be obtained by calculating the total life time of all the atoms of the element and dividing it by the total number of atoms present initially in the sample.

Let the sample of radioactive element containing N_0 atoms at $t=0$. The number of atoms left at time t be N . Suppose a small number of atoms $= dN$ disintegrate further in a small time dt .

Therefore the life time of each of these dN atoms is $(t+dt)$. If dt is taken very small, then the age of each of the dN atoms can be taken as t .

$$\therefore \text{Total age of } dN \text{ atoms} = t dN$$

$$\text{Total life time of all the atoms in sample} = \int_0^{N_0} t dN$$

$$\therefore \text{Average life time of the element } (\tau) = \frac{1}{N_0} \int_0^{N_0} t \cdot dN$$

$$\text{We know that - } dN = -\lambda N dt = -\lambda (N_0 e^{-\lambda t}) dt$$

Hence we get -

$$\tau = \frac{1}{N_0} \int_0^{\infty} (-\lambda N_0 e^{-\lambda t} dt) \times t$$

or

$$\tau = \lambda \int_0^{\infty} t e^{-\lambda t} dt$$

by Integrating by parts, we get -

$$\tau = \lambda \left[\left(t \frac{e^{-\lambda t}}{-\lambda} \right)_0^{\infty} - \int_0^{\infty} 1 \times \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$\tau = \lambda \left[0 + \frac{1}{\lambda} \int_0^{\infty} e^{-\lambda t} dt \right] = \int_0^{\infty} e^{-\lambda t} dt$$

$$\tau = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^{\infty} = \left[0 - \left(-\frac{1}{\lambda} \right) \right] = \frac{1}{\lambda}$$

$$\boxed{\tau = \frac{1}{\lambda}}$$

Hence average life of a radioactive element is reciprocal of the decay constant.

Q. In a given sample two radioactive elements A and B are initially present in the ratio of 1:4. The half lives of A and B are respectively 100 years and 50 years. Find the time after which the amount of A and B become equal.

CBSE
2012

Sol. Given that - $\frac{N_{0A}}{N_{0B}} = \frac{1}{4}$, $T_A = 100 \text{ Yr}$, $T_B = 50 \text{ Yr}$

so that - $\lambda_A = \frac{0.693}{100}$ and $\lambda_B = \frac{0.693}{50}$

$$\text{and } \frac{\lambda_A}{\lambda_B} = \frac{1}{2}$$

Let after t years, $N_A = N_B$

so that - $N_{0A} e^{-\lambda_A t} = N_{0B} e^{-\lambda_B t}$

$$e^{-\lambda_A t} = 4 e^{-\lambda_B t} \quad [N_{0B} = 4 N_{0A}]$$

$$\text{or } e^{-(\lambda_A - \lambda_B)t} = 4$$

$$-(\lambda_A - 4\lambda_B)t = \log_e 4 \Rightarrow \lambda_A t = \log_e 4$$

$$t = \frac{\log_e 4}{\lambda_A} = \frac{2 \times 0.693}{0.693/100}$$

$$t = 200 \text{ Years.}$$

Q. The half life of a radioactive element is 20 s. Calculate
(i) Decay Constant, (ii) time taken for the sample to decay $\frac{7}{8}$ th of the initial value.

CBSE
2009

Sol. Given that - $T = 20 \text{ s.}$

$$(i) \text{ Hence decay constant } \lambda = \frac{0.693}{T} = \frac{0.693}{20}$$

$$\lambda = 0.03465 \text{ s}^{-1}$$

$$(ii) N = N_0 - \frac{7}{8} N_0 \Rightarrow N = \frac{N_0}{8}$$

$$\text{by equation - } \left(\frac{N}{N_0}\right) = \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n \Rightarrow n = 3 \quad \text{so that time } t = nT$$

$$t = 3 \times 20 = 60 \text{ sec.}$$

Units of Radioactivity -

The activity of a radioactive source is measured by the rate of disintegration of the source.

Commonly used units of radioactivity are:

(1) Curie (Ci) -

The activity of a radioactive sample is said to be one curie when 3.7×10^{10} decays take place in every one second.

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/sec.}$$

(2) Becquerel (Bq) -

It is the SI unit of activity. The radioactivity is said to be one Becquerel when 1 decay take place in every one second.

$$1 \text{ Bq} = 1 \text{ decay/sec.}$$

(3) Rutherford (Rd) -

The activity of a radioactive sample is said to be one rutherford, when 10^6 decays take place in every one second.

$$1 \text{ Rd} = 10^6 \text{ decays/sec.}$$

Alpha, Beta and Gamma decay -

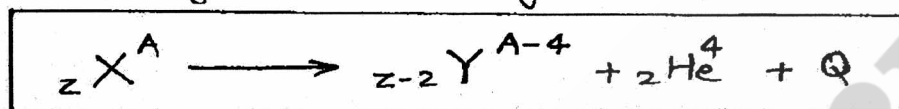
The radioactive nuclei are unstable and emit radiations to achieve states of greater stability. The emission of any of the α , β and γ radiations changes the original nucleus (Parent nucleus) into a new nucleus, called daughter nucleus.

The process of radiation continues till a stable nucleus of lead ($Z=82$) is formed.

Alpha decay -

It is the phenomena of emission of an α particle from a radioactive nucleus.

When a nucleus emits an α -particle, its mass number decreases by 4 and charge number decreases by 2.



The total mass energy of the decay products is less than the mass energy of the original nuclei. The difference between the initial mass energy and total mass energy of decay products is called disintegration energy (Q) of the process.

Hence

$$Q = [m_X - (m_Y + m_{\text{He}})] c^2$$

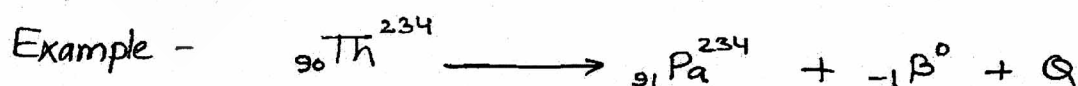
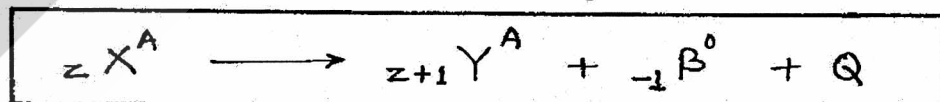
The energy released (Q) is shared by daughter nucleus Y and α -particle.



Beta decay -

Beta decay is the phenomena of emission of an electron from a radioactive nucleus.

When a nucleus emits a β -particle, its mass number remains same because mass of electron is negligibly low, and charge number increases by 1.



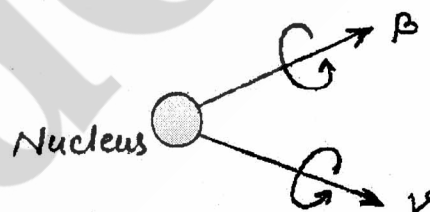
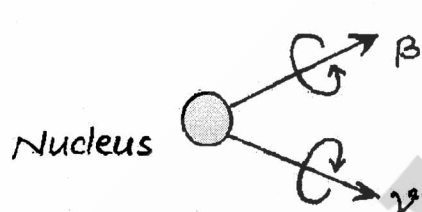
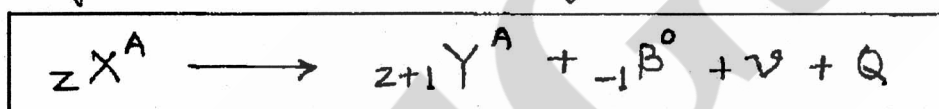
* The principle of conservation of energy Conservation of angular momentum fails in β -decay

Neutrino hypothesis -

In order to save the law of conservation of energy 'Pauli' postulated the existence of an electrically neutral low mass particle that would be emitted along with the beta particle, which is called 'Neutrino'.

The neutrino hypothesis implies that a low energy β -particle is accompanied by a high energy neutrino and vice-versa. So that total energy Q of a β particle and neutrino remains constant. Thus the law of Conservation of energy is followed in β -decay.

Thus β -decay of a nucleus may be represented as:



Further neutrino has a spin $\frac{1}{2} \left(\frac{h}{2\pi} \right)$. If both the β particles and neutrino spin in opposite direction, then the spin or angular momentum of daughter nucleus will be equal to spin/angular momentum of parent nucleus.

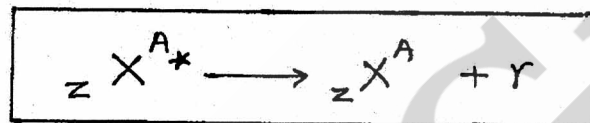
However when both the β particle and neutrino spin in the same direction then there will be integral change in spin or angular momentum of the daughter nucleus as observed experimentally.

Gamma decay-

Gamma decay is the phenomenon of emission of gamma ray photon from a radioactive nucleus.

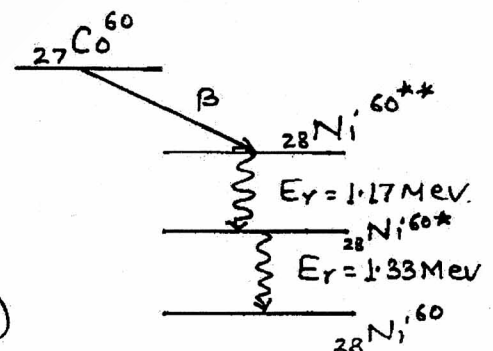
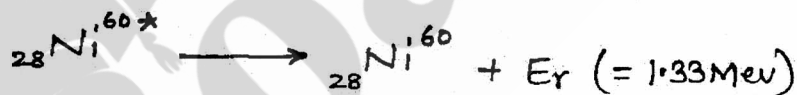
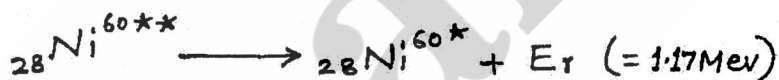
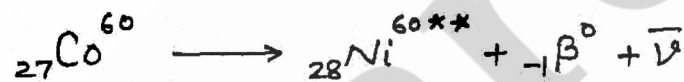
This occurs when an excited nucleus makes a transition to a state of lower energy. Hence the electromagnetic waves emitted by excited nuclei are called γ -rays.

Since photons do not have any charge or rest mass, therefore in γ -decay daughter nucleus has the same charge number and same mass number as those of parent nucleus.



After an α decay or a β decay, the daughter nucleus is usually in an excited state and it achieves stability by the emission of one or more γ -ray photons

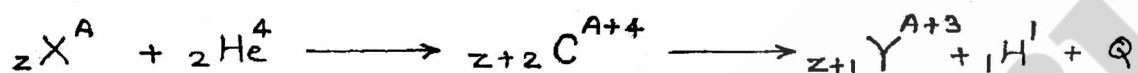
Example -



Nuclear Reaction -

A nuclear reaction represents the transformation of one stable nucleus into another nucleus by bombarding with suitable high energy particles.

Nuclear reaction can be represented as -



Here X is the target nucleus, bombarded by an α -particle. C is the compound nucleus which is unstable. It disintegrates to give a product nucleus Y and a product particle (Proton).

The reaction is named (α, P) reaction after the name of the bombarded particle (α) and product particle (P). In the nuclear reaction Q is the total energy change in the reaction.

Examples :

- (i) ${}_3 \text{Li}^7 + {}_1 \text{H}^1 \longrightarrow {}_4 \text{Be}^{8*} \longrightarrow {}_2 \text{He}^4 + {}_2 \text{He}^4$ (P, α reaction)
- (ii) ${}_5 \text{B}^{11} + {}_1 \text{H}^1 \longrightarrow {}_6 \text{C}^{12*} \longrightarrow {}_6 \text{C}^{12} + {}_0 \text{n}^1$ (P, n reaction)
- (iii) ${}_6 \text{C}^{12} + {}_1 \text{H}^1 \longrightarrow {}_7 \text{N}^{13*} \longrightarrow {}_7 \text{N}^{13} + \gamma$ (P, γ reaction)
- (iv) ${}_1 \text{H}^2 + \gamma \longrightarrow {}_1 \text{H}^1 + {}_0 \text{n}^1$ (Photo disintegration)

Note:-

The Q value of nuclear reaction is calculated from Einstein's mass energy equivalence relation $E = (\Delta m)c^2$. It may be positive or negative.

- * A nuclear reaction in which Q value is positive, energy is released, such a reaction is called exoergic.
- * A nuclear reaction in which Q value is negative, energy has to be supplied before the reaction occurs. Such a reaction is called endoergic.

Nuclear Fission -

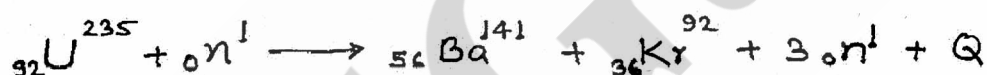
Nuclear fission is the phenomenon of splitting of a heavy nucleus ($A > 230$) into two or more lighter nuclei.

In this process, certain mass disappears i.e. sum of the masses of final product is found to be slightly less than the sum of the masses of the reactant components. This difference in masses is called mass defect (Δm).

Hence Energy released in nuclear fission (E) = $\Delta m \times c^2$

This energy appears in the form of γ -rays and Kinetic energy of fission fragment.

Example -

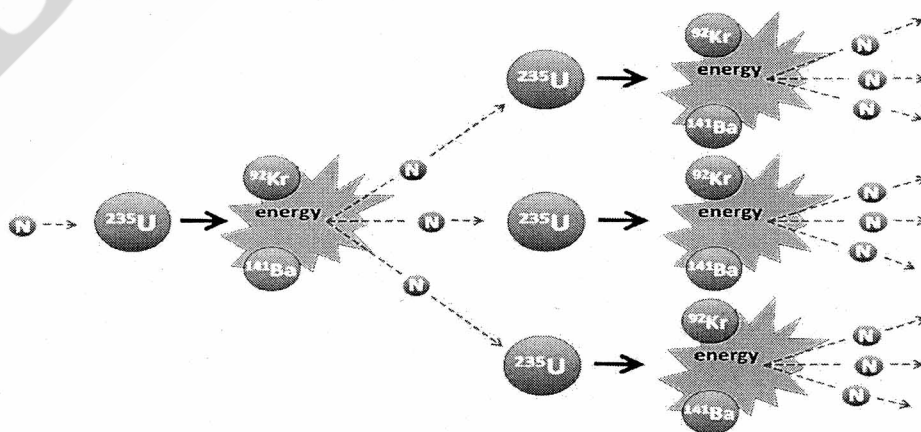


★ The disintegration energy (Q) in fission, first appears as the Kinetic energy of fragments. After that it is transferred to the surrounding matter in the form of heat.

Nuclear Chain Reaction -

Nuclear chain reaction refers to a process in which neutron released in fission produces an additional fission in at least one further nucleus. This nucleus in turn produces neutrons and the process repeats.

The process may be controlled (nuclear power) or uncontrolled (nuclear bomb).



The following points should be considered for a self propagating nuclear chain reaction :

- (1) Leakage of neutrons from the system.
- (2) Absorption of neutrons by impurity.
- (3) Absorption of neutrons by Uranium-238.
- (4) Critical size of the fissionable sample.
- (5) Reproduction factor (K) = $\frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$

- * If $K = 1$, the chain reaction will be steady and continuous.
- * If $K > 1$, the chain reaction accelerates resulting in explosion.
- * If $K < 1$, the chain reaction gradually stops.

Nuclear Reactor -

A nuclear reactor is a device to initiate and control a nuclear chain reaction. Nuclear reactors are used at nuclear power plants for electricity generation and in propulsion of ships.

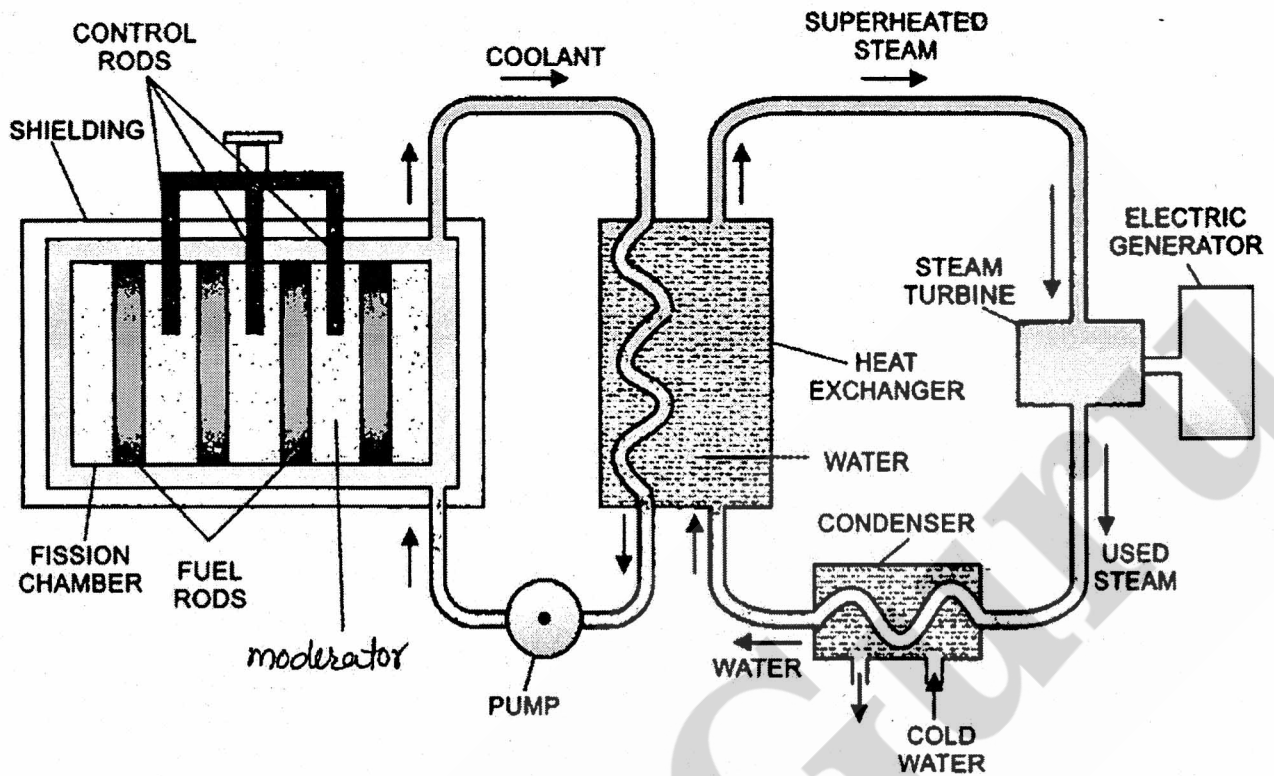
A nuclear reactor is based upon controlled nuclear chain reaction.

Construction :-

The main components of nuclear reactor are:

(1) Nuclear Fuel :-

It is a fissionable material to be used for the fission process to take place. Commonly used fuel in a nuclear reactor are U^{235} , U^{238} , Pu^{239} etc. When slow neutrons interact with the fuel the fission starts and the energy is released.



(2) Moderator -

Its function is to slow down the fast moving secondary neutrons produced during the fission. The material of moderator should be light and it should not absorb neutrons.

Usually heavy water, graphite, deuterium and paraffin etc. can be used as moderator

(3) Control Rods -

The control rods have the ability to capture the slow neutrons. To control the chain reaction from becoming violent, rods of cadmium or boron are inserted in the holes of reactor core, up to a desirable length, and the desired number of neutrons are absorbed and only limited number of neutrons are left to produce fission.

(4) Coolant :-

A substance which is used to remove the heat produced and transfer it from the core of the nuclear reactor to the surroundings is called coolant.

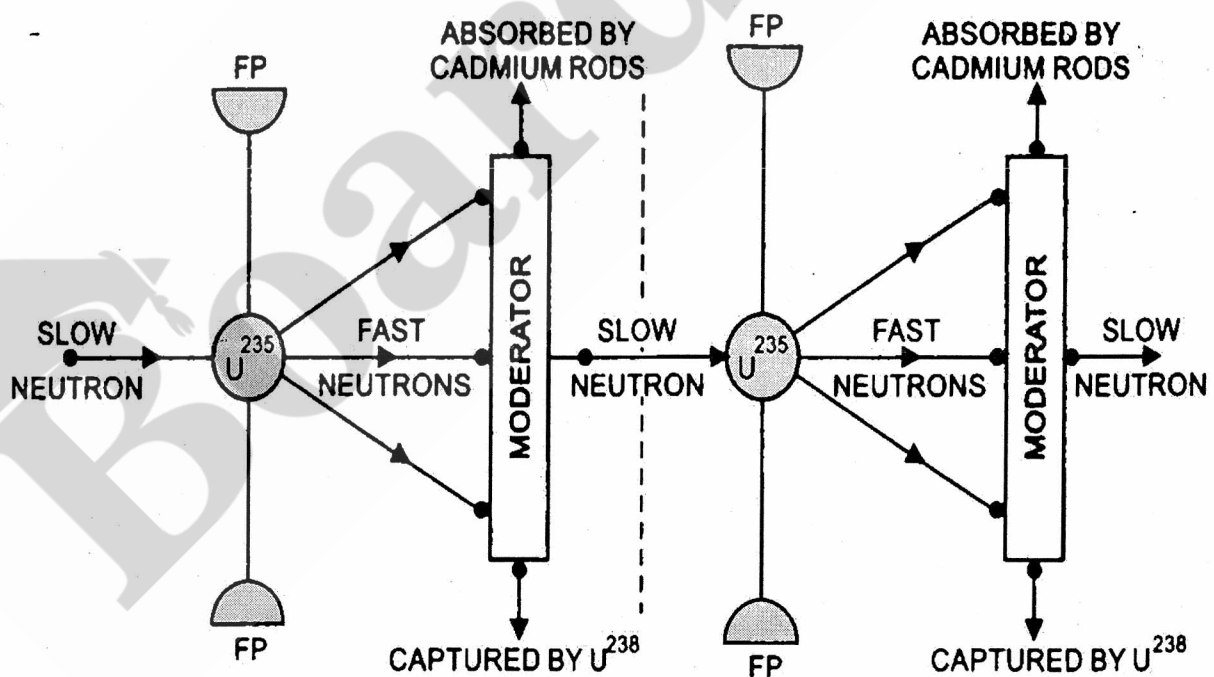
At ordinary temperature, water and heavy water serve as coolants but at high temperatures, generally liquid sodium is used as coolant.

(5) Shielding:-

The whole reactor is protected with concrete walls, 2 to 2.5 m. thick, so that radiations emitted during nuclear reactions may not produce harmful effects on the persons working on the reactor.

Working:-

As shown in figure slow neutrons cause the fission of U^{235} nuclei. To start the nuclear reaction, the cadmium rods are slowly removed and to stop it, they are inserted. This brings out a controlled nuclear chain reaction and hence the energy produced can be used for constructive purposes.



The nuclear reactors working in India are Apsara, Zerlina, CIR (Canada India Reactor) and Dhruva.

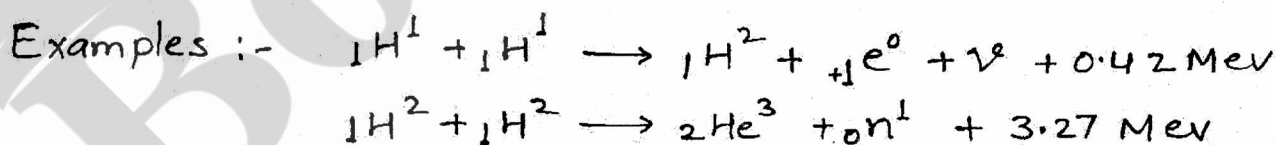
Uses of Nuclear Reactors-

- (i) Nuclear reactors are used to produce radioactive isotopes which in turn are used in medicine, industry and agriculture.
- (ii) They are used in electric power generation.
- (iii) They can be used for the propulsion of ships, submarines and air crafts.
- (iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Nuclear Fusion -

Nuclear Fusion is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

The mass of the product nucleus is slightly less than the sum of the masses of the lighter nuclei fusing together. This difference in masses (Δm) results in the release of tremendous amount of energy, in accordance with Einstein's mass energy relation $E = (\Delta m) c^2$.



In nuclear reactions, the coulomb repulsion between the nucleus prohibits them to come close enough to be within the range of their attractive nuclear forces and fuse. The height of coulomb barrier depends on the charges and the radius of the interacting nuclei.

The essential condition for carrying out nuclear fusion is to raise the temperature of the material so that particles have enough energy due to their thermal motions alone and they can penetrate the Coulomb barrier. This process is called Thermonuclear fusion.

Difference between Nuclear Fission and Nuclear Fusion -

- (1) In fission a heavy nucleus splits into two or more lighter nuclei. In fusion, two or more lighter nuclei fuse together to form a heavier nucleus.
- (2) For nuclear fission a fast moving neutron is needed. For carrying out nuclear fusion the lighter nuclei have to be brought very close to each other against Coulomb repulsion. For this suitable energy is to be made available, often by raising the temperature of the order of 10^7 K.
- (3) In a nuclear fusion reaction, energy liberated per unit mass of their nuclei is many times larger than the energy liberated per unit mass in nuclear fission reaction. Therefore for a given weight, hydrogen bomb (based on nuclear fusion) is far more dangerous than an atom bomb (based on nuclear fission).
- (4) The product of nuclear fission are radioactive. They produce environmental pollution and hence require very careful disposal. However the products of nuclear fusion are not radioactive. They are harmless and can be disposed off easily.

Very Important Questions

1 Mark Questions -

- Q.1. Write any two characteristic properties of nuclear forces. (CBSE 2011)
- Q.2. How is the mean life time of a radioactive sample related to its half life? (CBSE 2011)
- Q.3. How is the radius of a nucleus related to its mass number? (CBSE 2011)
- Q.4. A nucleus undergoes β -decay. How does its
(a) Mass number (b) atomic number change? (Delhi 2011)
- Q.5. Two nuclei have mass numbers in the ratio 1:8 what is the ratio of their nuclear radius? (CBSE 2009)
Answer: 1:2
- Q.6. Assuming the nuclei to be spherical in shape, how does the surface area of a nucleus of mass number A_1 compare with that of a nucleus of mass number A_2 ? (CBSE 2008)
Answer: $(A_1/A_2)^{2/3}$
- Q.7. Define the term 'activity' of a radionuclide. Write its SI unit. (RBSE 2006, CBSE 2007)
- Q.8. The radioactive isotope D decays according to the given sequence -
$$D \xrightarrow{\beta\text{-Particle}} D_1 \xrightarrow{\alpha\text{-Particle}} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what is (a) the mass number (b) atomic number of D? (Delhi 2007)

Q.9. Name the absorbing material used to control the reaction rate of neutrons in a nuclear reactor.

(CBSE 2008)

Q.10 A radioactive sample has N_0 nuclei at $t=0$. Its number of undecayed nuclei get reduced to $\frac{N_0}{e}$ at $t=\tau$. What does the term τ stand for? Write in term of τ , the time interval T in which half of the original number of nuclei of this radionuclide would have got decayed.

(CBSE 2004)

2 Marks Questions-

Q.1. How does the size of a nucleus is experimentally determined? Write the relation between the radius and mass number of the nucleus. Show that the density of nucleus is independent of its mass number.

(CBSE 2008)

Q.2. A radioactive nucleus A undergoes a series of decays according to the following scheme-



The mass number and atomic number of A_4 are 172 and 69 respectively. What are these numbers for A?

Answer : 180 and 72

(CBSE 2003)

Q.3. Draw a plot representing the law of radioactive decay. Define the activity of a sample of a radioactive nucleus. Write its SI unit.

(RBSE 2008)

Q.4. Why is it necessary to slow down the neutrons in nuclear fission of ${}_{92}\text{U}^{235}$ nuclei by neutrons? What type of nuclei are needed for slowing down fast neutrons?

(CBSE 2002, 08)

- Q.5. Using the binding energy curve per nucleon as a function of mass number A , explain how the release in energy in the processes of nuclear fission and nuclear fusion can be explained. (CBSE 2011)
- Q.6. Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions you can draw regarding the nature of nuclear forces. (CBSE 2003, 10)
- Q.7. Draw a plot of the binding energy per nucleon as a function of mass number for a large number of nuclei $20 < A < 240$. How do you explain the constancy of binding energy per nucleon in the range $30 < A < 170$. (CBSE 2010)
- Q.8. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV is split into two fragments Y and Z of mass numbers 110 and 130. The binding energy of nucleons in Y and Z is 8.5 MeV per nucleon. Calculate the energy released per fission in MeV. (CBSE 2005, 10)
- Q.9. (a) The mass of a nucleus in its ground state is always less than the total mass of its constituents neutrons and protons. Explain.
- (b) Plot a graph showing the variation of Potential energy of a pair of nucleons as a function of their separation. (RBSE 2006, CBSE 2009)
- Q.10. State the Law of radioactive decay. Establish a mathematical relation between half life time and disintegration constant of a radioactive nucleus.

3 Marks Questions-

Q.1. State the Law of radioactive decay.

Plot a graph showing the number (N) of undecayed nuclei as a function of time (t) for a given radioactive sample having half life time T.

(RBSE 2002, CBSE 2011)

Q.2. Give an example of decay process for β emission.

(a) Is the energy of the emitted β -particles continuous or discrete?

(b) Is the daughter nucleus obtained through β decay an isotope or an isobar of the parent nucleus?

(RBSE 2004, CBSE 2010)

Q.3. What is meant by half life of a radioactive element?

The half life of a radioactive substance is 30 sec.

Calculate - (i) The decay constant (ii) Time taken for the sample to decay by $\frac{3}{4}$ th of the initial value.

(RBSE 2005, CBSE 2009)

Q.4. Calculate the energy released if ${}_{92}\text{U}^{238}$ nucleus emits an α -particle. Given that -

Mass of $\text{U}^{238} = 238.0508 \text{ u}$, Mass of $\text{U}^{234} = 234.04363 \text{ u}$
mass of α -particle = 4.0026 u and $1 \text{ u} = 931 \text{ Mev}$.

Answer: $Q = 4.25 \text{ Mev}$ (CBSE 2001, 07)

Q.5. Distinguish between nuclear fission and fusion -

In a fusion reaction ${}_1\text{H}^2 + {}_1\text{H}^3 \rightarrow {}_2\text{He}^4 + {}_0\text{n}^1$

Calculate the amount of energy in Mev released

Given $m({}_1\text{H}^2) = 2.014102 \text{ u}$, $m({}_1\text{H}^3) = 3.016049 \text{ u}$.

$m({}_2\text{He}^4) = 4.002603 \text{ u}$, $m_n = 1.00867 \text{ u}$.

Answer = $E = 17.5848 \text{ Mev}$ (CBSE 2002, 07)

Q.1. Assuming the nuclei to be spherical in shape, how does the surface area of a nucleus of mass number A_1 compare with that of a nucleus of mass number A_2 ?

Ans. $\frac{S_1}{S_2} = \left(\frac{A_1}{A_2}\right)^{2/3}$

Q.2. Two nuclei have mass number in the ratio of 2:5. What is the ratio of their nuclear densities ?

Ans. 1:1

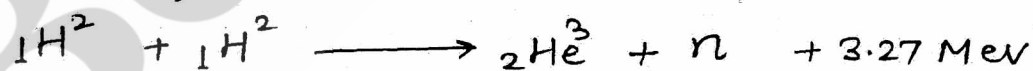
Q.3. The binding energy per nucleon of the two nuclei A and B are 4 Mev and 8.2 Mev. Which of the two nuclei is more stable ?

Q.4. Draw a graph showing the variation of decay rate with number of active nuclei.

Q.5. Define half life of a radioactive sample. Which of the following radiations : α -rays, β -rays and γ -rays (i) are similar to x ray (ii) are easily absorbed by matter (iii) travel with the greatest speed. (iv) are similar in nature to Cathode ray?

Ans. (i) γ -rays (ii) α -rays (iii) γ -rays (iv) β -rays

Q.6. In a typical nuclear reaction, e.g.



Although number of nucleons is conserved, yet energy is released. How ? Explain

Q.7. Why is heavy water used as a moderator in a nuclear reactor ?

Q.8. What is the longest wavelength photon that can ionize a hydrogen atom in its ground state?

Ans. $\lambda = 0.91 \times 10^{-10} \text{ m}$.

Q.9. ${}^3_2\text{He}$ and ${}^3_1\text{H}$ nuclei have the same mass number. Do they have the same binding energy?

Q.10. The ground state energy of hydrogen atom is -13.6 eV . What is the Kinetic energy and potential energies of the electron in this state?

Q.11. The ground state energy of hydrogen atom is -13.6 eV . If an electron makes a transition from an energy level -0.85 eV to -3.4 eV , calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?

Ans. $\lambda = 4862 \text{ \AA}$, Balmer Series

PHYSICS

Unit - 14. Semiconductor Electronics

Class - 12th

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15. SEMICONDUCTOR ELECTRONICS

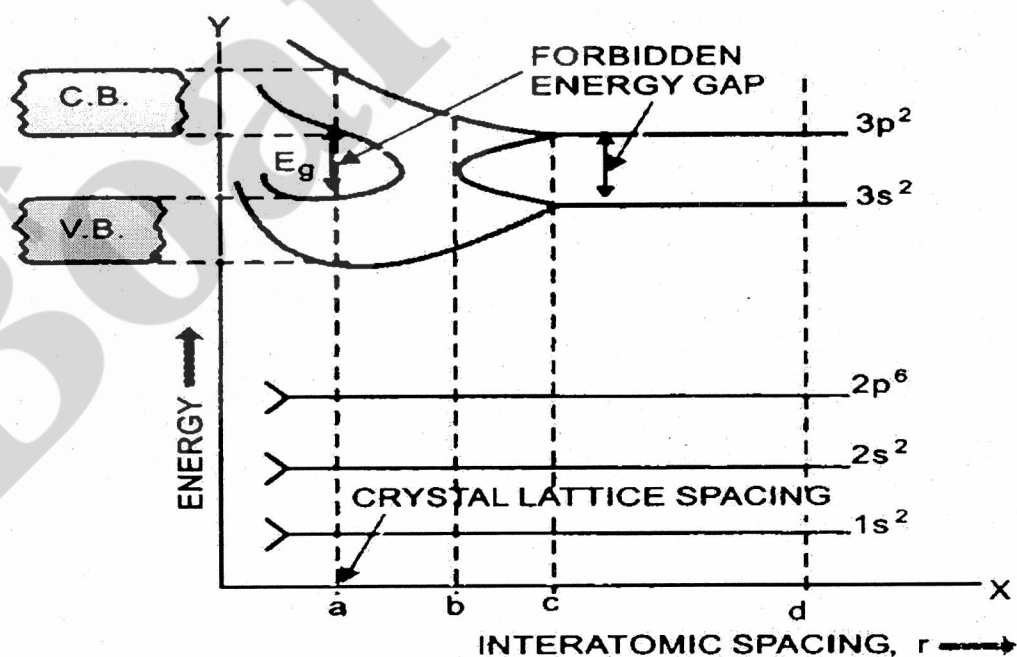
Semiconductors -

The materials which have conductivity and resistivity in between conductors and insulators are called semiconductors.

Examples - Silicon (Si), Germanium (Ge) etc.

Energy Band Theory of Solids -

According to Bohr's theory there are well defined energy levels of electrons in an atom. If large number of atoms are brought close to one another to form a crystal, they begin to influence each other. Due to this interatomic interaction there is no modification in the energy levels of the electrons in the inner shell but there is a considerable modification in the energy levels of the electrons in the outer shells.



To understand modification in energy levels of electrons consider a silicon crystal containing N atoms.

- Silicon (Si) atoms have four valence electrons i.e. number of electrons in the outermost orbit is 4. Therefore the total number of valence electrons in the crystal of Si is $4N$.
- (i) If the interatomic spacing of the Si atoms is very large ($r = d$), then there is no interatomic interaction.
- (ii) When the interatomic spacing r is less than d but greater than c , then there is no visible splitting of energy levels.
- (iii) When the interatomic spacing r is equal to c , the energy of outermost shell electrons of neighbouring silicon atoms start changing i.e. the splitting of these energy levels occurs. Whereas there is no change in the energy levels of electrons in the inner shells.
- (iv) When interatomic spacing r lies in between b and c ($b < r < c$), instead of a single $3s$ or $3p$ level, we get a large number of closely packed levels. Where $2N$ levels corresponding to a single $3s$ level and $6N$ levels for a single $3p$ level of an isolated atom. This spreading of energy levels reduces the energy gap between $3s$ and $3p$ levels of free atom.
- This collection of closely spaced levels is called an energy band.
- (v) When the interatomic spacing r becomes equal to b but greater than a ($r = b > a$), the energy gap between $3s$ and $3p$ levels completely disappears. In such a situation, it is not possible to distinguish between the electrons belonging to $3s$ and $3p$ subshells. We can only say that $4N$ levels are filled and $4N$ levels are empty.

(vi) when the interatomic spacing r becomes equal to a ($r=a$) then the band of $4N$ filled energy levels is separated from the band of $4N$ unfilled energy levels by an energy gap called energy band gap, which is denoted by E_g .

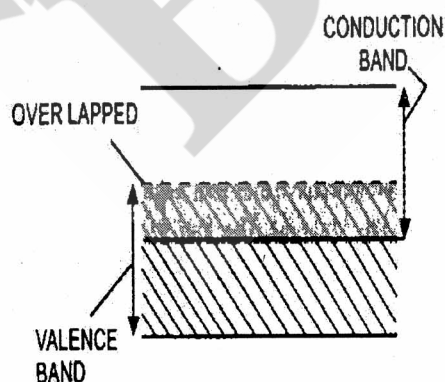
The lower completely filled band is called valance band and the upper unfilled band is called conduction band. The minimum energy required for shifting electrons from valance band to conduction band is equal to energy band gap (E_g).



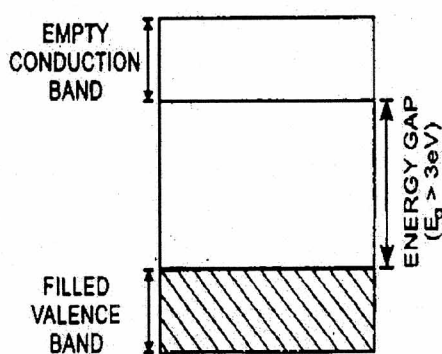
Difference between Metals (Conductors), Insulators and semiconductors on the basis of Energy Bands :-

Conductors (Metals) -

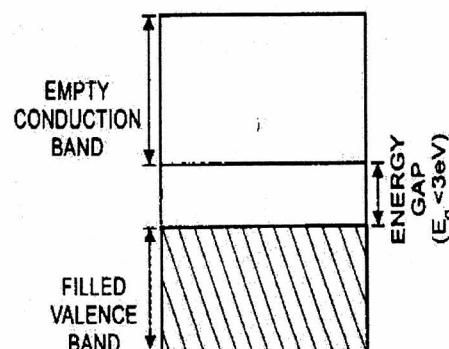
In conductors the conduction and valance band partly overlap each other and there is no energy gap in between.



Conductors



Insulators



Semiconductors

Semiconductors -

In semiconductors the valance band is totally filled and the conduction band is empty but the energy gap between conduction band and valance band is quite small. It is less than 3 eV. E_g for germanium is 0.72 eV and for silicon it is 1.1 eV.

At zero kelvin temperature semiconductor behaves as insulator.

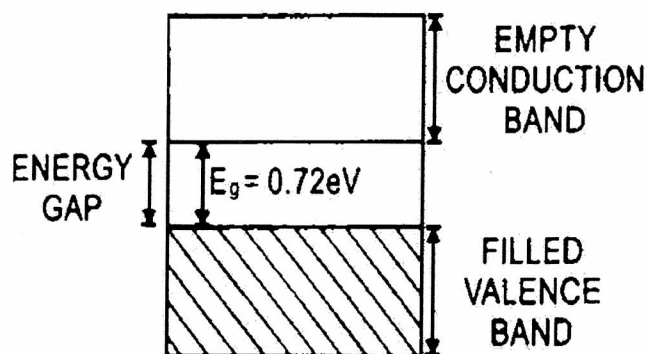
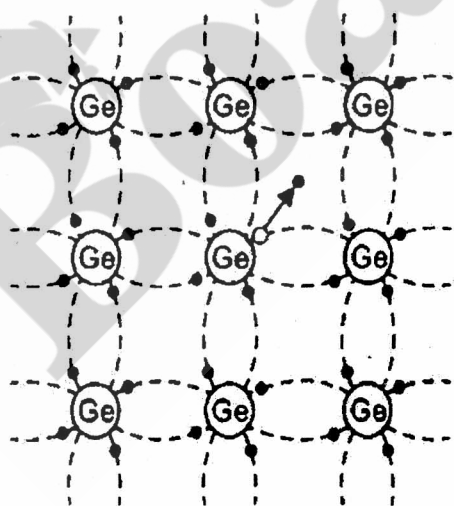
Insulators -

In insulators the energy gap is quite large ($E_g > 3 \text{ eV}$). Due to large energy gap no electron is able to go from the valance band to the conduction band. Hence electrical conduction in these materials is impossible and they behave as insulators.

INTRINSIC SEMICONDUCTORS -

A pure semiconductor which is free of every impurity is called intrinsic semiconductor.

Examples - Silicon (Si) and Germanium (Ge).



Silicon (14) $1s^2 2s^2 2p^6 3s^2 3p^2$

Germanium (32) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^2$

Both the atoms (Si and Ge) have four valance electrons.

The four valance electrons of a germanium atom form four covalent bonds by sharing the electrons of neighbouring four germanium atoms.

The minimum energy required to break a covalent bond is 0.72 eV for Ge and 1.1 eV for Si. At room temperature when an electron breaks away from a covalent bond, the empty place or vacancy left in the bond is called a hole.

When an external electric field is applied, these free electrons and holes move in opposite directions and constitute a current. The number of free electrons and holes are exactly equal in an intrinsic semiconductor.

$$n_e = n_h$$

Also the conductivity of intrinsic semiconductor is very low. The hole is considered as an active particle in the valance band, having a positive charge equal to that of an electron.

Doping :-

Doping is a process of addition of a desirable impurity atoms to a pure semiconductor to modify its properties in a controlled manner. The impurity atoms added are called dopants.

Doping of a semiconductor increases its electrical conductivity to a great extent.

Methods of doping :-

- (i) Add the impurity atoms in the melt of semiconductor.
- (ii) Implant dopant atoms by bombarding the semiconductor with their ions.
- (iii) Heat the semiconductor in atmosphere of dopant atoms.

EXTRINSIC SEMICONDUCTORS -

A doped semiconductor or a semiconductor with suitable impurity atom added to it is called extrinsic semiconductor.

Extrinsic semiconductors are of two types -

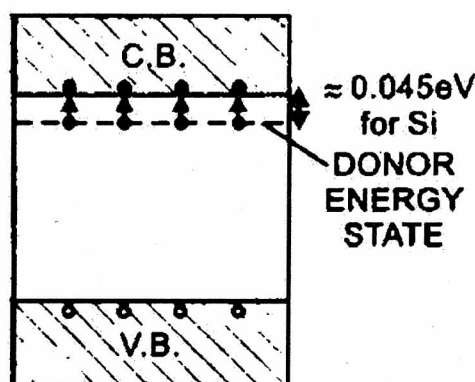
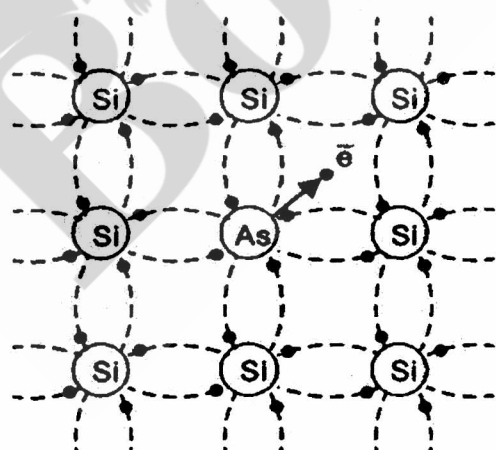
(i) n-type Semiconductor (ii) P-type Semiconductor

(i) n-type Semiconductor -

When a pure semiconductor of silicon (Si) or Germanium (Ge) is doped with pentavalent atoms which have five valance electrons (Phosphorous, Arsenic, antimony or Bismuth) then it is called n-type semiconductor.

The four of the five valance electrons of the impurity atoms will form covalent band with the adjoining four atoms of the silicon, while the fifth electron is free to move. Thus each impurity atom added donates one free electron to the crystal. These impurity atoms are called donor atoms.

Since the conduction of electricity is due to the motion of electrons (i.e. negative charges), so that the resulting semiconductor is called n-type or donor type semiconductor.



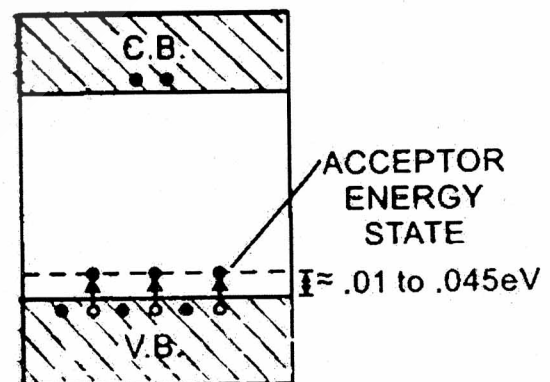
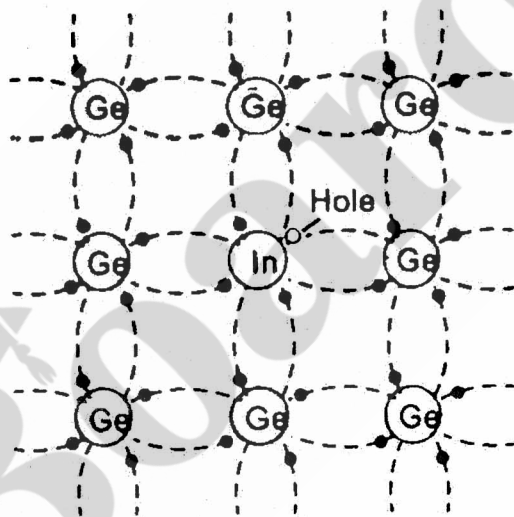
In n-type semiconductors electrons are majority carriers and holes are minority carriers.

(ii) P-type Semiconductor -

When a pure semiconductor of silicon (Si) or Germanium (Ge) is doped with a controlled amount of trivalent atoms, which have three valance electrons (Boron, Aluminium, Gallium, Indium) then it is called p-type semiconductor.

The three valance electrons of the impurity atom will form Covalent bonds with the adjoining three atoms of germanium (Ge), while there will be one incomplete covalent bond with a neighbouring Ge atom, this deficiency of an electron creates a 'hole'.

The trivalent atoms are called acceptor atoms and the conduction of electricity occurs due to motion of holes (i.e. positive charges), so the resulting semiconductor is called p-type or acceptor type semiconductor.



In p-type semiconductor electrons are minority carriers and holes are majority carriers.

Distinction between Intrinsic and extrinsic semiconductors-

INTRINSIC SEMICONDUCTOR	EXTRINSIC SEMICONDUCTOR
1. It is pure semiconducting material and no impurity atoms are added to it. 2. Examples are crystalline forms of pure silicon and germanium. 3. The number of free electrons in conduction band and the number of holes in valence band is exactly equal and very small indeed. 4. Its electrical conductivity is low. 5. Its electrical conductivity is a function of temperature alone.	1. It is prepared by doping a small quantity of impurity atoms to the pure semiconducting material. 2. Examples are silicon and germanium crystals with impurity atoms of arsenic, antimony, phosphorous etc. or indium, boron, aluminium etc. 3. The number of free electrons and holes is never equal. There is excess of electrons in <i>n</i>-type semiconductors and excess of holes in <i>p</i>-type semiconductors. 4. Its electrical conductivity is high. 5. Its electrical conductivity depends upon the temperature as well as on the quantity of impurity atoms doped in the structure.

Difference between N-type and P-type semiconductors-

n-TYPE SEMICONDUCTOR	p-TYPE SEMICONDUCTOR
1. It is an extrinsic semiconductor which is obtained by doping the impurity atoms of Vth group of periodic table to the pure germanium or silicon semiconductor. 2. The impurity atoms added, provide extra electrons in the structure, and are called donor atoms. 3. The electrons are majority carriers and holes are minority carriers. 4. The electron density (n_e) is much greater than the hole density (n_h), i.e., $n_e \gg n_h$. 5. The donor energy level is close to the conduction band and far away from valence band.	1. It is an extrinsic semiconductor which is obtained by doping the impurity atoms of III group of periodic table to the pure germanium or silicon semiconductor. 2. The impurity atoms added, create vacancies of electrons (i.e. holes) in the structure and are called acceptor atoms. 3. The holes are majority carriers and electrons are minority carriers. 4. The hole density (n_h) is much greater than the electron density (n_e), i.e., $n_h \gg n_e$. 5. The acceptor energy level is close to valence band and is far away from conduction band.

P-N JUNCTION -

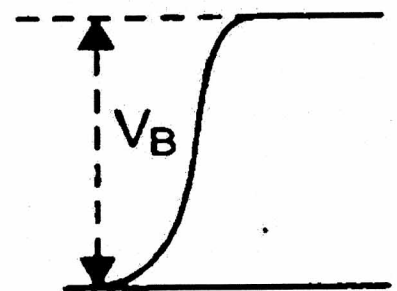
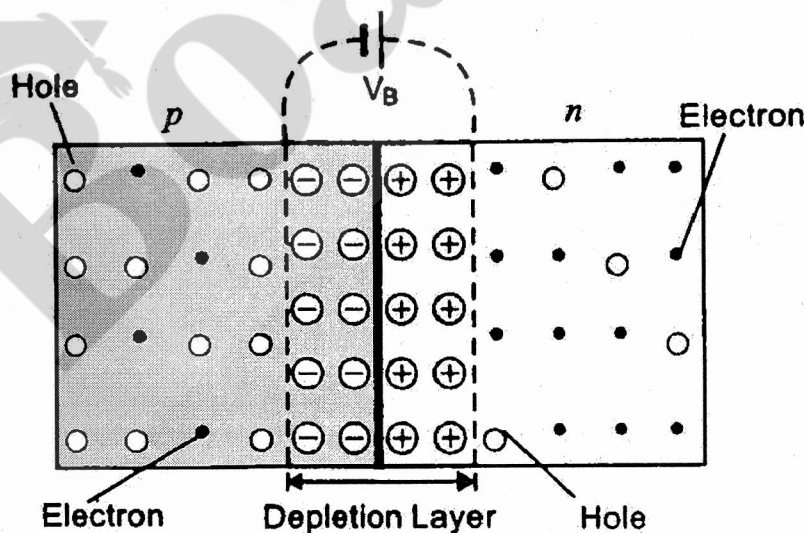
When a p-type semiconductor crystal is joined with an n-type semiconductor crystal, then the resulting arrangement is called a p-n junction or junction diode.

Formation of PN-Junction -

To make a p-n junction, the n type and p-type silicon crystals are cut into thin wafers. If on a wafer of n-type silicon, an aluminium film is placed and heated to a high temperature 580°C , aluminium diffuses into silicon and a p-type semiconductor is formed on an n-type semiconductor. Such a formation of p-region on n-region is called p-n junction.

Diffusion and Drift -

When p-n junction is formed due to difference in concentration of charge carriers in two regions of p-n junction, the electrons from n-region diffuse through the junction into p-region and holes from p-region diffuse into n-region. The motion of charge carriers gives rise to diffusion current across the junction.



Due to diffusion of electrons and holes a layer of positively charged donor atoms in n-region and a layer of

- negatively charged acceptor atoms in p-region are created.
- This positive and negative space charge regions on both sides of p-n junction will form a region which has immobile ions and is called depletion region.

Due to positive and negative space charge region at p-n junction, an electric field is set up across the junction. Due to this electric field developed at the junction, an electron on p-side of the junction moves to n-side and a holes on n-side of junction moves to p-side of junction. The motion of these charge carriers due to electric field is called drift. As a result of it, a drift current starts, which is opposite in direction to the diffusion current.

In the begining the diffusion current is large but drift current is small. As the diffusion process continues, the strength of electric field across the junction increases and thereby drift current increases. This process continues until the diffusion current becomes equal to the drift current. Now the p-n junction is said to be in equilibrium state and there is no current across the p-n junction. At this stage, the potential barrier across the junction has maximum value V_B . Now the movement of majority charge carriers across the junction stops and the potential acts as a barrier, hence known as potential barrier.

At room temperature (300 K) V_B is about 0.3 V for Ge and 0.7 V for Si. The value of V_B increases with rise in temperature for Ge and Si.

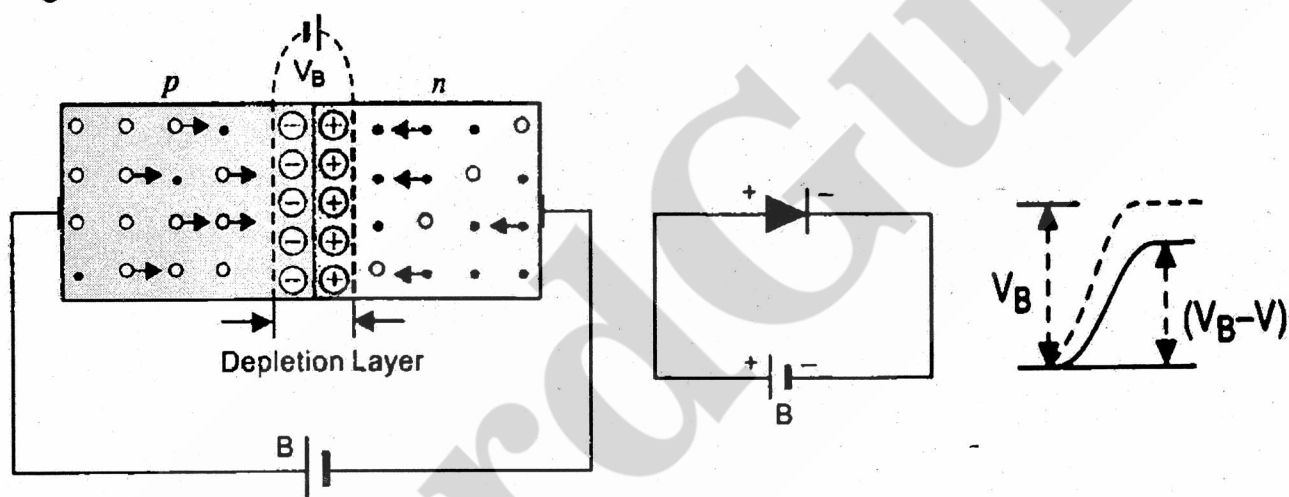
BIASING OF THE PN JUNCTION -

There are two methods of biasing the p-n junction-

- (i) Forward biasing (ii) Reverse biasing

(1) Forward Biasing :-

A p-n junction is said to be forward biased if the positive terminal of the external battery is connected to p-side and the negative terminal to the n-side of p-n junction.



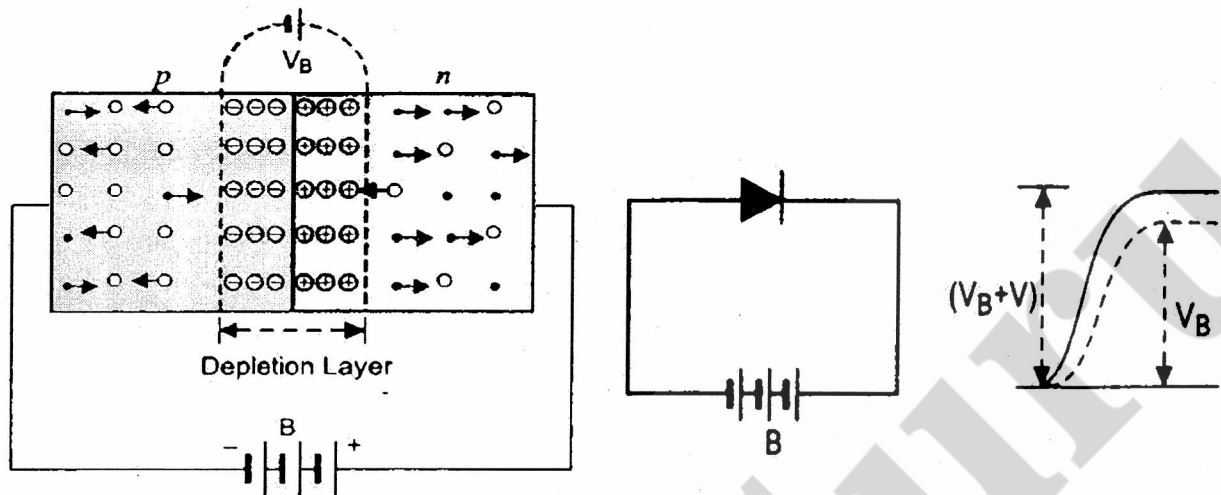
In forward biasing the forward voltage opposes the potential barrier V_B . As a result of it potential barrier and width of depletion layer decreases. The effective value of potential barrier in forward biasing is $(V_B - V)$.

(2) Reverse Biasing :-

A p-n junction is said to be reverse biased if the positive terminal of the external battery is connected to n-side and the negative terminal to p-side of the p-n junction.

In reverse biasing the reverse bias voltage supports the potential barrier V_B . As a result of it barrier potential and width of depletion region increases.

The effective value of barrier potential under reverse bias is $(V_B + V)$.

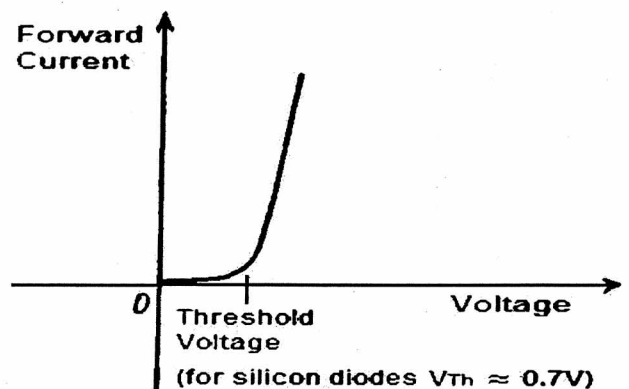
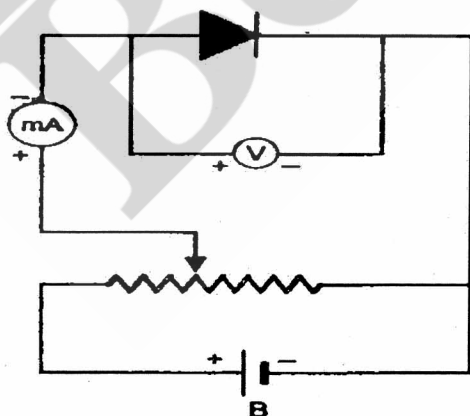


In reverse biasing, there is no conduction across the junction due to majority carriers. However a few minority carriers of p-n junction diode cross the junction after being accelerated by high reverse bias voltage. They constitute a current which is called reverse current or leakage current.

CHARACTERISTICS OF PN JUNCTION DIODE -

(i) Forward Characteristics -

On plotting a graph between forward bias voltage and forward current we get the following graph.

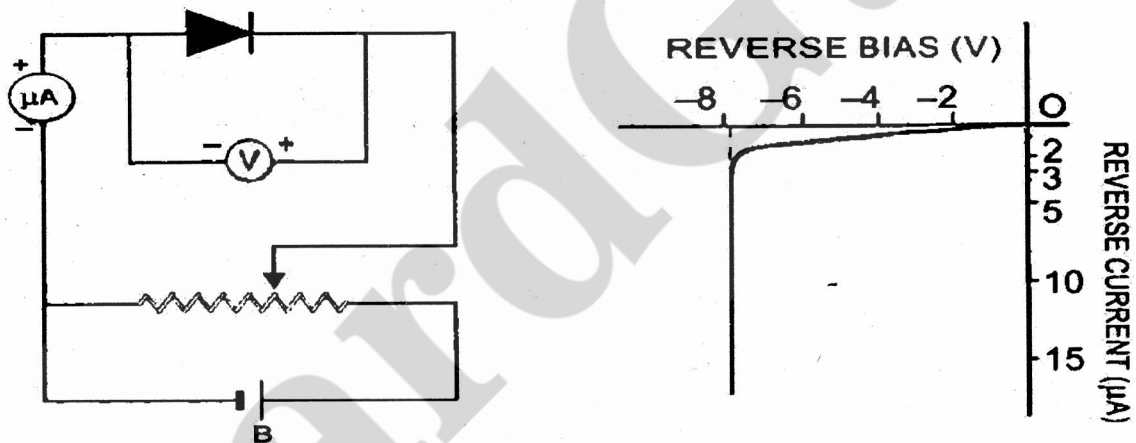


It is found that beyond forward voltage $V = V_K$, called Knee voltage (0.3V for Ge and 0.7V for Si) the current

through the junction starts increasing rapidly with voltage and showing the linear variation. But below the knee voltage the variation in current is negligible and the curve is non-linear.

(ii) Reverse Characteristics -

On plotting a graph between reverse bias voltage and reverse current, we get the reverse characteristics as shown in fig. From the curve we note that in reverse biasing of p-n junction, the current is very small (μA) and is independent on voltage upto certain reverse bias voltage, known as breakdown voltage.



If the reverse bias voltage exceeds the breakdown voltage, the reverse current through the p-n junction will increase abruptly. If this current exceeds the rated value of p-n junction (specified by manufacturer), the p-n junction will get damaged.

PN-DIODE AS A RECTIFIER -

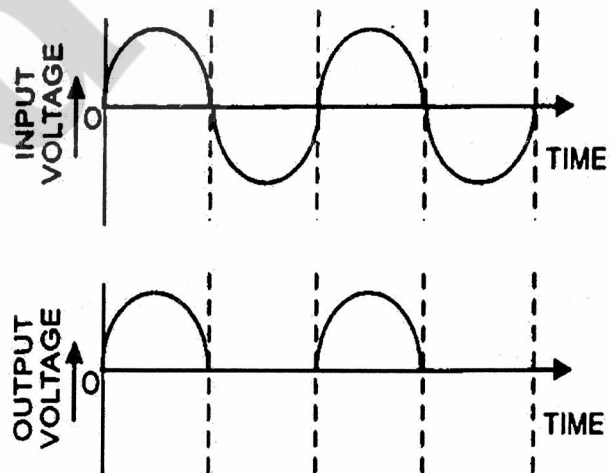
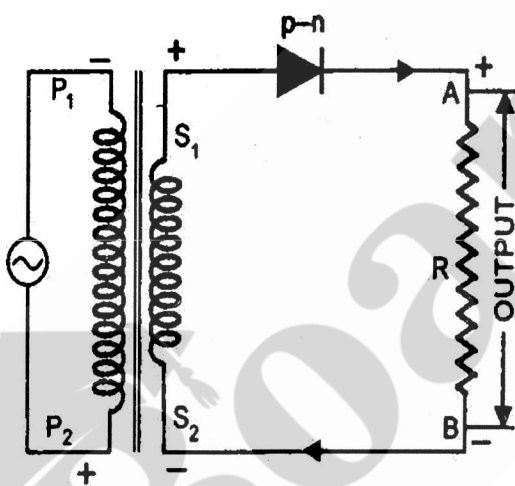
Rectifier is a device which is used for converting alternating current/voltage into direct current/voltage.

There are two types of rectifier -

- (i) Half wave rectifier (ii) Full Wave Rectifier

(1) HALF WAVE RECTIFIER -

A.C. voltage to be rectified is connected to the primary winding P_1P_2 of a stepdown transformer. S_1S_2 is the secondary coil of the transformer. S_1 is connected to the portion P of the $p-n$ junction. S_2 is connected to the portion n through load resistance R . Output is taken across the load resistance R .



WORKING -

During positive half cycle of A.C., suppose S_1 becomes positive, S_2 becomes negative and the $p-n$ junction is forward biased. The resistance of $p-n$ junction becomes low. The maximum forward current flows in the circuit and we get output across - Load.

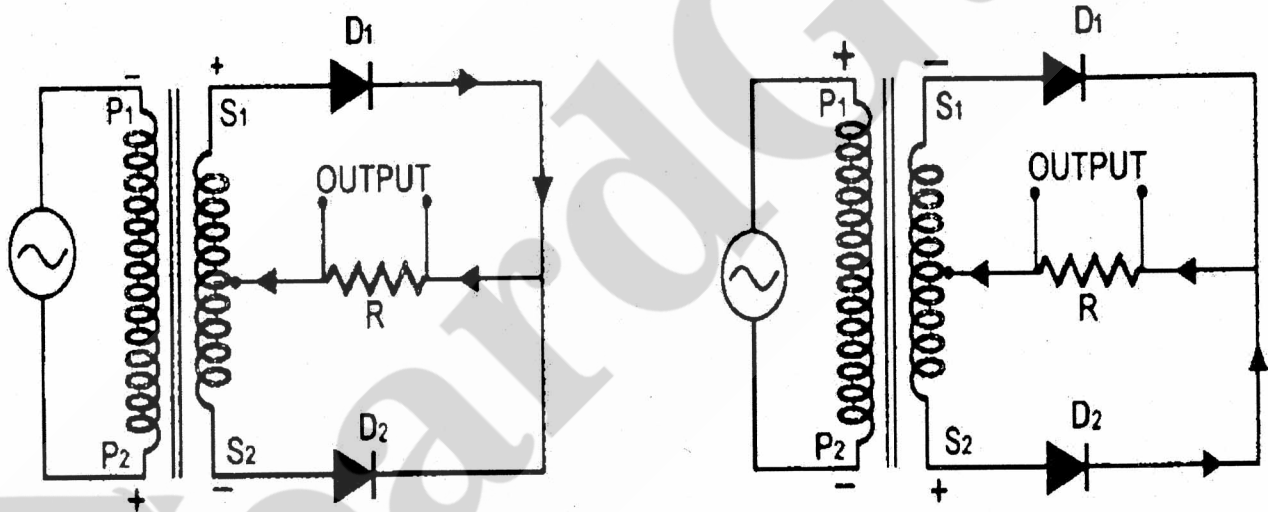
During negative half cycle of A.C. S_1 becomes negative, and S_2 is positive. The $p-n$ junction is reverse

biased. It offers high resistance and hence there is no flow of current and thus no output across Load.

Hence in the output, we have current corresponding to one half cycle of the wave and the other half is missing. That is why the process is called half wave rectification.

(2) FULL WAVE RECTIFIER -

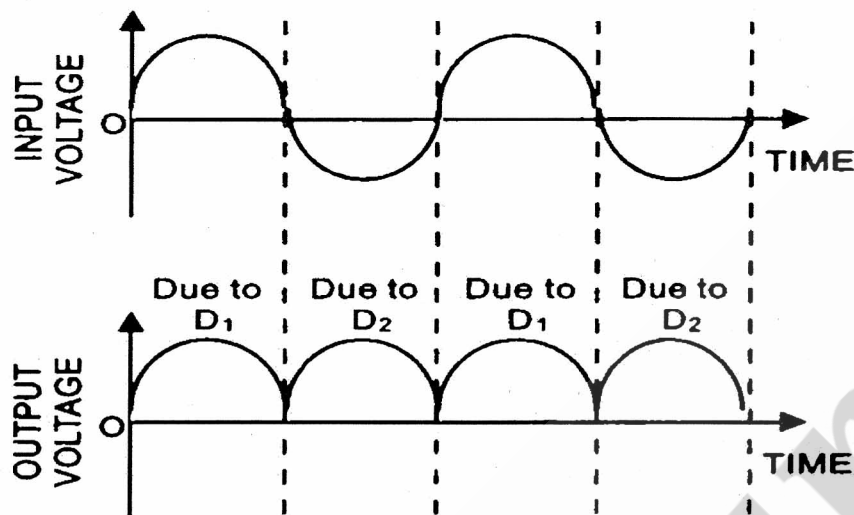
For full wave rectification, we have to use two P-n junction diodes D_1 and D_2 . The circuit is shown in the given fig.



WORKING -

During the positive half cycle of A.C. the diode D_1 is forward biased and the diode D_2 is reverse biased. The forward current flows through diode D_1 in the direction as shown in fig.

During the negative half cycle of A.C. the diode D_1 is reverse biased and the diode D_2 is forward biased. The forward current flows through diode D_2 . We observe that during both the half cycles of A.C., current through R flows in the same direction.



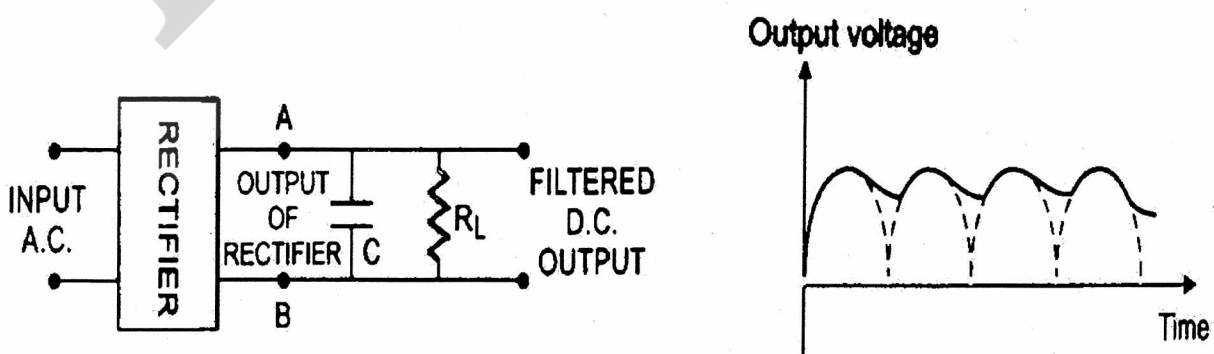
Hence the output signal voltage is unidirectional having ripple contents (i.e. d.c. components and a.c. components of voltage). It can be converted into d.c. voltage by filtering through a filter circuit.

FILTER-

A single capacitor of high value of capacitance connected across the output of rectifier can work as the filter.

The capacitor offers low impedance to a.c. component ($X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$) and offers infinite impedance to d.c.

Due to it, the a.c. component is bypassed or filtered out. It produces a voltage drop across load resistance R_L as a filtered d.c. output, which is almost d.c. voltage. Such filter is widely used in power supplies.

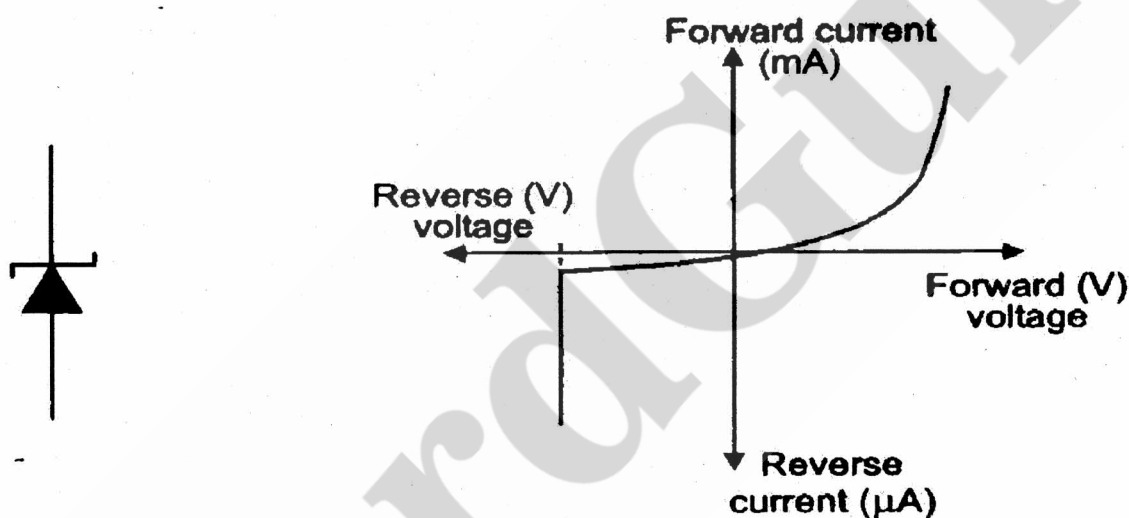


SPECIAL PURPOSE P-N JUNCTION DIODE -

Some devices which use basically p-n junction diodes are developed for different applications.

(i) ZENER DIODE -

It is designed specially to operate in reverse breakdown voltage region continuously without being damaged. In zener diode both p-side and n-side of p-n junction are heavily doped.



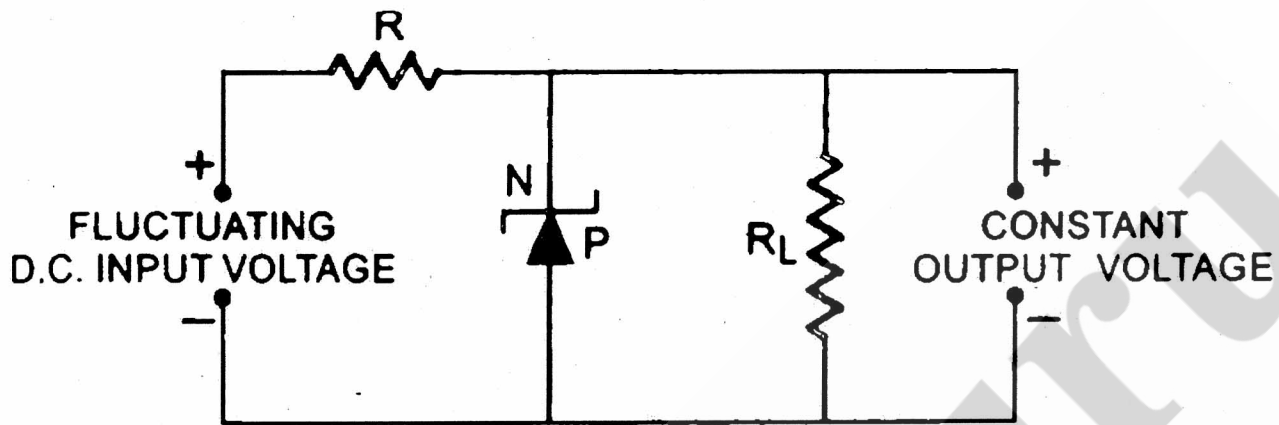
A zener diode has a unique feature that the voltage drop across it is independent of current flowing through it.

Zener diode as a Voltage Regulator -

Zener diode is used in making the constant voltage power supply. Its working is based on the fact that in reverse breakdown region, a very small change in voltage across the zener diode produces a very large change in current through the circuit but the voltage across the zener diode remains constant.

The zener diode is joined in reverse bias to the fluctuating d.c. input voltage through a resistance R of suitable value.

The constant output voltage is taken across a load resistance R_L connected in parallel with zener diode.



WORKING -

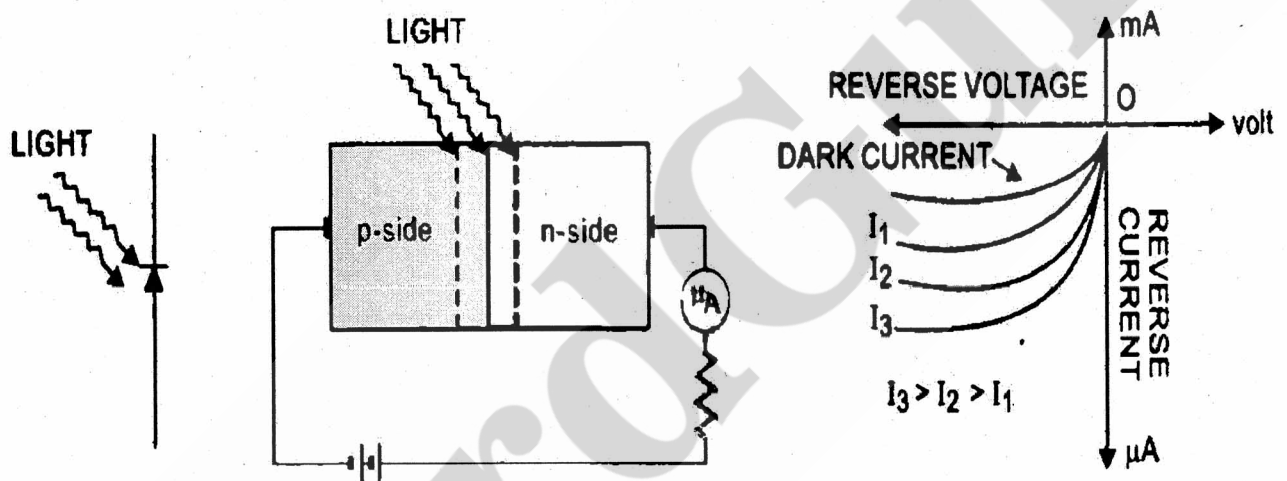
When the input d.c. voltage across zener diode increases beyond a certain limit (i.e. zener breakdown voltage) the current through the circuit rises sharply, causing a sufficient increase in the voltage drop across the dropping resistor R . As a result of it, the voltage across the zener diode remain constant and hence the output voltage lowers back to normal value.

When the input d.c. voltage across the zener diode decreases, the current through the circuit goes down sharply causing sufficient decrease in the voltage across the dropping resistor R . As a result of it, the voltage across the zener diode remains constant and hence the output voltage is raised to normal. Hence the output voltage remains constant.

(ii) PHOTODIODE -

Photodiode is an optoelectronic device in which current carriers (electrons and holes) are generated by photons through photo excitation.

In photodiode a transparent window is made to allow the light of suitable frequency to fall on it. It is operated under reverse bias. The conductivity of p-n junction photodiode increases with the increase in intensity of light falling on it.



Working :-

When a light of energy greater than forbidden energy gap ($h\nu > E_g$) is incident on a reverse biased p-n junction photodiode, an additional electron hole pairs are created in the depletion layer. These charge carriers flow across the junction and generate a reverse current across the junction.

It is found that the reverse saturation current through the photodiode varies almost linearly with the light intensity.

When the photodiode is reverse biased, then a certain current exists in the circuit even when no light is incident on the p-n junction of photodiode. This current is called dark current.

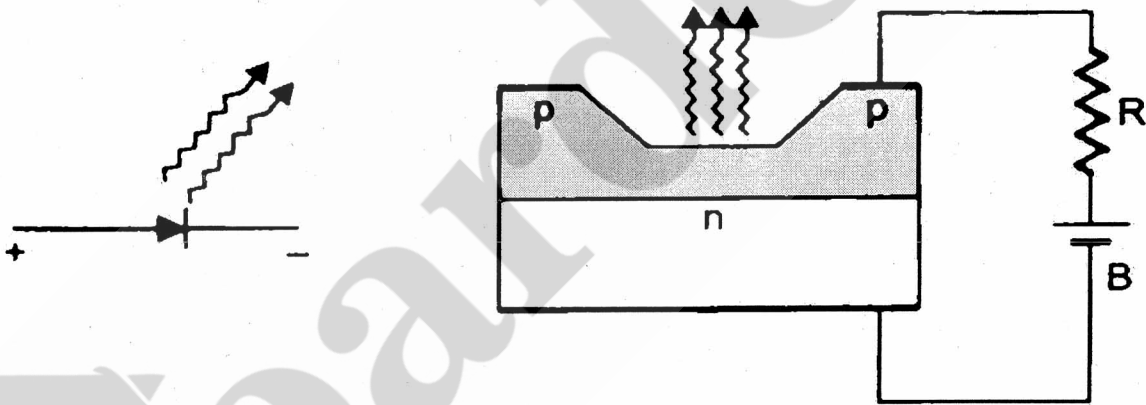
Uses of Photodiodes -

- (i) In photodetection for optical signals.
- (ii) In demodulation for optical signals.
- (iii) In switching the light on and off.
- (iv) In reading of computers, punched cards and tapes.

(iii) LIGHT EMITTING DIODE (LED) -

LED is a photoelectronic device which converts electrical energy into light energy.

It is a heavily doped p-n junction diode which emits spontaneous radiation (light) under forward bias.



In an LED the upper layer of p-type semiconductor is deposited by diffusion on n-type layer of semiconductor. The metallised contacts are provided for applying the forward bias voltage to the p-n junction diode from battery through a resistance (R) which controls the brightness of light emitted.

WORKING -

When p-n junction is forward biased, the movement of majority charge carriers take place across

the junction. The electrons move from n-side to p-side through the junction and holes move from p-side to n-side through the junction. As a result of it, the concentration of minority carriers increases rapidly at the junction boundary.

These minority carriers recombine with majority carriers near the junction. On recombination of electron and hole the energy is given out in the form of heat and light. In p-n junction diodes made of materials like gallium arsenide (GaAs), gallium phosphide (GaP) and gallium-arsenide-phosphide (GaAsP) a greater percentage of energy released during the recombination is in the form of visible light.

Advantages of LED over Bulb :-

- (1) LED has Less power and low operational voltage.
- (2) LED has fast action and requires no warm up time.
- (3) LED is cheap and easy to handle.
- (4) LED can be used for variety of use e.g. in burglar alarm system, in optical communication, in digital watches etc.

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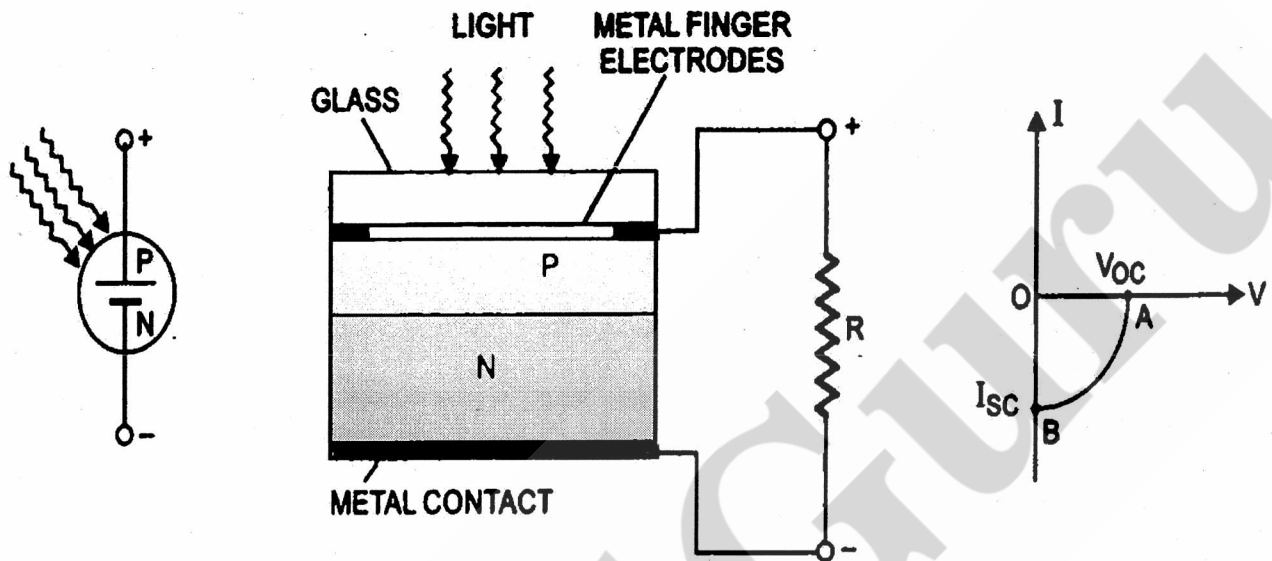
Explain giving reason, why the semiconductor used for fabrication of visible light LEDs must have a band gap of at least (nearly) 1.8 eV.

Sol. semiconductors with band gap (E_g) close to 1.8 eV are preferred to make LEDs because the emitted light falls in the visible region of EM wave spectrum.

The other reason to select these materials are high optical absorption and low cost.

(iv) SOLAR CELL -

A solar cell converts solar energy into electrical energy. It is also a p-n-junction diode,



A solar cell consists of a silicon or gallium-arsenide p-n junction diode packed in a can with glass window on top. The upper layer is of p-type semiconductor. It is very thin so that the incident light photons may easily reach the p-n junction.

On the top face of p-layer, the metal finger electrodes are prepared in order to have enough spacing between the fingers for the light to reach the p-n junction through P-layer.

Working :-

When photons of light (of energy $h\nu > E_g$) fall at the junction, electron-hole pairs are generated in the depletion layer. The electrons and holes produced move in opposite direction due to junction field.

The photo generated electrons move towards n-side and holes move towards p-side of p-n junction. They will be collected at the two sides of the junction, giving rise to a photo voltage between the top and bottom

metal electrodes. When an external load is connected across metal electrodes a photo current flows.

The V-I characteristics of a solar cell lying in fourth quadrant of the coordinate axes. It is so because solar cell does not draw current but supplies to the load. In graph point A represents open circuit voltage and point B represents short circuit.

Uses :-

- (i) Solar cells are used for charging storage batteries in day time, which can supply the power during night.
- (ii) Solar cells are used in satellites to operate the various electrical instrument kept inside the satellite.
- (iii) Solar cells are used in calculators, watches etc.
- (iv) Solar cells are used to power traffic signals.

Q. State the reason, why GaAs is most commonly used in making solar cells.

Sol. The energy of the maximum intensity of the solar radiation is nearly 1.5 eV. In order to have photo excitation the energy of radiation ($h\nu$) must be greater than energy band gap (E_g).

Therefore the semiconductor with energy band gap about 1.5 eV or lower and with higher absorption coefficient is likely to give better solar conversion efficiency. The energy band gap for Si is 1.1 eV and for GaAs it is about 1.53 eV.

JUNCTION TRANSISTOR -

A junction transistor is obtained by growing a thin layer of one type semiconductor (P or n) in between two thick layers of other type semiconductor (n or P). Thus a junction transistor is a semiconductor device having two junctions and three terminals.

The three layers of junction transistors are-

- (i) Emitter (E) - It is a heavily doped thick layer of transistor.
- (ii) Base (B) - It is lightly doped thin layer of transistor, which is in middle.
- (iii) Collector (C) - It is a moderately doped thick layer of transistor.

Size	→	$C > E > B$
Doping	→	$E > C > B$

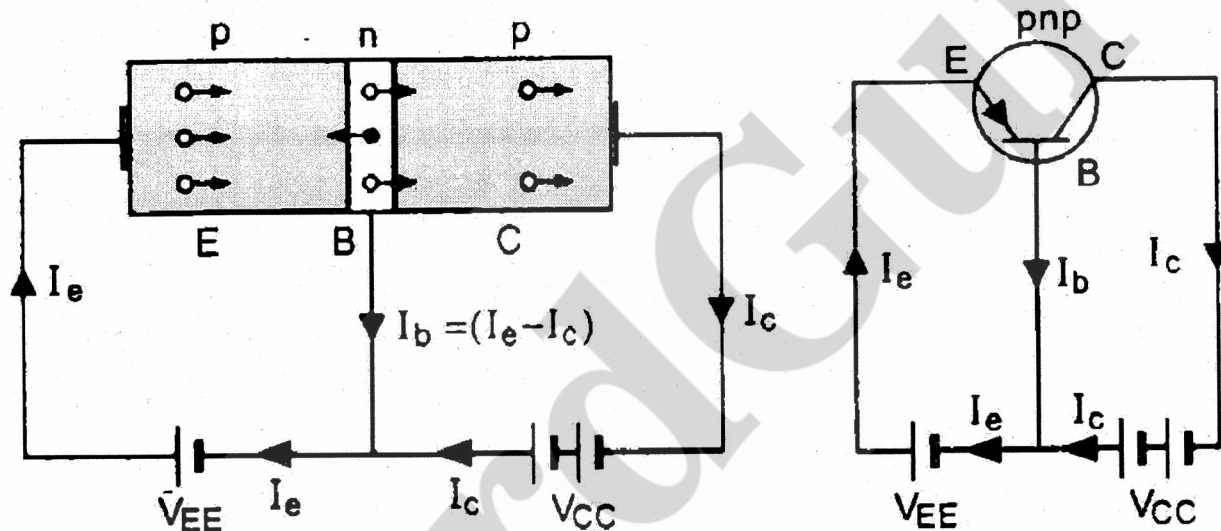
The function of emitter is to emit the majority charge carriers and the function of collector is to collect the majority carriers. Base provides the proper interaction between the emitter and collector.

WORKING OF JUNCTION TRANSISTOR -

(1) PNP Transistor -

A PNP transistor is obtained by growing a thin layer of n-type semiconductor in between two relatively thick layers of p-type semiconductor.

To study the working of p-n-p transistor the experimental arrangement is shown in fig.



The emitter-base junction is forward biased and collector-base junction is reverse biased and this arrangement is known as Common base Configuration.

Holes which are majority carriers in emitter are repelled towards base by positive potential on emitter due to battery V_{EE} , resulting in emitter current I_e .

The base being thin and lightly doped has low number density of electrons. When holes enter the base region, then only a few holes (say 5%) get recombine with electrons, resulting in base current I_b . Here $I_b = 5\% I_e = 0.05 I_e$. The remaining 95% holes pass over to the collector on account of high negative potential due to battery V_{CC} , resulting in collector current I_c . Where $I_c = 95\% I_e = 0.95 I_e$

As one hole reaches the collector, it is neutralised by the flow of one electron from the negative terminal of the battery V_{CC} to collector through connecting wire. At the same time a covalent bond is broken in the emitter, the electron goes to the positive terminal of the battery V_{EE} through connecting wire and hole produced begins to move towards base.

Then one electron flows from negative terminal of battery V_{EE} to positive terminal of battery V_{CC} . When the hole coming from emitter combines with the electron in base, the deficiency of electron in base is compensated by the flow of electron from negative terminal of battery V_{EE} to the base through connecting wire.

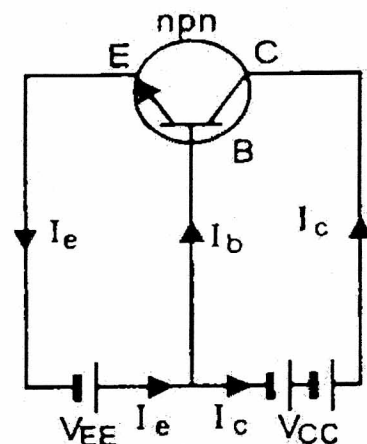
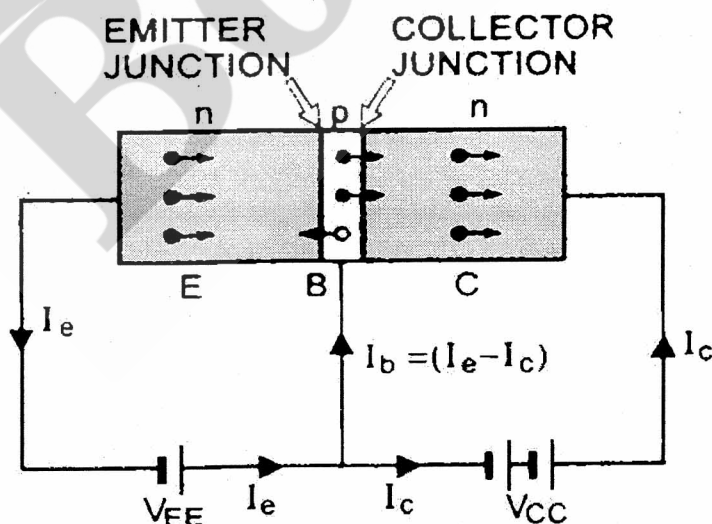
Thus the current in PNP transistor is carried by holes and at the same time their concentration is maintained. But in the external circuit the current is due to flow of electrons.

Here we can see that

$$I_e = I_b + I_c$$

(2) NPN-TRANSISTOR -

A NPN junction transistor is obtained by growing a thin layer of p-type semiconductor in between two relatively thick layers of n-type semiconductor.



In this case also the emitter base junction is forward biased and the collector base junction is reverse biased.

Electrons which are majority charge carriers in emitter are repelled towards base by negative potential of V_{EE} on emitter, resulting emitter current I_e . The base being thin and lightly doped has low number density of holes. When electrons enter the base region then only a few holes (say 5%) get neutralised by the holes, resulting in base current $I_b (= 5\% I_e = 0.05 I_e)$. The remaining 95% electrons pass over to the collector, on account of high positive potential of collector due to battery V_{CC} , resulting in collector current $I_c (= 95\% I_e = 0.95 I_e)$.

As one electron reaches the collector, it flows to the positive terminal of battery V_{CC} through connecting wire. At the same time, one electron flows from negative terminal of V_{CC} to positive terminal of V_{EE} and one electron flows from negative terminal of V_{EE} to emitter.

When the electron coming from emitter combines with the hole in base, the deficiency of hole in base is compensated by the breakage of covalent bond there. The electron so reached flow to the positive terminal of battery V_{EE} , through connecting wire.

Thus in n-p-n transistor the current is carried inside the transistor as well as in external circuit by the electrons.

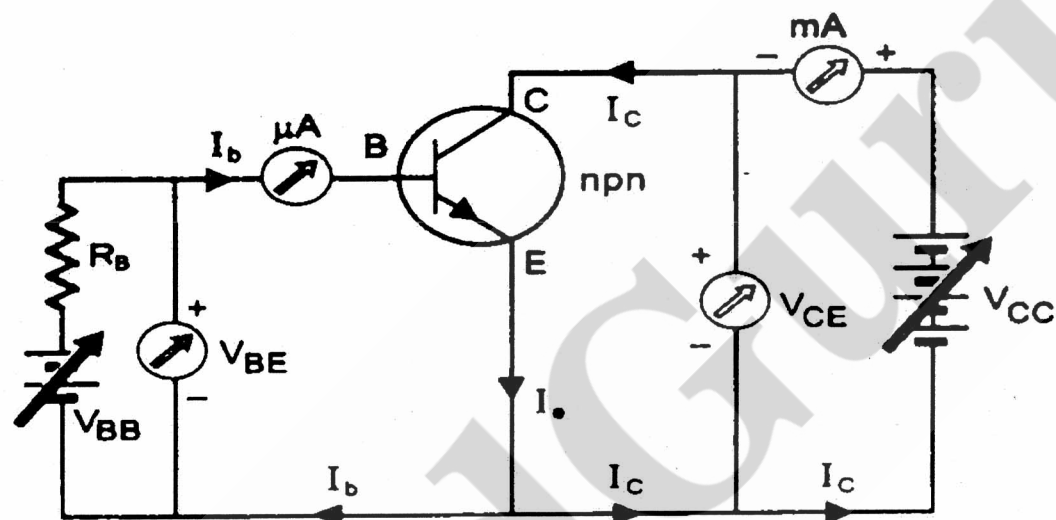
Thus here we can see that

$$I_e = I_b + I_c$$

COMMON EMITTER TRANSISTOR CHARACTERISTICS

The experimental set up to study the characteristics of a transistor, when emitter is kept as a common terminal is shown in fig.

Base is the input terminal and collector is the output terminal. Here also $I_e = I_b + I_c$.



Input characteristics -

It represents the variation of the base current I_b with base emitter voltage V_{BE} , keeping V_{CE} fixed. As shown in fig. the current is small as long as V_{BE} is less than the barrier voltage. When V_{BE} is greater than the barrier voltage the curve look similar to that of a forward biased diode.

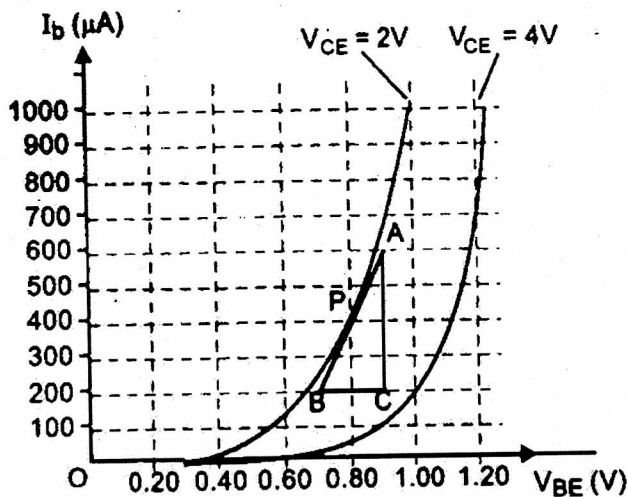
At a fixed point on the characteristics, we can find input dynamic resistance of transistor.

Input dynamic resistance -

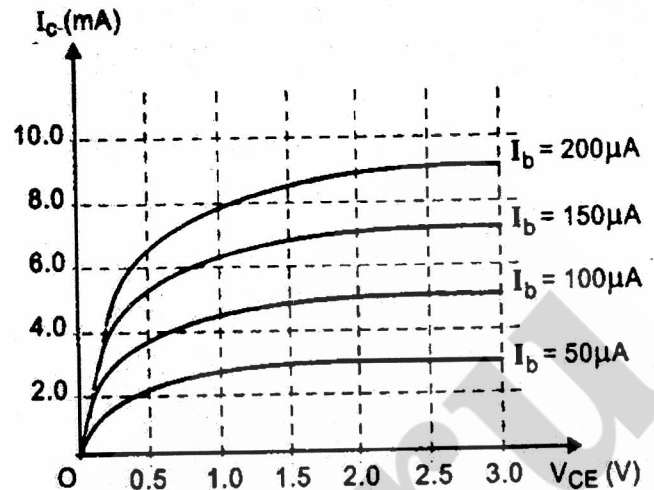
It is defined as the ratio of change in base-emitter voltage (ΔV_{BE}) to the resulting change in the base current (ΔI_b) at fixed collector emitter voltage (V_{CE}).

$$R_i = \left(\frac{\Delta V_{BE}}{\Delta I_b} \right)_{V_{CE} = \text{Constant}}$$

R_i ranges from few hundreds to a few thousand ohm.



Input Characteristics



Output Characteristics

Output Characteristics -

It represents the variation of collector current (I_c) with collector emitter voltage V_{CE} , keeping base current (I_b) fixed.

From the curve, we find that for a given value of V_{CE} , I_c is large for larger value of I_b . This happens because the transistor is in saturation state and not in active state. Further as long as the collector emitter junction is reverse biased ($V_{CE} > V_{BE}$) we get I_c which is almost independent of V_{CE} . The value of I_c however controlled by I_b .

Output Dynamic resistance (R_o) -

It is defined as the ratio of change in collector-emitter voltage (ΔV_{CE}) to the change in collector current (ΔI_c), when base current I_b is kept constant.

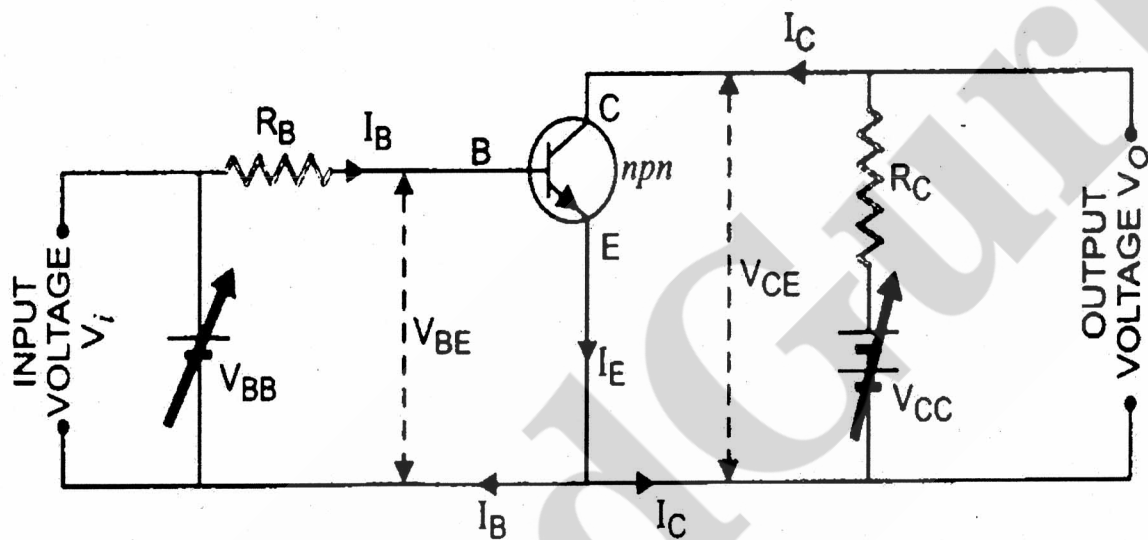
$$R_o = \left(\frac{\Delta V_{CE}}{\Delta I_c} \right)_{I_b = \text{Constant}}$$

R_o ranges from 50 to 100 $k\Omega$

TRANSISTOR AS A SWITCH -

Switch is a device, used to 'on' or 'off' the current in the circuit.

To understand the operation of the transistor as a switch, we use npn transistor with common emitter configuration, as shown in fig.



Using Kirchhoff's voltage Law - for the input circuit we have -

$$-V_{BB} + I_B R_B + V_{BE} = 0$$

$$\text{or } V_{BB} = I_B R_B + V_{BE} \quad \text{--- (1)}$$

For Si transistor V_{BE} is 0.6V to 0.7V.

For the output characteristics, we have -

$$-V_{CC} + I_C R_C + V_{CE} = 0$$

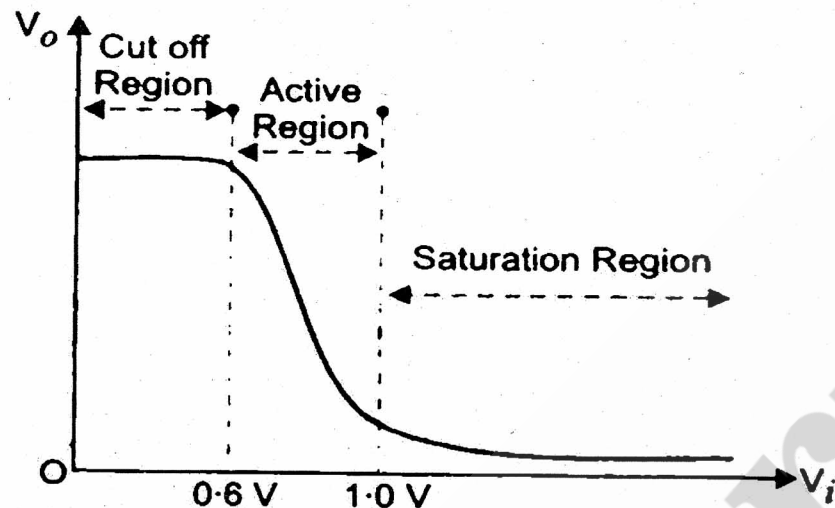
$$\text{or } V_{CE} = V_{CC} - I_C R_C \quad \text{--- (2)}$$

We can assume V_{BB} as the d.c. input voltage V_i and V_{CE} as the output voltage V_o . Then

$$\text{from eq. (1)} \quad V_i = I_B R_B + V_{BE} \quad \text{--- (3)}$$

$$\text{from eq. (2)} \quad V_o = V_{CC} - I_C R_C \quad \text{--- (4)}$$

(i) In case of si transistor as $V_i < 0.6V$, there is no collector current ($I_C = 0$). Now the transistor will be in cut off state. Also $V_o = V_{CC}$.



(ii) When $V_i > 0.6\text{ V}$ but nearly less than 1 V , there will be some collector current I_c . From eq. (4) it is clear that as I_c increases, the value of output voltage V_o decreases. With the increase of V_i beyond 0.6 V , I_c increases almost linearly and so V_o decreases linearly till V_i becomes nearly 1 V . In this situation the transistor is in active state.

(iii) When $V_i \geq 1\text{ V}$, the variation of V_i and V_o is non linear, because with the increase in V_i , V_o is found to decrease towards zero but never become zero. In this case the collector current I_c becomes maximum and the transistor is in saturation state.

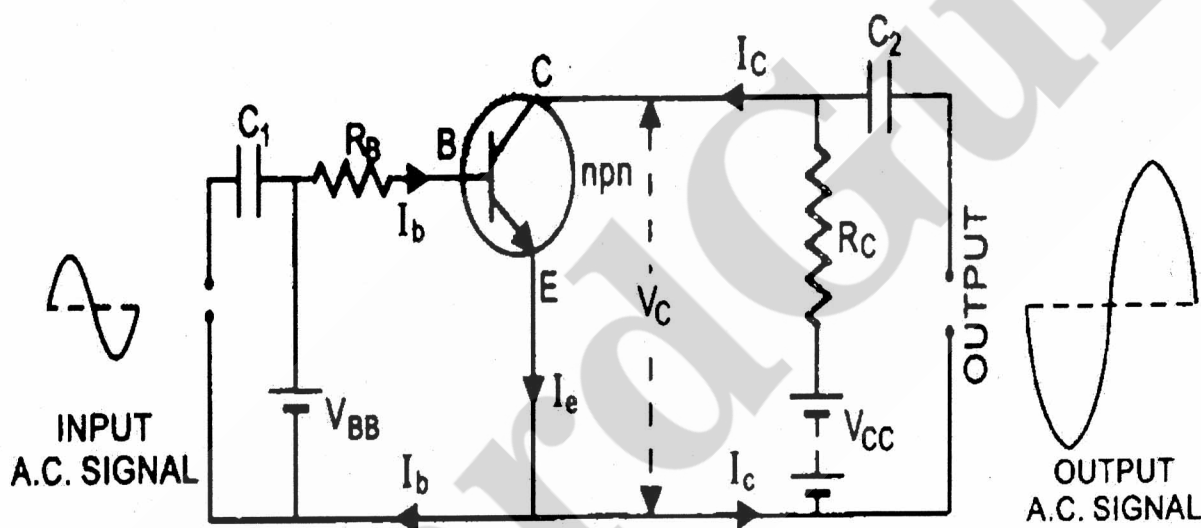
From the above we note that as long as V_i is low ($V_i < 0.6\text{ V}$), V_o is high ($= V_{cc}$) and I_c is nearly equal to zero. The transistor is not working and is said to be in switched off state.

If V_i is high ($V_i > 1\text{ V}$), V_o is low and is nearly equal to zero. In this case I_c is maximum. The transistor is now fully conducting and is said to be in switched on state.

TRANSISTOR AS COMMON EMITTER AMPLIFIER-

An amplifier is a device which is used for increasing the amplitude of variation of alternating voltage or current or power.

The circuit of a common emitter amplifier is shown in fig. The input circuit is forward biased and the output circuit is reverse biased. Here the resistance of input circuit is low and that of output circuit is high.



When no a.c. signal voltage is applied to the input circuit but emitter-base circuit is closed, then

$$I_e = I_b + I_c$$

Due to collector current I_c , voltage drop across R_c is $I_c R_c$. If V_c is collector voltage (potential diff. between collector and emitter) then-

$$V_{CE} = V_c + I_c R_c \quad \text{or} \quad V_c = V_{CE} - I_c R_c$$

When the input signal voltage is fed to the emitter base circuit, it will change the emitter voltage and hence to the emitter current, which in turn will change the collector current.

Due to it the collector voltage V_c will vary in accordance with relation $V_c = V_{CE} - I_c R_c$. This variation in collector voltage appear as amplified output.

Phase relationship between input and output voltages-

- (i) During Positive half cycle of input a.c. voltage, it supports the forward biasing of the emitter-base circuit. Due to it I_E increases and consequently I_c increases. As a result of it V_c decreases.

Thus during positive half cycle of input A.c. voltage, the output signal voltage varies through a negative half cycle.

- (ii) During negative half cycle of input a.c., it opposes the forward biasing of emitter base circuit, due to it I_E decreases and hence I_c decreases, As a result of it V_c increases, i.e. the collector becomes more positive.

Thus during negative half cycle of input A.c. signal voltage, the output signal voltage varies through positive half cycle.

Hence the input signal voltage and output signal voltage are in opposite phase (i.e. 180° out of phase) in common emitter amplifier circuit.

VARIOUS GAINS OF COMMON EMITTER AMPLIFIER -

(1) Current Gain / Current Amplification Factor -

D.C. Current Gain - (β)

It is defined as the ratio of the collector current (I_c) to the base current (I_b).

$$\beta = \frac{I_c}{I_b} = \frac{I_c}{I_e - I_c}$$

$$I_e = I_b + I_c$$

$$\beta > 1$$

A.C. Current Gain (β_{ac}) -

It is defined as the ratio of change in collector current (ΔI_c) to the change in base current (ΔI_b) at constant collector voltage.

$$\beta_{ac} = \frac{\Delta I_c}{\Delta I_b}$$

For a transistor β lies between 15 to 50.

(2) Trans conductance (g_m) -

It is defined as the ratio of change in collector current (ΔI_c) to the change in base emitter voltage (ΔV_i). Its unit is Ω^{-1} .

$$g_m = \frac{\Delta I_c}{\Delta V_i} = \frac{\Delta I_c}{\Delta I_b} \times \frac{\Delta I_b}{\Delta V_i}$$

or

$$g_m = \frac{\beta_{ac}}{R_i}$$

$$R_i = \frac{\Delta V_i}{\Delta I_b}$$

(3) A.C. Voltage Gain (A_v) -

It is defined as the ratio of the change in output voltage (ΔV_c) to the change in input voltage (ΔV_i).

$$A_v = \frac{\Delta V_c}{\Delta V_i} = \frac{\Delta I_c \times R_o}{\Delta I_b \times R_i} = \beta_{ac} \times \frac{R_o}{R_i}$$

(4) A.C. Power Gain -

It is defined as the ratio of the change in output power to the change in input power.

$$\text{A.C. Power Gain} = \frac{\Delta P_o}{\Delta P_i} = \frac{\Delta I_c^2 \times R_o}{\Delta I_b^2 \times R_i}$$

$$= \beta_{a.c.}^2 \times \text{resistance gain}$$

$$= \beta_{a.c.} \times (\beta_{a.c.} \times \text{resistance gain})$$

$$\text{A.C. Power Gain} = \beta_{a.c.} \times A_v$$

(5) Relationship between α and β -

$$\beta = \frac{\alpha}{1 + \alpha}$$

$$\text{and } \alpha = \frac{\beta}{1 + \beta}$$

Where β is current gain in CE Configuration. $\beta = \frac{I_c}{I_b}$

and α is current gain in CB Configuration. $\alpha = \frac{I_c}{I_e}$

where $0 < \alpha < 1$

Q. The AC current gain of a transistor is 120. What is the change in the Collector current in the transistor whose base current changes by $100 \mu\text{A}$?

Sol. Given that - $\beta = 120$, $\Delta I_b = 100 \mu\text{A}$

We know that - $\beta = \frac{\Delta I_c}{\Delta I_b}$

$$120 = \frac{\Delta I_c}{100 \times 10^{-6}} \Rightarrow \Delta I_c = 120 \times 100 \times 10^{-6}$$

$$\Delta I_c = 12 \text{ mA}$$

Q. The input resistance of a CE-transistor is 1000Ω . On changing its base current by $10 \mu\text{A}$, the collector current increases by 2 mA . If a load resistance of $5 \text{ k}\Omega$ is used in the circuit, calculate

(a) The current gain (b) Voltage gain of amplifier

Sol. Given that - $R_i = 1000 \Omega$, $\Delta I_b = 10 \mu\text{A}$
 $\Delta I_c = 2 \text{ mA}$, $R_L = R_o = 5 \text{ k}\Omega = 5000 \Omega$

(a) Current gain $\beta = \frac{\Delta I_c}{\Delta I_b}$

$$\beta = \frac{2 \times 10^{-3}}{10 \times 10^{-6}} = 200$$

(b) Voltage gain $A_v = \beta \times \frac{R_o}{R_i}$

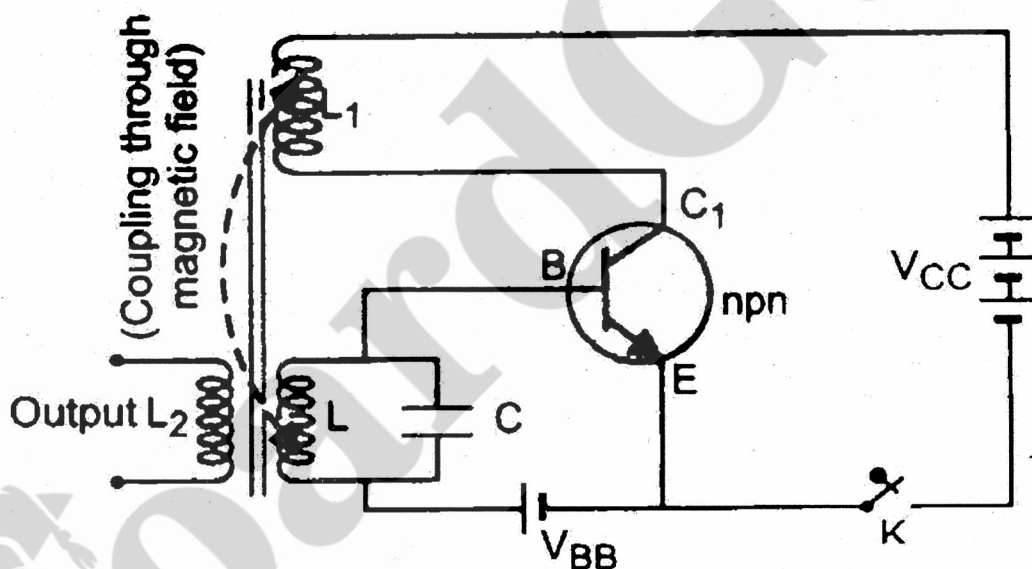
$$A_v = 200 \times \frac{5000}{1000} = 1000$$

TRANSISTOR AS AN OSCILLATOR -

A simple L-C circuit can be used to produce electrical oscillations of desired frequency. The radio wave which are used as carrier waves in radio communication are produced by these circuits.

Circuit diagram of a transistor as an oscillator is shown in fig. Here L-C circuit is connected with emitter base circuit of transistor which is forward biased with battery V_{BB} .

The collector emitter circuit is reverse biased with battery V_{CC} . A coil L_1 is inserted in collector emitter circuit, which is coupled with L .



Working:-

If we close the key K , the collector attains positive potential due to which there will be a weak collector current which will start rising with time due to inductance L_1 . As a result of it, an increasing magnetic flux is linked with L_1 and hence with L . Due to mutual induction, an e.m.f. is induced in L which will charge the upper plate of capacitor with positive charge, consequently there will be support to the forward biasing of emitter base circuit.

This results in an increase in the emitter

current and hence an increase in the collector current. Due to it, more increasing magnetic flux is linked with L_1 and hence with L . As a result of this more e.m.f. is induced in L , charging the upper plate of capacitor with more positive charge and hence providing more support to the forward biasing of emitter-base circuit, which results further increase in the emitter current and hence in the collector current. The process continues till the collector current becomes maximum or saturated.

Now the mutual induction stops playing its part. The capacitor C gets discharged through inductance L . As a result of it, the support to the forward biasing of emitter-base circuit is withdrawn, thereby the emitter current decreases and hence collector current also decreases. Due to it, a decreasing magnetic flux is linked with L_1 and hence with L .

Due to mutual induction an e.m.f. is induced in L which will charge the lower plate of capacitor C with positive charge and upper plate of capacitor with negative charge; consequently there will be opposition to the forward biasing of emitter base circuit. This results in further decrease in emitter current and hence in collector current. This process continues till the collector current becomes zero.

Now again mutual induction stops playing its part. The capacitor gets discharged through inductance L . As a result of it, the opposition to the forward biasing of emitter base circuit is withdrawn, thereby the emitter current increases and hence collector current also increase and the process repeats. Thus the collector current oscillates between a maximum and zero value.

We know that in common emitter circuit, a signal voltage applied to the emitter base circuit suffers a phase change of 180° in the collector emitter circuit. By coupling L_1 with L , we bring about a further phase change of 180° . Thus the energy which is fed to the LC circuit is in proper phase at proper timings and hence we get undamped oscillations in LC circuit of frequency

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

Therefore we get the carrier waves from the oscillator.

Note- In this oscillator, the energy is being supplied by the battery V_{cc} to the LC circuit at proper time and in proper phase, therefore the battery gets consumed in oscillator. It means in oscillator D.C. is converted into A.C.

Advantages of Semiconductor devices-

- (1) Semiconductor devices are much smaller in size and weight as compared to vacuum tubes.
- (2) Semiconductor devices start operating instantly. This save a lot of electrical power.
- (3) Semiconductor devices does not get heated, while operating so that no cooling system is required.
- (4) The semiconductor devices can withstand rough handling as compared to the vacuum tubes.
- (5) Semiconductor devices have much longer life as compared to the life of vacuum tubes.
- (6) Semiconductor devices are cheaper than vacuum tube devices.
- (7) Semiconductor devices are low power devices.

Disadvantages of semiconductor devices -

- (1) Semiconductor devices are more sensitive to changes of temperature whereas the vacuum tubes are less sensitive.
- (2) It is difficult to produce semiconductor devices with exactly identical characteristics.
- (3) The noise level is higher in semiconductor devices as compare to the vacuum tubes.
- (4) Semiconductor devices cannot handle as much power as vacuum tubes.

LOGIC GATE -

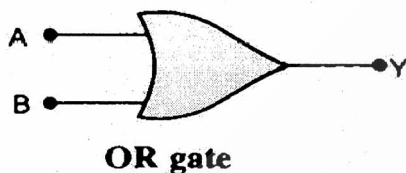
A digital circuit which either allows a signal to pass through or stops it, is called a gate. Such gate allows the signal to pass through only when some logical conditions are satisfied. Hence they are called logic gates.

Basic Logic gates are given below :-

(1) OR Gate -

The OR gate is a device which combines A with B to give Y as the result.

The OR gate has two or more inputs and one output.



OR gate

$$Y = A + B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

In Boolean algebra, addition symbol (+) is referred to as OR. The Boolean expression $A + B = Y$ indicates that Y equals A OR B.

(2) AND Gate -

The AND gate is a device that combines A with B to give result Y. The AND gate has two or more inputs and one output.

Multiplication sign (X or $\cdot$) is referred to as AND in Boolean algebra. The Boolean expression $A \cdot B = Y$, indicates Y equals A AND B.



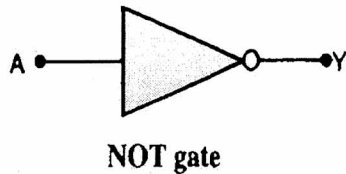
AND gate

$$Y = A \cdot B$$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(3) NOT Gate-

The NOT gate is a device which inverts the input. If input is 1 then the output is not 1 but is zero and when input is 0 then output is not zero but is 1. The NOT gate is one input and one output device.



$$\bar{A} = Y$$

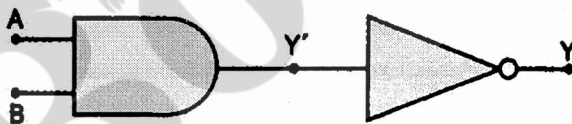
A	Y
0	1
1	0

The bar symbol (—) is referred to as NOT in Boolean algebra. The Boolean expression $\bar{A} = Y$ indicates Y equals NOT A

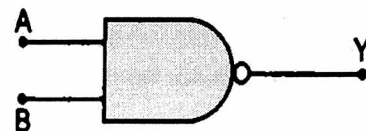
Combination of Gates -

(1) NAND Gate-

If we connect that output Y' of AND gate to the input of a NOT gate, then the gate so obtained is called NAND gate.



A	B	Y'	Y
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

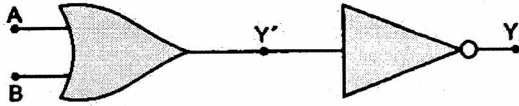


A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

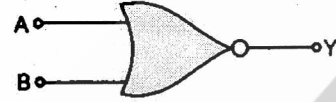
The Boolean expression, the NAND gate is given as $Y = \overline{A \cdot B}$ and is being read as A AND B negated.

(2) NOR Gate-

If we connect the output Y' of an OR gate to the input of the NOT gate then the gate so obtained is called the NOR gate.



A	B	Y'	Y
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0



A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

In Boolean expression the NOR gate is expressed as $Y = \overline{A+B}$ and is being read as A OR B negated.

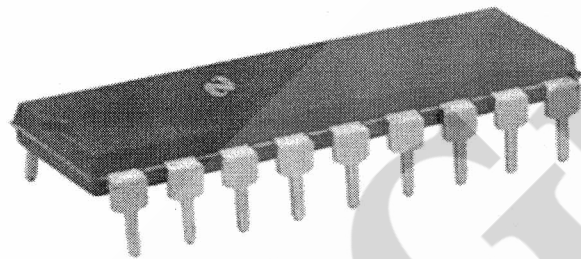
BOOLEAN ALGEBRA -

- | | | |
|--|-----|--|
| (1) $A + 1 = 1$ | and | $A + 0 = A$ |
| (2) $A \cdot 1 = A$ | and | $A \cdot 0 = 0$ |
| (3) $A + \overline{A} = 1$ | and | $A \cdot \overline{A} = 0$ |
| (4) $\overline{A+B} = \overline{A} \cdot \overline{B}$ | and | $\overline{A \cdot B} = \overline{A} + \overline{B}$ |
| (5) $\overline{\overline{A}} = A$ | | |
| (6) $A + A = A$ | and | $A \cdot A = A$ |

INTEGRATED CIRCUIT -

Integrated circuit (IC) is that circuit in which the circuit components such as resistors, diodes, capacitors and transistors etc. are part of a small semiconductor chip.

IC consists of number of circuit components and their inter connections in single small package to perform a complete electronic function.



The following processes are involved in the fabrication of integrated circuits -

(1) Epitaxial Growth -

This process is used to have a layer of n-type or p-type on silicon chip as and when required.

(2) Oxidation -

This process is used to have a layer of insulating SiO_2 on silicon chip which is able to separate one region of silicon chip from the other.

(3) Photolithography -

By this process the different regions of silicon chip are selected photographically where the different components to be used in integrated circuits can be installed.

(4) Diffusion of different impurities -

This process is used to obtain different device structure on silicon chip.

(5) Metalisation -

This process involves the deposition of metal films which may interconnect different components installed on chip to obtain the required circuit.

Advantages of ICs -

- (1) They are highly reliable due to lesser number of connections.
- (2) They require very small place.
- (3) They have lesser weight due to very small circuits.
- (4) Their total cost is low.
- (5) They require low power to operate
- (6) They have greater ability to operate at extreme values of temperatures.

Q. What will be the values of inputs A and B for the Boolean expression $\overline{(A+B)} \cdot \overline{(A \cdot B)} = 1$?
CBSE 2007

Sol. Given that $\overline{(A+B)} \cdot \overline{(A \cdot B)} = 1$

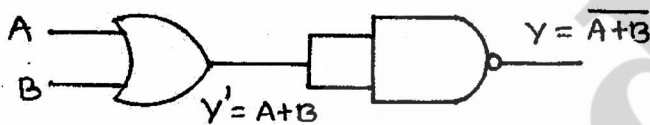
LHS, $\overline{(A+B)} \cdot \overline{(A \cdot B)}$

If $A=0$, $B=0$ then $\overline{(0+0)} \cdot \overline{(0 \cdot 0)} = \overline{(0)} \cdot \overline{(0)} = 1$

Hence the value of $A=B=0$.

Q. The output of an OR gate is connected to both the inputs of a NAND gate. Draw the logic circuit of this combination of gates and write truth table.
CBSE 2007

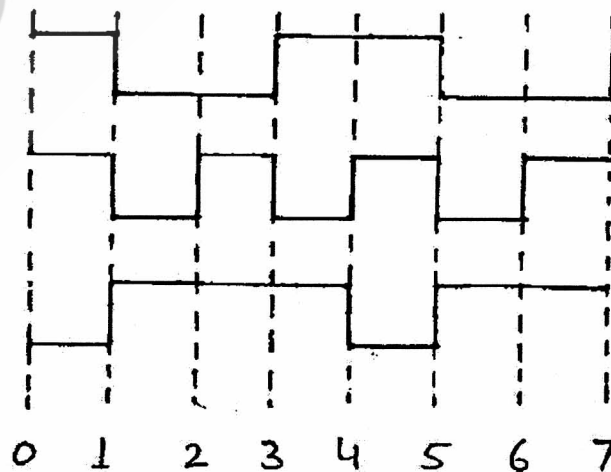
Sol. Circuit



Truth Table

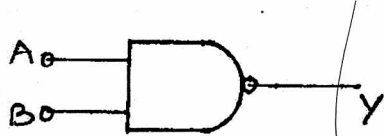
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Q. The following figure shows the input waveforms A, B and the output waveform Y of a gate. Identify the gate, write its truth table and draw its logic symbol.
CBSE 2006 2009



Sol. From the given output (Y) waveform it is clear that output is new only when both inputs are 1, so the gate is NAND gate.

Logic Symbol



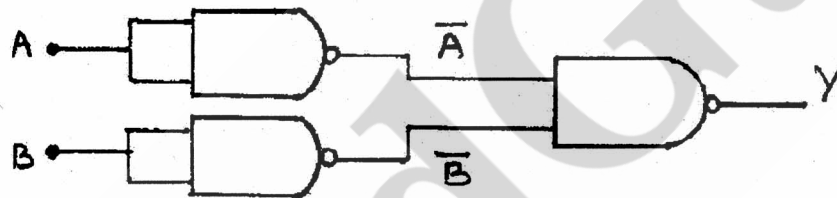
Truth Table

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

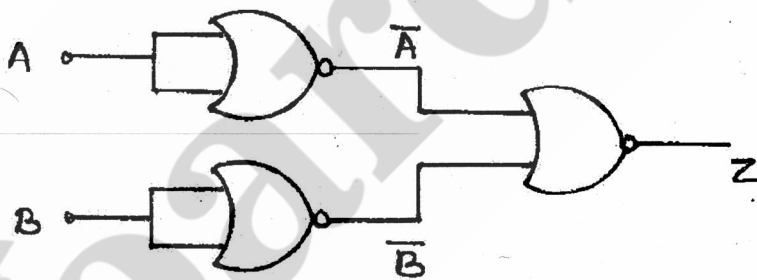
Q.
CBSE
2009

The two given circuits are a combination of NAND gates and NOR gates. Write truth table for each of these combinations.

(a)



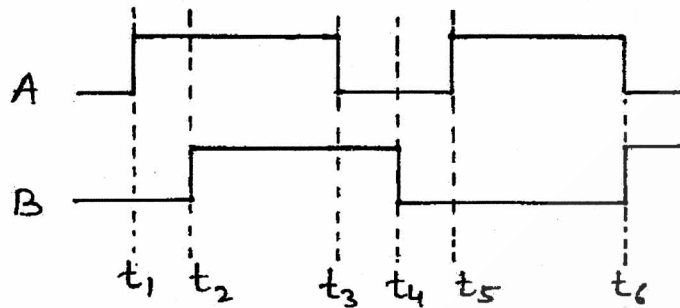
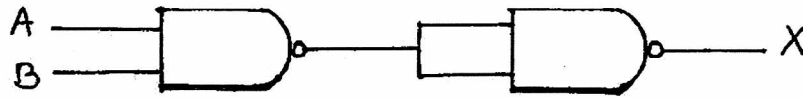
(b)



Sol.

Q.
CBSE
2008

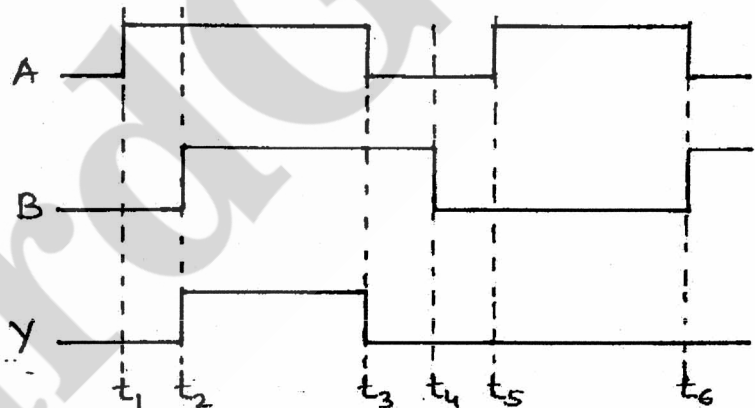
Draw the output waveform at X, using the given inputs A, B for the logic circuit shown below. Also identify the gate.



Sol. Given logic circuit behaves like an AND gate.

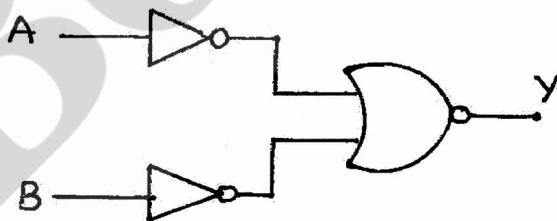
Truth Table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1



Q.
CBSE
2002
2011

Why are NAND gate called universal gate? Identify the equivalent gate for the given circuit.



Sol. NAND gate is called universal gate because all three basic gates (OR, AND and NOT) can be made using NAND gates.

The equivalent gate is AND gate.

Very Important Questions

1 Mark Questions-

Q.1. What happens to the width of depletion layer of P-n junction when it is -
(a) Forward biased ? (b) Reverse biased

(CBSE 2011, 06)

Q.2. Why cannot we take one slab of p-type semiconductor and physically join it to another slab of n-type semiconductor to get P-n junction?

(CBSE 2010)

Q.3. State the relation between the frequency ν of radiation emitted by LED and the band gap energy E of the semiconductor used to fabricate it.

(CBSE 2009)

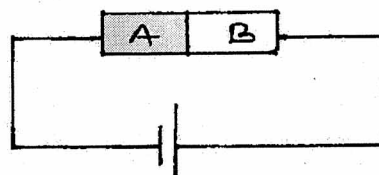
Q.4. At what temperature would an intrinsic semiconductor behave like a perfect insulator?

(Delhi 2008)

Q.5. Two semiconductor materials A and B shown in figure are made by doping germanium crystal with arsenic and indium respectively. The two are joined end connected to a battery as shown-

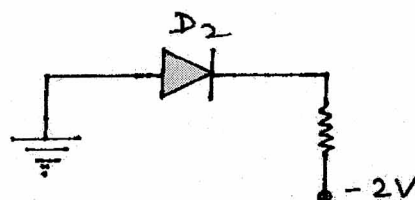
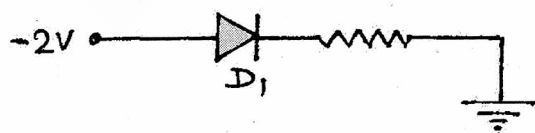
(a) Will the junction be forward or reverse biased?

(b) Sketch a V-I graph for this arrangement.



(CBSE 2007)

Q.6. Which one of the two diodes D_1 and D_2 in the given figure is -



(a) Forward Biased ?
(b) Reverse Biased.

(CBSE 2007)

Q.7. In a transistor, doping level in base is increased slightly. How will it affect
 (a) Collector Current
 (b) Base Current

(Delhi 2011)

Q.8. Draw the logic circuit of a NAND gate and write its truth table.

(CBSE 2011)

Q.9. Draw the logic circuit of AND gate and NOT gate and write its truth table.

(Delhi 2008, 11)

Q.10. Write the truth table for the given circuit. Name the equivalent gate that this circuit represents.

(CBSE 2010)



Q.11. The truth table of a logic gate has the form given here. Name this gate and draw its symbol.

(CBSE 2007, 10)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Q.12. Give the logic symbol of NOR gate, NAND gate and AND gate.

(CBSE 2009)

Q.13. Write the full form of the terms (a) MSI
 (b) LSI used for different types of integrated circuits.

(CBSE 2005, 08)

2 Marks Questions -

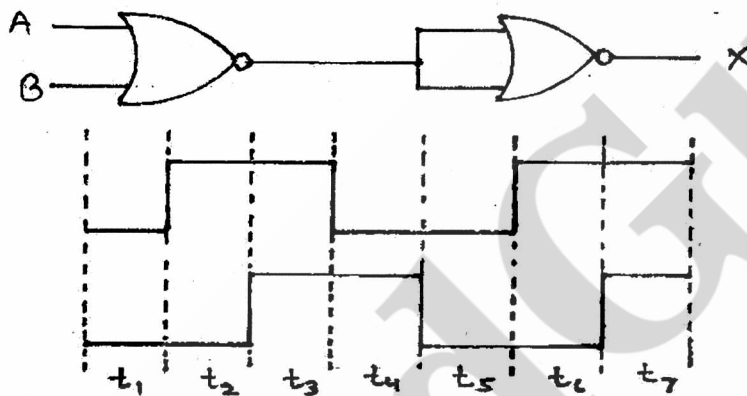
- Q.1. Write two characteristic feature to distinguish between n-type and p-type semiconductors.
(CBSE 2007)
- Q.2. Name the semiconductor device that can be used to regulate an unregulated DC power supply. With the help of V-I characteristics of this device, explain its working principle.
(Delhi 2008/11)
- Q.3. Explain how a depletion region is formed in a junction diode?
(Delhi 2011)
- Q.4. Draw the circuit diagram of an illuminated photodiode in reverse bias. How is photodiode used to measure light intensity?
(Delhi 2010)
- Q.5. Write the main use of the (a) Photodiode
(b) Zener diode.
(CBSE 2010)
- Q.6. Distinguish between an intrinsic semiconductor and p-type semiconductor. Give reason, why a p-type semiconductor crystal is electrically neutral although $n_h \gg n_e$?
(Delhi 2008)
- Q.7. Draw a circuit diagram showing the biasing of an LED. State the factor which controls -
(a) Wavelength of light
(b) Intensity of light emitted by the LED.
(RBSE 2008)
- Q.8. How is that the reverse current in a zener diode starts increasing suddenly at a relatively low breakdown voltage of 5V or so?
(CBSE 2008)
- Q.9 Distinguish between 'analog and digital signals'.
(CBSE 2005)

Q.10. Describe briefly with the help of a circuit diagram, how the flow of current carriers in a p-n-p transistor is regulated with emitter-base junction forward biased and base-collector junction reverse biased.

(CBSE 2012)

Q.11. Draw the output waveform at X, using the given inputs A and B for the logic circuit shown below. Also identify the logic operation performed by this circuit.

(Delhi 2011)

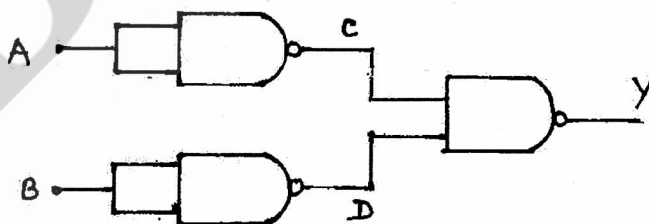


Q.12. Draw the transfer characteristic curve of a base biased transistor in CE configuration. Explain clearly how the active region of the V_o versus V_i curve in a transistor is used as an amplifier.

(Delhi 2011)

Q.13. Write the truth table for the logic circuit shown below and identify the logic operation performed by this circuit.

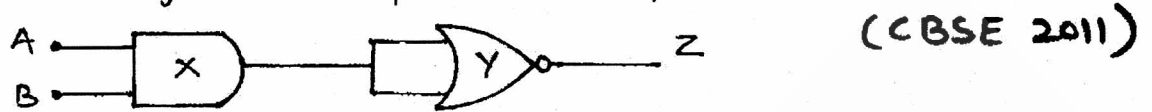
(Delhi 2011)



Q.14. Define the terms (a) Input resistance (R_i) (b) Current amplification factor β of a transistor used in its CE configuration.

(CBSE 2007, 11)

Q.15. Identify the logic gates 'X' and 'Y' in the given figure. Write down the truth table for output Z for all possible inputs 'A' and 'B'.



Q.16. (a) Identify the logic gates marked P and Q in the given circuit.

(b) Write down the output at x for the inputs $A=0$, $B=0$ and $A=1$, $B=1$.



Q.17. The output of a 2-input AND Gate is fed to a NOT gate. Give the name of the combination and its logic symbol. Write down its truth table. (Delhi 2009)

Q.18. Draw the logic symbol of the gate whose truth table is given below.

(RBSE 2009)

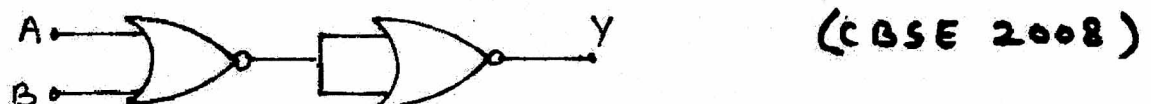
Inputs		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

If this logic gate is connected to a NOT gate, what will be the output when -

(a) $A=0$, $B=0$ (b) $A=1$, $B=1$

Draw the logic symbol of the combination.

Q.19. Write the truth table and draw the logic symbol of the gate for the circuit given below.



3 Marks Questions.

Q.1. Draw a labelled diagram of a full wave rectifier circuit. State its working principle. Show the input output waveforms.

(CBSE 2006, II)

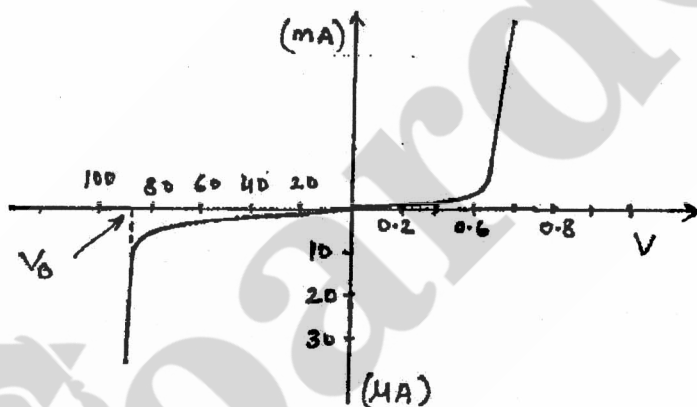
Q.2. With the help of a suitable diagram, explain the formation of depletion region in a P-n junction. How does its width change when the junction is
(a) Forward Bias (b) Reverse bias.

(CBSE 2005, 09)

Q.3. How is a zener diode fabricated so as to make it a special purpose semiconductor diode? Draw the circuit diagram of a zener diode as a voltage regulator and explain its working.

(CBSE 2009)

Q.4. The given figure shows the V-I characteristic of a semiconductor diode.



- Identify the semiconductor diode used.
- Draw the circuit diagram to obtain the given characteristic of this device

(Delhi 2008)

Q.5. Draw transfer characteristics of a CE n-p-n transistor. Point out the region in which the transistor operates as an amplifier. Define the following terms used in transistor amplifiers:

- Input Resistance
- Output resistance
- Current amplification factor

RBSE 2007, II)

Q.6. Explain briefly with the help of a circuit diagram, the working principle of a transistor amplifier as an oscillator. (CBSE 2011, 08)

Q.7. Draw the labelled circuit diagram of a common-emitter transistor amplifier. Explain clearly how the input and output signals are in opposite phase? (CBSE 2008)

5 Marks Questions-

Q.1. Why is a zener diode considered as a special purpose semiconductor diode? Draw the V-I characteristics of zener diode and explain how reverse current suddenly increase at the breakdown voltage?

Describe briefly with the help of a circuit diagram how a zener diode works to obtain a constant DC voltage from the unregulated DC output of a rectifier. (CBSE 2012)

Q.2. Describe briefly, with the help of a diagram, the role of the two important processes involved in the formation of a P-n junction.

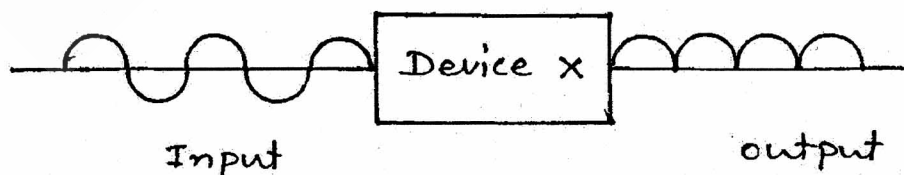
Draw a circuit arrangement for studying V-I characteristics of a P-n junction diode in

(a) Forward bias and (b) Reverse bias.

(Delhi 2010)

Q.3. Explain the formation of depletion layer and potential barrier in a P-n junction.

In the given figure the input waveform is converted into the output waveform by a device X. Name the device and draw its circuit.



(CBSE 2008)

Q.4. With the help of a labelled circuit diagram explain the use of a p-n junction diode as a full wave rectifier. Draw the input and output waveforms.

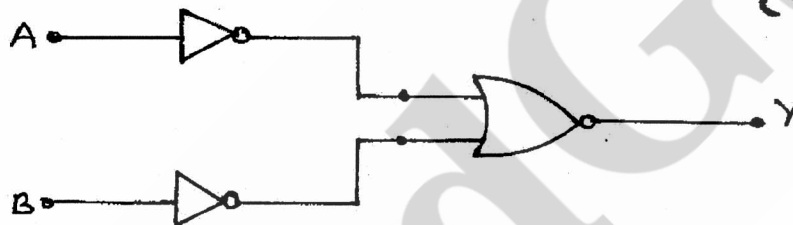
Draw V-I characteristics of a zener diode.

(Delhi 2007, 10)

Q.5. Explain briefly the principle on which a transistor amplifier works as an oscillator. Draw the necessary circuit diagram and explain its working.

Identify the equivalent gate for the given circuit and write its truth table.

(CBSE 2012)

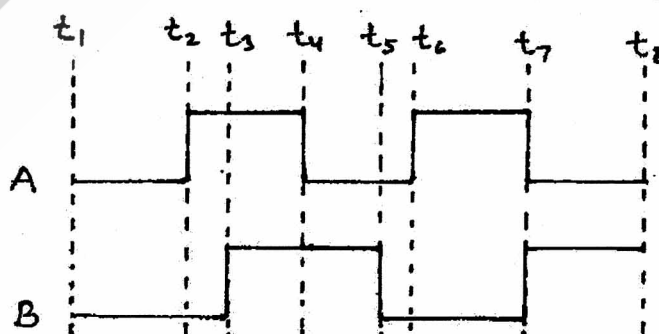


Q.6. Draw a simple circuit of a CE transistor amplifier. Explain its working. Show that the voltage gain, A_v of the amplifier is given by $A_v = -\frac{\beta_{ac} R_L}{R_i}$, where β_{ac} is the ac current gain R_L is the load resistance and R_i is the input resistance of the resistor. What is the significance of the negative sign in the expression for the voltage gain.

(Delhi 2012)

Q.7. Show the output waveforms (Y) for the given inputs A and B. of (a) OR gate (b) NAND gate.

(Delhi 2012)



Q.8. Draw a circuit diagram of an n-p-n transistor with its emitter base junction forward biased and base collector junction reverse biased. Describe briefly its working.

Draw a circuit diagram and explain the operation of a transistor as a switch.

(CBSE 2009)

Q.9. Draw the symbolic representation of a
(a) p-n-p (b) n-p-n transistor.

Why is the base region of transistor thin and lightly doped? With proper circuit diagram, show the biasing of a p-n-p transistor in common base configuration. and show that $I_E = I_C + I_B$.

(CBSE 2007)

Q.10. Draw a circuit diagram of a common-emitter amplifier. Deduce the expression for its voltage gain. Explain briefly how the output voltage is out of phase by 180° with the input voltage.

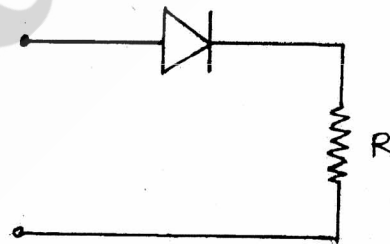
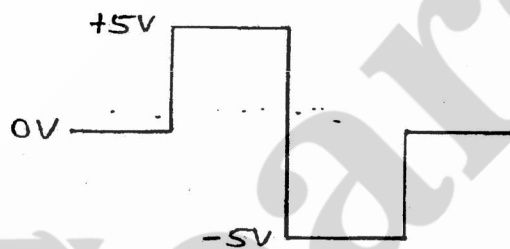
(Delhi 2006, 10)

Q.11. (a) The same signal is applied to both the input terminals of a given logic gate. If the output is the -

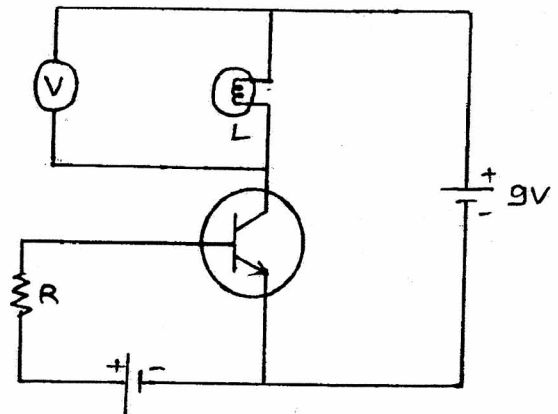
- (i) Same as the input signal
 - (ii) Inverted with respect to the input signal.
- Identify the logic gates involved in each case.

(CBSE 2008)

- Q.1. State with reason why a photodiode is usually operated at a reverse bias.
- Q.2. State the factor which controls (i) wavelength of light and (ii) Intensity of light emitted by a LED.
- Q.3. In a transistor, doping level in base is increased slightly. How will it affect (i) Collector current and (ii) base current?
- Q.4. Can the potential barrier across a p-n junction be measured by simply connecting a voltmeter across the junction?
- Q.5. What is an ideal diode? Draw the output waveform across R for the input waveform given below.

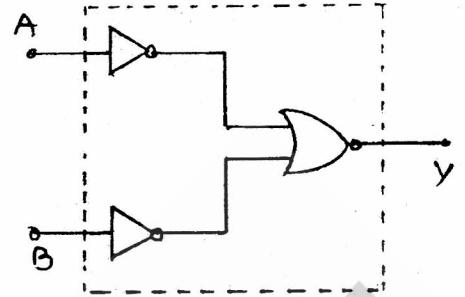


- Q.6. In the given circuit, a voltmeter 'V' is connected across a lamp L. How would (i) the brightness of the lamp and (ii) Voltmeter reading 'V' be affected, if the value of resistance 'R' is decreased?



Ans. (i) Increases (ii) Increases

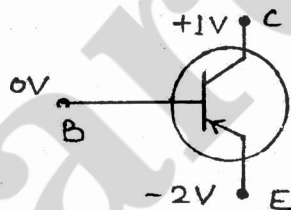
Q.7. Identify the gate equivalent to the 'dotted box' shown here and give the symbol and truth table.



Q.8. Three photo diodes D_1 , D_2 and D_3 are made of semiconductors having band gaps of 2.5 eV, 2 eV and 3 eV, respectively. Which one will be able to detect light of wavelength $6000 \text{ \AA}$?

Q.9. Explain how the heavy doping of both p-and n- sides of a p-n junction diode results in the electric field of the junction being extremely high even with a reverse bias voltage of a few volts.

Q.10 In the given figure is (i) the emitter and (ii) the collector forward or reverse biased?



Q.11. A semiconductor has equal electron and hole concentration of $2 \times 10^8 / \text{m}^3$. On doping with a certain impurity, the hole concentration increases to $4 \times 10^{10} / \text{m}^3$.

- What type of semiconductor is obtained on doping?
- Calculate the new electron and hole concentration of the semiconductor.
- How does the energy gap vary with doping.

Ans. (i) p-type (ii) $n_e = 10^6 / \text{m}^3$, $n_h = 4 \times 10^{10} / \text{m}^3$
 (iii) Energy gap decreases on doping.

PHYSICS

Unit- 15. Communication System

Class - 12th

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15. Communication System

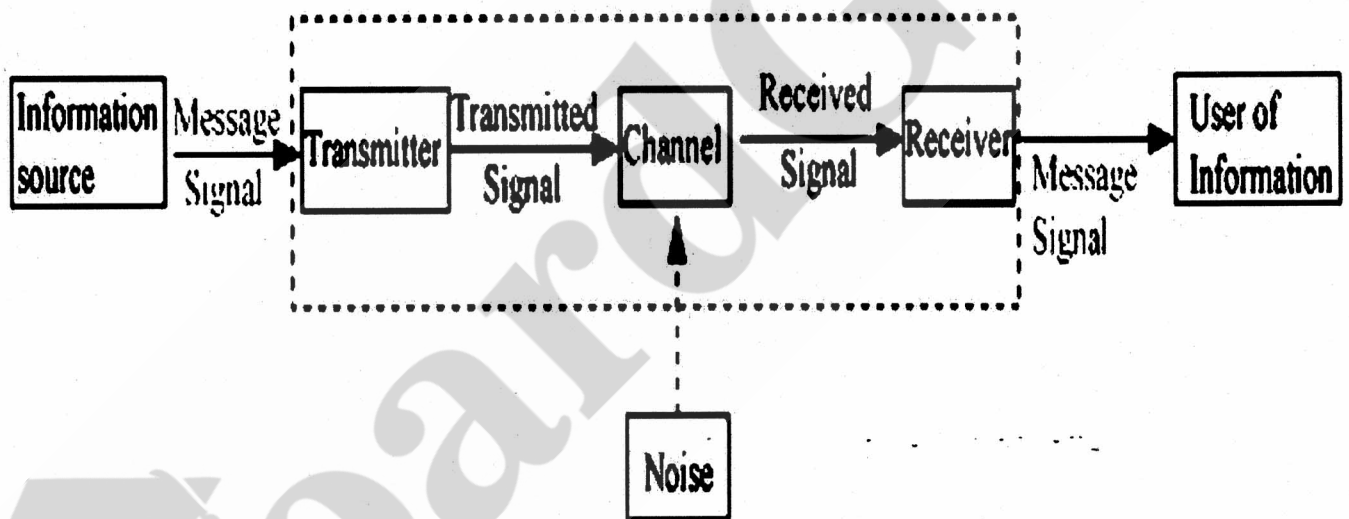
The act of transmission & reception of information is called communication.

Elements of Communication System -

The setup used in the transmission and reception of information from one place to another is known as Communication System.

Every Communication system has the following three essential elements :

(1.) Transmitter (2.) Communication channel (3.) Receiver



The information source provides the message signal to be transmitted. Usually the message signal is a non electrical signal like a voice signal. A transducer converts the message signal into electrical form which is suitable for transmission through the channel.

These message signals are superimposed on a high frequency wave, which acts as the carrier wave. This process is known as modulation. The resultant wave is called modulated wave, which is radiated with the help of transmitting antenna.

The communication channel carries the modulated wave from the transmitter to the receiver. In case of telephony and telegraphy, communication channel is the transmission lines, which connect the transmitter and the receiver. In radio communication or wireless communication the free space through which the modulated signal travels serves as the communication channel.

As the transmitted signal propagates through the communication channel, it gets distorted due to noise.

The receiver therefore receives a corrupted version of the transmitted signal. At the receiver end, a demodulator separates the low frequency audio signal from the modulated signal.

The low frequency signal so separated is then amplified and fed to a transducer. The transducer converts the low frequency signal (in the form of electrical pulses) into audio visual waves.

There are two basic modes of communication -

(i) Point to Point (ii) Broadcast.

(i) Point to Point -

In this mode message is transmitted over a link between a single transmitter and a receiver.

Example - Telephony, Telegraphy etc.

(ii) Broadcast -

In this mode there are large number of receivers corresponding to a single transmitter.

Example - Radio & T.V. etc.

Basic Terminology used in Communication system-

(1.) Signal -

It is a single valued function of time that carries the information. It is usually in electrical form and is suitable for transmission.

The electrical signals are of two types :

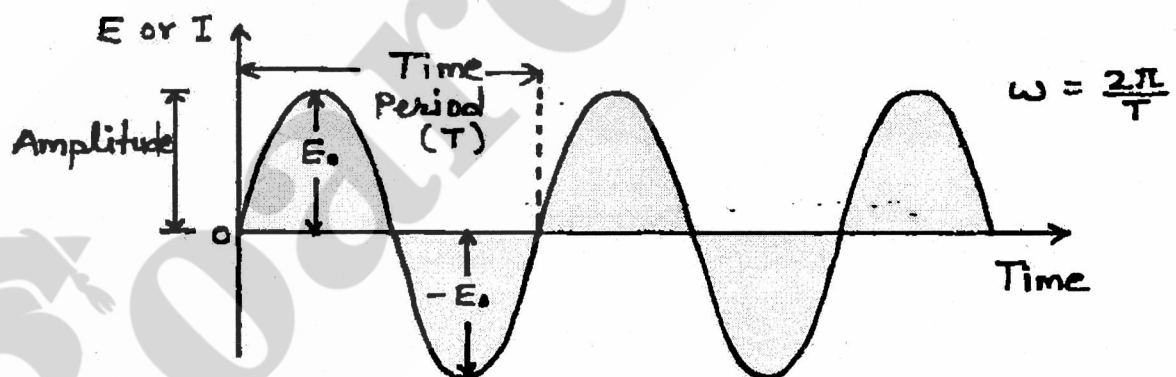
(a) Analog Signal -

An analog signal is that in which current or voltage value varies continuously with time.

It is represented by the equation -

$$E = E_0 \sin(\omega t + \phi)$$

Where E_0 is the maximum value of voltage called amplitude, T is time period and $\omega = \frac{2\pi}{T}$ is angular frequency of the signal and ϕ represents the phase angle.



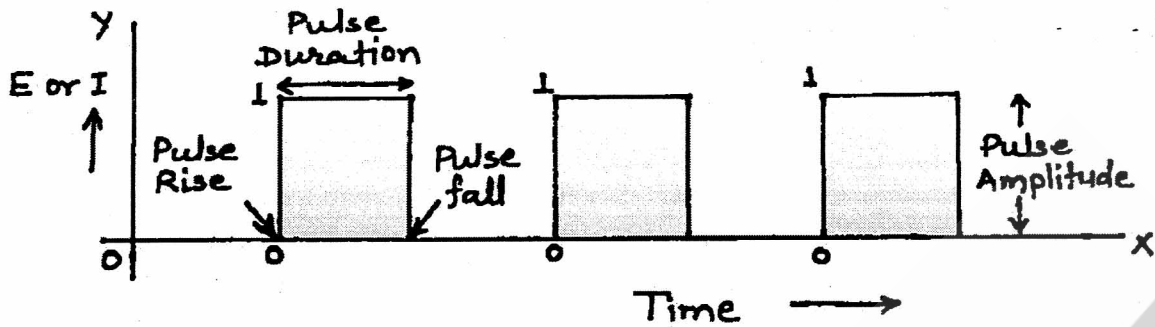
(b) Digital signal -

A digital signal is a discontinuous function of time.

Such a signal is usually in the form of pulses.

Each pulse has two levels of voltage / current represented by 0 or 1. Zero (0) of a digital signal refers to open circuit and 1 of digital signal refers to closed circuit.

Both 0 and 1 are called bits.



(2.) Transducer -

Any device / arrangement that converts one form of energy into another is called a transducer.

The device like microphone, piezoelectric sensors and photodetectors, which convert a physical quantity into electrical signals are transducers.

On the contrary, a Loud speaker which convert electrical signal into sound waves is also a transducer.

(3.) Noise -

The disturbance or distortion in the transmission and processing of message signals in a communication system is called noise.

(4.) Transmitter -

It is an arrangement that converts the message signal to a form suitable for transmission and then transmits it through some suitable communication channel.

(5.) Receiver -

A receiver is an arrangement that picks up the transmitted signal at the channel output and processes it to produce the message signal in the suitable form.

(6.) Attenuation -

The loss of strength of a signal during its propagation through the communication channel is called attenuation.

(7.) Amplification -

It is the process of increasing the strength of the transmitted signal using some suitable electronic circuit.

Amplification can be done anywhere between the transmitter and receiver when signal strength becomes weaker than the required strength. The energy required for additional signal strength is obtained from a d.c. power source.

(8.) Bandwidth -

The range of frequencies over which the communication system works is called bandwidth.

(9.) Modulation -

Modulation is the phenomenon of superimposing the low frequency message signal (called the modulating signal) on a high frequency wave (called the carrier wave).

The resulting wave is called the modulated wave, which is transmitted.

(10.) Demodulation -

It is the reverse process of modulation. Demodulation is the phenomena of retrieval of information from the modulated wave at the receiver.

(11.) Repeater -

A repeater is a combination of receiver, amplifier and transmitter. Its function is to extend the range of communication.

Each repeater receives the transmitted signal, amplifies it properly keeping its original form and then transmits it to the next repeater.

Modulation and its Necessity -

Suppose we want to transmit an electrical signal in the audio frequency range (20 Hz to 20 KHz) over a long distance.

We cannot do it, because of the given reasons:

(1) Size of antenna -

For transmission and reception each antenna should have a size comparable to the wavelength of the signal (atleast $\frac{1}{4}$ in size), so that time variation of the signal is properly sensed by the antenna.

For an audio frequency signal of frequency 15 KHz the wavelength

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{15 \times 10^3} = 20000 \text{ m.}$$

The length of the antenna = $\frac{\lambda}{4} = \frac{20000}{4} = 5000 \text{ m}$

To set up an antenna of vertical height 5000 m is practically impossible to construct and operate.

If transmission frequency were raised to 1 MHz then

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^6} = 300 \text{ m.}$$

and the length of the antenna would be-

$$\Rightarrow \frac{300}{4} = 75 \text{ m. which is reasonable.}$$

(2) Effective Power Radiated by Antenna-

Theoretical studies shows that power P radiated from a linear antenna of length l is proportional to $(\frac{l}{\lambda})^2$.

$$P \propto \left(\frac{l}{\lambda}\right)^2$$

As high powers are needed for good transmission, therefore for given antenna length, wavelength λ should be small or frequency f should be high.

(3.) Mixing up of signals from different transmitter:

When many transmitters are transmitting baseband information signals simultaneously, they get mixed up and there is no way to distinguish between them.

The possible solution is, communication at high frequencies and allotting a band of frequencies to each transmitter so that there is no mixing.

Modulation -

It is the phenomenon of superimposing the low audio frequency information signals on a high frequency wave (carrier wave). The resultant wave is called the modulated wave, which is transmitted.

There are three types of modulation -

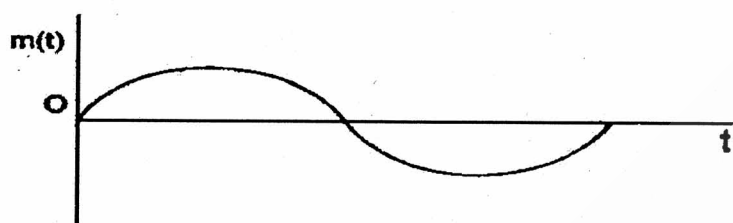
- (i) Amplitude modulation (AM)
- (ii) Frequency Modulation (FM)
- (iii) Phase Modulation (PM)

Again the significant characteristics of a Pulse are : pulse Amplitude, pulse duration or pulse width. Any one of these characteristics can be varied in accordance with the modulating baseband signal, giving rise to the respective Pulse Amplitude Modulation (PAM) and Pulse duration Modulation (PDM) or Pulse width modulation (PWM).

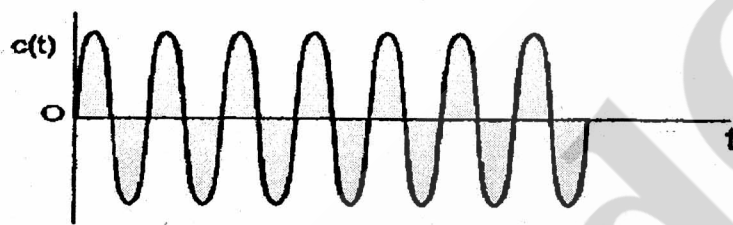
(1) Amplitude Modulation - (AM)

If the amplitude of the carrier wave is varied in accordance with the amplitude of the audio frequency modulating signal then it is called amplitude modulation.

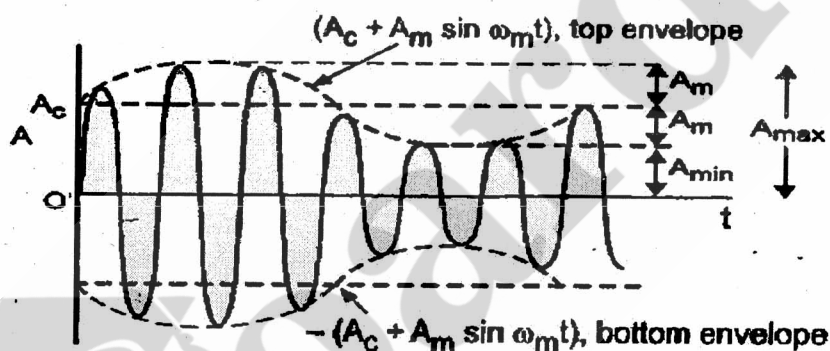
The frequency and phase of amplitude modulated wave remains the same as that of the carrier wave.



(a) MODULATING SIGNAL



(b) CARRIER WAVE



(c) AMPLITUDE MODULATED WAVE

To explain amplitude modulation, let us take a sinusoidal modulating signal, represented by -

$$m(t) = A_m \sin \omega_m t$$

Let the sinusoidal carrier wave be represented as -

$$c(t) = A_c \sin \omega_c t$$

In amplitude modulation the amplitude of the modulated wave $C_m(t)$ is $(A_c + A_m \sin \omega_m t)$. As its frequency is unchanged ($= \omega_c$), therefore we represent amplitude modulated wave as -

$$C_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$$C_m(t) = A_c \left(1 + \frac{A_m}{A_c} \sin \omega_m t \right) \sin \omega_c t$$

$$\text{or } C_m(t) = A_c (1 + \mu \sin \omega_m t) \sin \omega_c t$$

$$C_m(t) = A_c \sin \omega_c t + \mu A_c \sin \omega_m t \sin \omega_c t$$

Where μ is called amplitude modulation index.

* In practice, the value of μ is kept ≤ 1 to avoid distortion. Often, we represent μ in percent.

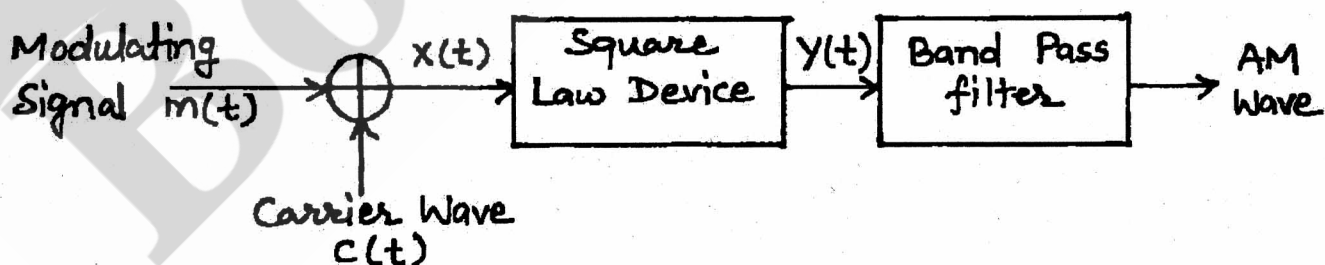
$$\text{Now } C_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

Equation shows that the amplitude modulated signal consist of the carrier wave of frequency ω_c with two sinusoidal waves, one of frequency $(\omega_c - \omega_m)$ and other of frequency $(\omega_c + \omega_m)$.

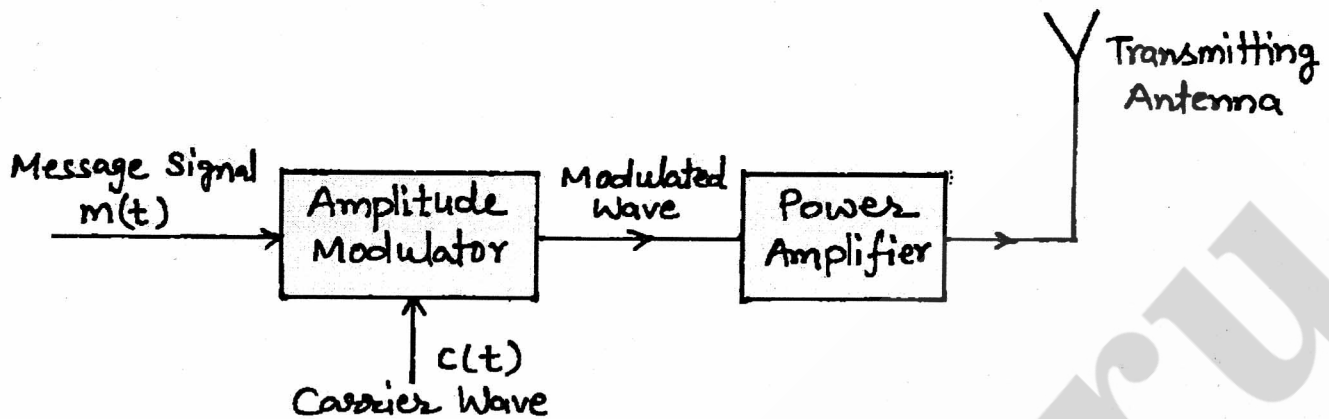
These two additional waves are called side bands. Their frequencies are called side band frequencies, which are $(f_c - f_m)$ and $(f_c + f_m)$.

Production of Amplitude Modulated Wave -

The simplest method for producing amplitude modulated wave is shown as a block diagram in the given fig.



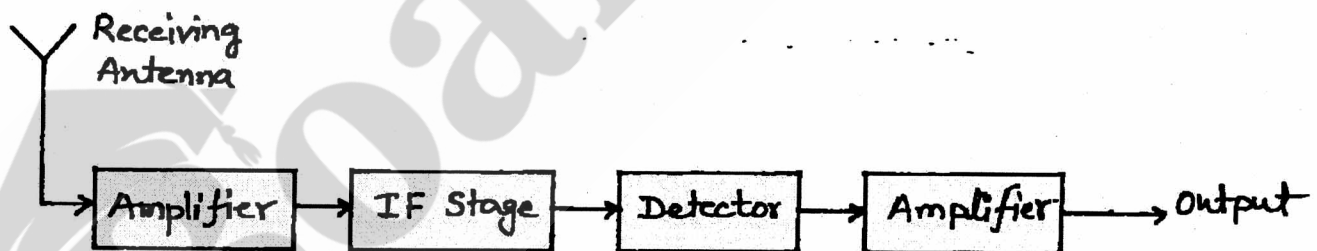
Block diagram of a transmitter -



The amplitude modulator is basically a common emitter amplifier for the carrier wave signal. The base biasing voltage in this case is the sum of d.c. and the modulating signal.

As the base biasing voltage changes, amplification changes. Therefore output voltage will be a carrier wave varying in amplitude in accordance with the biasing modulating voltage. This is the amplitude modulated wave.

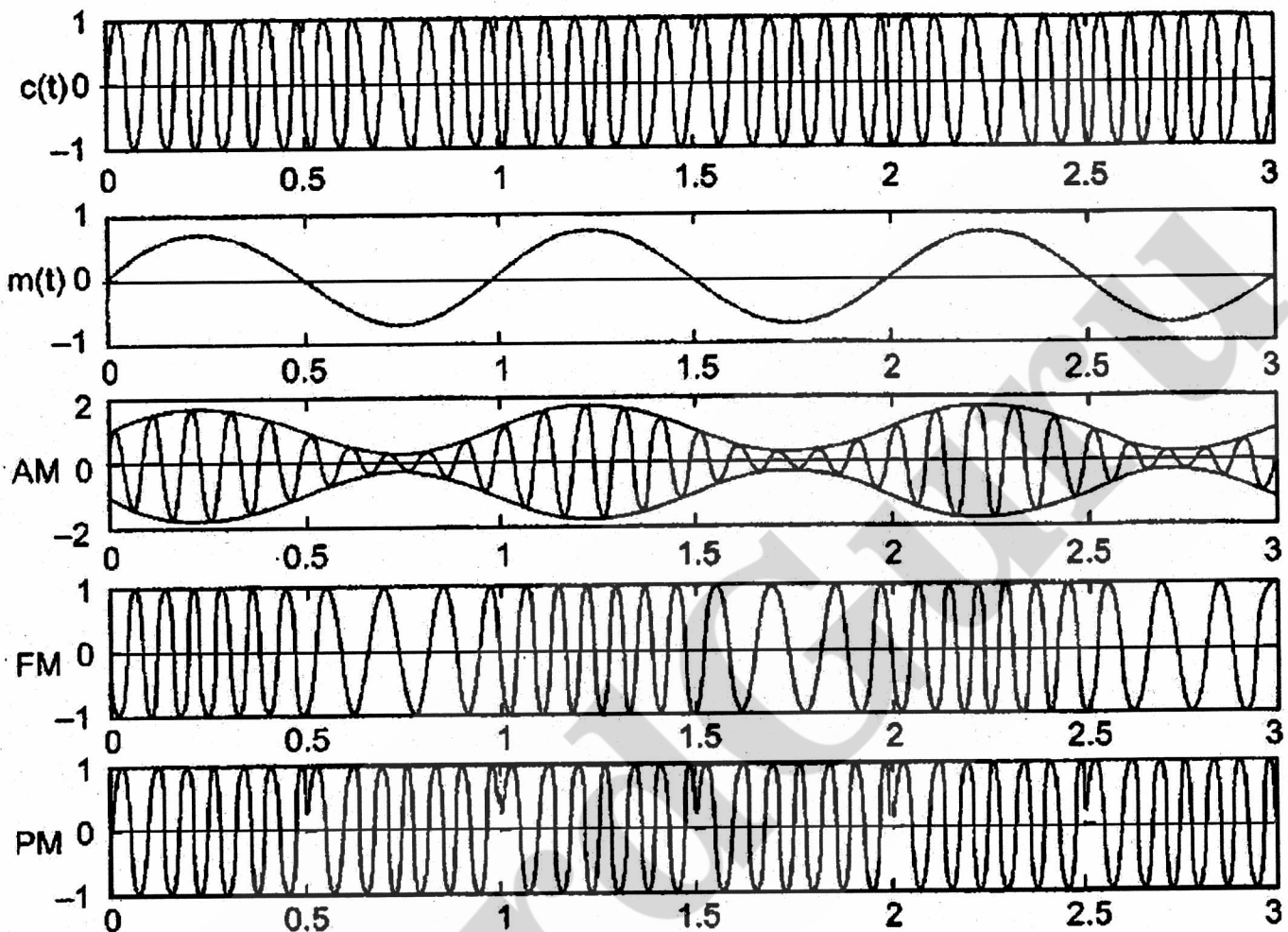
Block diagram of a Receiver -



The received signal is first amplified and given to the IF stage. IF stage represents intermediate frequency stage. The carrier frequency is usually changed to a lower frequency by the IF stage.

The low frequency signal is then given to the detector, which selects the desired signal and rejects the unwanted signals. The desired signal is then amplified and demodulated to recover the original modulating signal.

Comparison of AM, FM and PM



Q. Why is frequency modulation preferred over amplitude modulation for transmission of music?

Sol. In amplitude modulation the instantaneous voltage of carrier wave is varied by the instantaneous voltage of modulating waves. On transmission through a channel noise signal can also be added, resulting a change in the amplitude of modulated wave. Due to it, the receiver receives the modulating signal having a part of noise.

However, in frequency modulation, the frequency of carrier waves is changed as per instantaneous voltage of modulating waves. This can only be done at the mixing or modulating stage and not while signal is transmitted in channel. Hence noise does not effect FM signal. That is why FM signal is less susceptible to noise than an AM signal.

Q. What should be the length of dipole antenna for a carrier wave of frequency 5×10^8 Hz?

CBSE
2007

Sol. Length of dipole antenna $= \frac{\lambda}{4}$

$$\text{Here } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^8} = 0.6 \text{ m.}$$

$$\text{So that the length of dipole antenna} = \frac{0.6}{4} = 0.15 \text{ m.} = 15 \text{ cm.}$$

Q. A carrier wave of peak voltage 12 V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 75%?

CBSE
2010

Sol. % Modulation Index (μ) $= \frac{A_m}{A_c} \times 100$

where A_m and A_c are peak voltage of modulating signal and carrier wave voltage.

$$\text{Here } A_m = ? , A_c = 12 \text{ V} , \mu = 75 \%$$

$$\therefore 75 = \frac{A_m}{12} \times 100$$

$$A_m = \frac{12 \times 75}{100} = 9 \text{ V}$$

Q. Why the amplitude of a modulating signal is kept less than the amplitude of carrier wave.

Delhi
2011

Sol. It is so to avoid making overmodulated carrier wave. In this situation, the negative half cycle of the modulating signal is dipped and distortion occurs in reception.

Q. For an amplitude modulated wave, the maximum amplitude is found to be 10V while the minimum amplitude is 2V. Calculate the modulation Index.

RBSE
2011

Sol. Given that - maximum amplitude $A_{max} = 10V$
minimum amplitude $A_{min} = 2V$

$$\therefore \text{Modulation Index } (M) = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$$M = \frac{10 - 2}{10 + 2} = \frac{2}{3}$$

Q. A message signal of frequency 10 KHz and peak voltage of 10V is used to modulate at frequency of 1 MHz and peak voltage of 20V. Determine (a) Modulation Index (b) the side band produced.

CBSE
2003

Sol. Peak voltage of modulating signal $A_m = 10V$
Peak voltage of carrier signal $A_c = 20V$.

$$\therefore \text{Modulation Index } (M) = \frac{A_m}{A_c} = \frac{10}{20} = \frac{1}{2}$$

Q. Distinguish between frequency modulation and amplitude modulation.

CBSE
2006

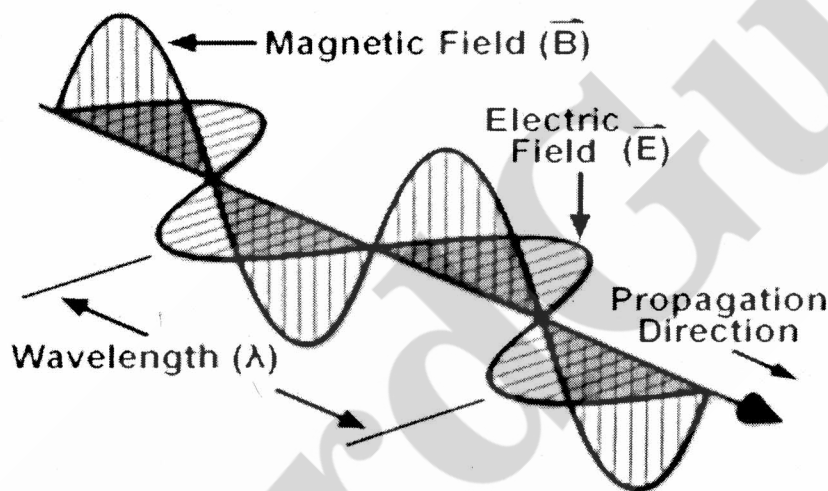
Sol.

Characteristic	FM	AM
Efficient Transmission	Highly Efficient	Most power goes waste
Bandwidth of Channel	It requires nearly 10 times to that of AM	Relatively smaller bandwidth
Fidelity	High fidelity due to presence of large number of side bands	Relatively low fidelity in reception
Area of Receptor	Being LOS, area of reception is much smaller	Relatively larger area of reception.

Electromagnetic Waves -

Electromagnetic waves are those waves in which there is a sinusoidal variation of electric and magnetic field vectors at right angles to each other as well as at right angles to the direction of propagation of waves.

In electromagnetic waves both the field vector ($\vec{E}$ and $\vec{B}$) vary with time and space and have the same frequency and same phase.



According to Maxwell the EM waves are of transverse in nature and they can pass through vacuum with the speed of light ($3 \times 10^8 \text{ m/s}$)

The velocity of EM waves in a medium is given by -

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

where μ and ϵ are permeability and permittivity of medium.

* The velocity of electromagnetic waves of different frequencies in vacuum is same but in any other medium, it is different. It is more for red light and less of violet light.

Behaviour of Atmosphere towards EM Waves-

The behaviour of atmosphere is different for electromagnetic waves of different frequencies.

The atmosphere is transparent to electromagnetic waves of visible region of wavelength range $4000 \text{ \AA}$ to $8000 \text{ \AA}$, as we can see the sun and the stars through it clearly.

The electromagnetic waves belonging to infrared region of wavelength range $8 \times 10^{-7} \text{ m}$ to $3 \times 10^{-5} \text{ m}$ are not allowed to pass through atmosphere rather they get reflected by atmosphere.

The ozone layer of earth's atmosphere blocks the EM waves of ultraviolet region of wavelength range $60 \text{ \AA}$ to $4000 \text{ \AA}$.

The lower part of atmosphere is more or less transparent to EM waves of wavelength 20 m and higher, used in radiocommunication.

But the top most layer, the ionosphere does not allow these waves to penetrate and reflects them back towards earth. Beyond certain frequency, above 40 MHz , the ionosphere bends any incident EM waves a little but does not reflect it back towards earth.

Radio Waves-

The EM waves of frequency ranging from 500 KHz to about 1000 MHz are called radio waves.

These waves are used in the field of radio communication.

Space Communication -

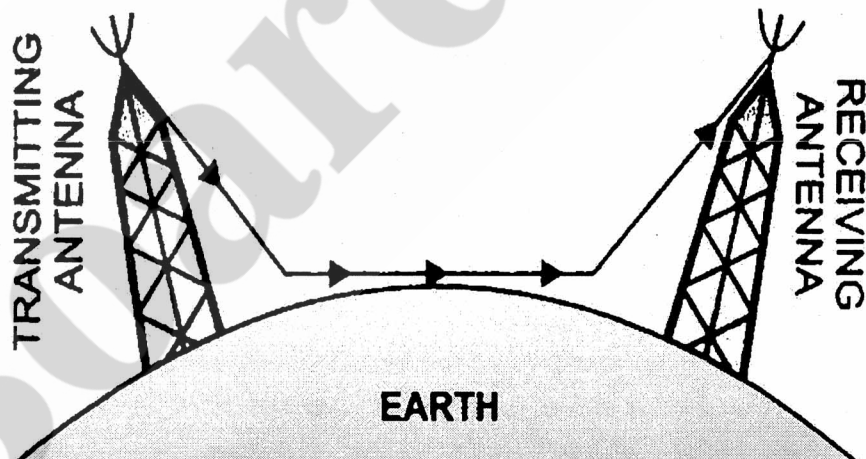
The process of sending, receiving and processing of information through space is known as space communication.

We have the following modes of space communication -

- (1.) Ground or surface wave propagation
- (2.) Space wave or tropospheric wave propagation
- (3.) Sky or Ionospheric wave propagation
- (4.) Satellite Communication

(1) Ground or Surface Wave Propagation -

It is a mode of wave propagation in which the ground has a strong influence on the propagation of signal waves from the transmitting antenna to receiving antenna.



While progressing along the surface of the earth, the ground wave induces current in the ground and bends round the corner of the objects on earth. Due to it the energy of ground wave is gradually absorbed by the earth and the power of ground wave decreases with the increase in distance from the transmitting station.

The attenuation of ground/surface wave increases very rapidly with the increase in its frequency.

For ground wave propagation, the maximum coverage range depends on -

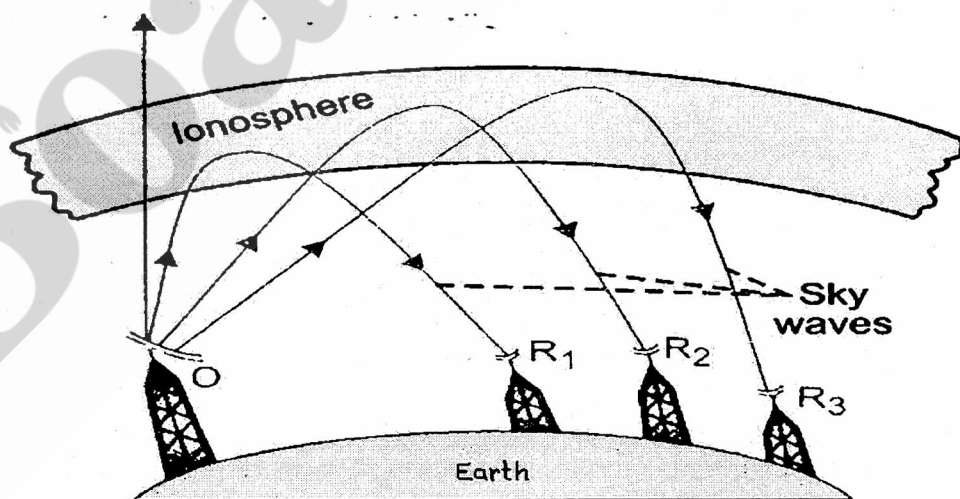
(i) The transmitting power (ii) Frequency of signal wave

The ground wave propagation is useful for low frequency signal waves (from 530 KHz to 1700 KHz). It is not suited for high frequency signal wave and for very long range communication.

* The ground wave propagation is generally used for local broadcasting as a medium wave broadcast service. and is not used for high frequency band.

(2) Sky Wave Propagation -

It is a mode of wave propagation in which the radiowaves emitted from the transmitter antenna reach the receiving antenna after reflection by the ionosphere.



In sky wave propagation, the radiowaves of frequency range from 1700 KHz to 40 MHz are used. This mode of propagation is used by short waves broadcast service.

The density of atmosphere decreases with height. At great heights, the solar radiations intensity is high but the density of earth's atmosphere is low. Therefore there are few air molecules to be ionised.

On the other hand, close to the earth, the density of earth's atmosphere is high but the radiation intensity is low. Due to it the ionisation is low.

However at some intermediate heights, there occurs maximum ionisation of earth's atmospheric molecules, resulting in ionised layers of maximum electrons and ions which are formed at different heights. Each ionised ionospheric layer act as a reflector for a range of radiowaves between 1700 KHz to 40 MHz.

The electromagnetic waves of frequencies greater than 40 MHz penetrate the ionosphere and escape.

The sky wave propagation is also known as ionospheric propagation, since the sky waves reach the receiver after reflection from the ionosphere.

In a single reflection from the ionosphere, the radio waves cover a distance about 4000 Km. That is why a very long distance round the globe communication is possible, with the help of sky wave propagation.

Fading -

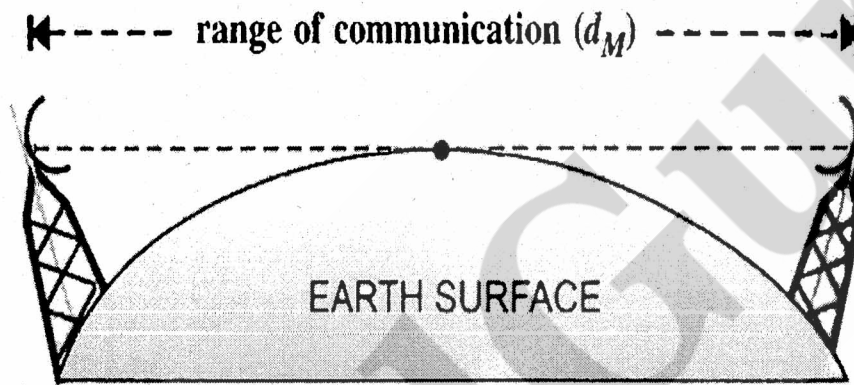
It is the variation in the strength of a signal at a receiver due to interference of waves. Fading is more at high frequencies. Fading causes an error in data transmission and retrieval.

The signal received due to sky wave propagation are subjected to fading, in which the strength of the signal varies with time. It is so because, at the receiver, a large number of waves reach, following different number of paths.

space Wave Propagation -

It is that mode of wave propagation in which the radiowaves emitted from the transmitter antenna reach the receiving antenna through space. These radio waves are called space waves.

The space waves are the radiowaves of frequency range from 54 MHz to 4.2 GHz.



The space waves travel in straight line from transmitting antenna to receiving antenna. Therefore the space waves are used for the line of sight communication as well as for the satellite communication.

The line of sight distance is that distance between transmitting antenna and receiving antenna at which they can see each other.

The range of space wave communication can be increased by increasing the heights of transmitting antenna and receiving antenna.

Relation between Coverage distance & Height of Transmitting antenna-

Suppose PQ is a TV transmitting antenna of height h located at P on the earth surface. Due to finite curvature of earth the TV signal transmitted from Q cannot be received beyond the tangent point T and S on earth.

Let $SP = TP = d$ (distance to the horizon). Therefore the TV signals will be received on earth in a circle of radius d .

Here radius of earth = $R = OS = OT = OP$

$PQ = h$ and $OQ = OP + PQ = R + h$

In right angle triangle QPS

$$SQ^2 = SP^2 + PQ^2 = d^2 + h^2$$

In right angle triangle OSQ

$$OQ^2 = OS^2 + SQ^2$$

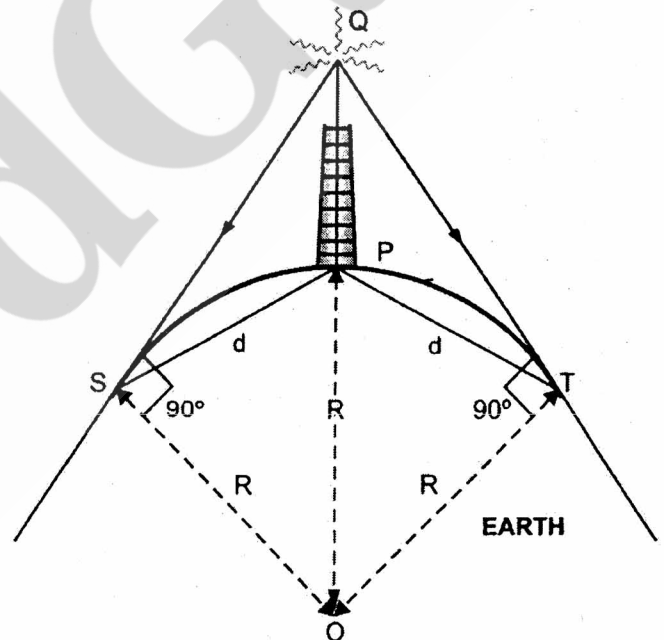
$$\text{or } (R+h)^2 = R^2 + (d^2 + h^2)$$

$$\text{or } R^2 + 2Rh + h^2 = R^2 + d^2 + h^2$$

$$\text{or } 2hR = d^2$$

or

$$d = \sqrt{2hR}$$



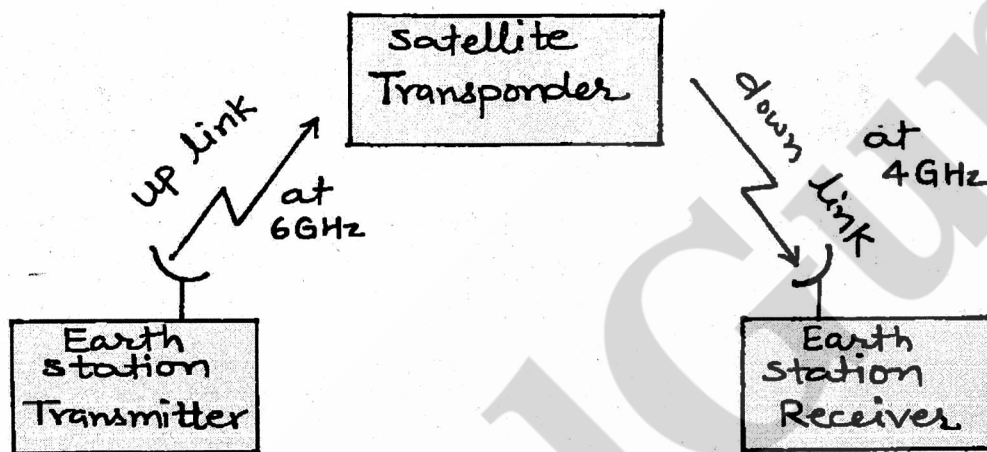
The maximum line of sight distance (d_m) between two transmitting antenna of height h_T and receiving antenna of height h_R above the earth is given by-

$$d_m = \sqrt{2h_T R} + \sqrt{2h_R R}$$

Satellite Communication -

It is a mode of communication of signal between transmitter and receiver through satellite.

A communication satellite is a space craft placed in an orbit around the earth provided with microwave transmitting and receiving equipment called a radio transponder.



In satellite communication a beam of modulated microwave from earth station transmitter is sent directly towards the satellite. The satellite receives the coming signal, amplifies and return it to the earth station receivers at a different frequency to avoid interference between the transmitted and received signal.

The frequency used in satellite communication for uplink is 6GHz and for downlink is 4GHz.

The signal received by receiver is generally weak. It is amplified by the receiver and then it is televised. Thus a communication satellite acts as a big microwave repeater in the sky.

The line of sight microwave communication through satellite is possible if the communication satellite is always at a fixed location with respect to the earth. This type of satellite is known as geo-stationary satellite.

Remote Sensing -

It is a technique of obtaining information about an object/area from a distance without being in physical contact with it. Any photography is a kind of remote sensing.

Remote sensing is widely used in forestry, in land use mapping, in agriculture, underground nuclear explosion, in climatology, in ground water survey, in pollution control, in tourism industry and in monitoring draught conditions.

Q. Why is it not possible to use sky wave propagation for transmission of TV signal.
CBSE 2009

Sol. The frequency of waves used for transmission of TV signals are of range 100 MHz - 220 MHz. But ionosphere may be able to reflect waves back on earth of frequency upto 40 MHz. Therefore, ionosphere is unable to reflect TV waves (space waves) back on earth.

Q. A transmitting antenna at the top of a tower has a height of 36 m and the height of the receiving antenna is 49 m. What is the maximum distance between them, for satisfactory communication in line of sight mode? (Radius of earth = 6400 km.)
CBSE 2006

Sol. Given that - $h_T = 36 \text{ m}$, $h_R = 49 \text{ m}$.

$\therefore$ Maximum line of sight distance $d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$d_m = \sqrt{2 \times 6.4 \times 10^6 \times 36} + \sqrt{2 \times 6.4 \times 10^6 \times 49}$$

$$d_m = 3.58 \times 6 \times 10^3 + 3.58 \times 7 \times 10^3$$

$$d_m = 46.54 \text{ km.}$$

Very Important Questions

1 Mark Questions -

- Q.1. What is the function of a repeater in a communication system ?
(CBSE 2010, 09, 11)
- Q.2. What is the function of transmitter in a communication system ?
(CBSE 2008, 11)
- Q.3. What is the sky wave propagation ?
(Delhi 2007, 09)
- Q.4. What is the ground wave propagation ?
(Delhi 2005, 09)
- Q.5. What is space wave propagation ?
(CBSE 2006, 09)
- Q.6. What does the term transducer mean in an electronic communication system ?
(Delhi 2004, 06)
- Q.7. Distinguish between sinusoidal and pulse shaped signals.
(CBSE 2007, 10)
- Q.8. What are the three basic units of a communication system ?
(Delhi 2002, 08)
- Q.9. What is the meaning of the term attenuation in communication system ?
(CBSE 2005, 08)
- Q.10. Name the mode of propagation of radio waves which travel in a straight line from the transmitting antenna to the receiving antenna.
(CBSE 2006, 11)
- Q.11. Why is frequency modulation preferred over amplitude modulation for transmission of music ?
(Delhi 2007)

Q.12. What is the purpose of modulating a signal in transmission ? (Delhi 2007, 10)

Q.13. Name the type of communication in which the signal is a discrete and binary coded version of the message or information. (CBSE 2006, 03)

Q.14. What is the length of a dipole antenna to transmit signals of frequency 200 MHz ?
Answer - 0.75 m. (Delhi 2006, 03)

Q.15. Identify the part X and Y in the following block diagram of a communication system. (Delhi 2008)



2 Marks Questions -

Q.1. What is sky wave communication? Why is this mode of propagation restricted to the frequencies only upto few MHz ? (CBSE 2011)

Q.2. Draw a block diagram showing the important component in a communication system. What is the function of a transducer? (Delhi 2007, 11)

Q.3. Write two factors justifying the need of modulation of a signal. (CBSE 2006, 10)

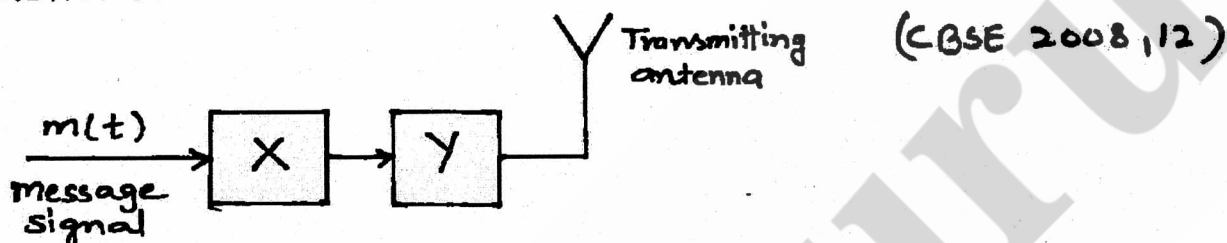
Q.4. What is the range of frequencies used in satellite communication? What is common between these waves and light waves ? (Delhi 2010)

Q.5. (a) What is line of sight communication ?
(b) Why is it not possible to use sky wave propagation for transmission of TV signal? (CBSE 2009)

Q.6. "A Communication satellite is essentially a repeater station in space". Justify this statement by analyzing the function of a repeater.

(CBSE 2006, 09)

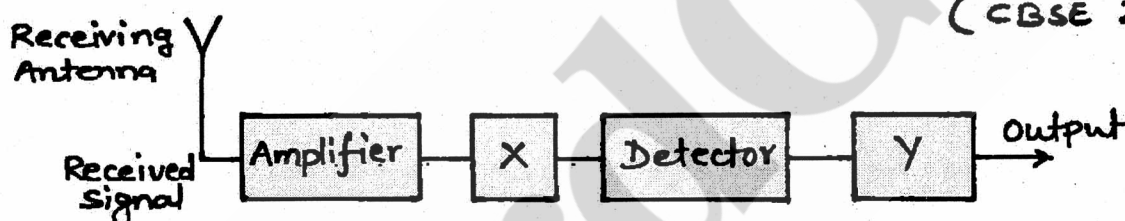
Q.7. Figure shows a block diagram of a transmitter identify the blocks X and Y and write their functions.



(CBSE 2008, 12)

Q.8. In the given block diagram of a receiver, identify the boxes X and Y and write their functions.

(CBSE 2009, 12)



Q.9. (a) Define modulation index.

(b) Why is the amplitude of modulating signal kept less than the amplitude of carrier wave?

(Delhi 2007, 11)

Q.10. Explain briefly the following terms used in Communication system:

(a) Transducer (b) Repeater (c) Amplification

(CBSE 2012)

Q.11. Draw a schematic diagram showing the -

(a) Ground wave

(b) Sky wave

(c) Space wave propagation modes for electromagnetic waves.

(CBSE 2011)

3 Marks Questions-

Q.1. Name the three different modes of propagation of electromagnetic waves. Explain, using a proper diagram, the mode of propagation used in the frequency range from a few MHz to 40 MHz.
(Delhi 2012)

Q.2. What is space wave propagation? Give two examples of communication system which use space wave mode.

A TV tower is 80 m tall. Calculate the maximum distance upto which the signal transmitted from the tower can be received.

Answer - $d = 32 \text{ km}$.

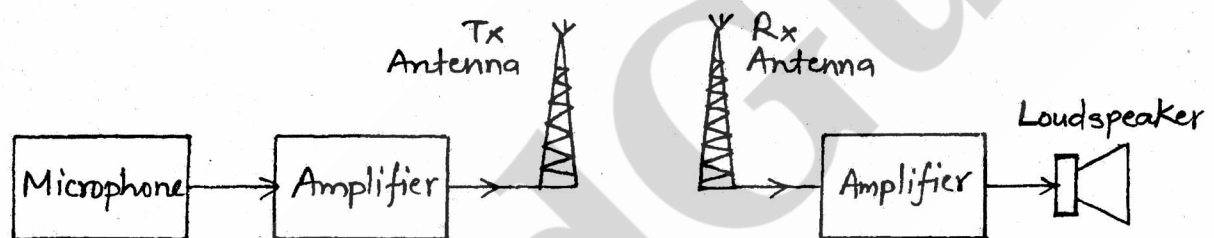
(Delhi 2010)

Q.3. Distinguish between sky wave and space wave propagation. Explain with the help of suitable diagram, how space wave are used for line of sight communication?
(CBSE 2005, 09)

Q.4. Draw a block diagram of a receiver for an amplitude modulated signal. Explain briefly the function of each of its components.
(Delhi 2006, 10)

Q.5. Draw the block diagram of a simple modulator for obtaining an amplitude modulated (AM) signal explaining briefly the function of each of its component.
(Delhi 2008)

- Q.1. Why audio signals converted into electromagnetic waves are not transmitted as such directly?
- Q.2. The amplitude of modulating signal is kept less than that of the carrier wave. Why?
- Q.3. Give two drawbacks from which the given arrangement suffers. Describe briefly with the help of a block diagram the alternative arrangement for the transmission and reception of message signal.



- Q.4. A carrier wave of peak voltage 12 V is used to transmit a message signal. What should be the peak voltage of the modulating signal, in order to have a modulation index of 75%.
- Ans. $A_m = 9\text{ V}$
- Q.5. By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21%?
- Ans. 10%.
- Q.6. What is sky wave communication? Why is this mode of propagation restricted to the frequencies only up to few MHz?
- Q.7. Why is an AM signal is more noisy than an FM signal upon transmission through a channel?